# Chaotic Friendship Round

Absolute Value

Spring 2023 Tournament

## Solutions

Team name: _____

Names of team members: _____

Put all answers in the following spaces. Guessing is not penalized. Work or answers written on other pages will not be graded. Submit only this page. Do not flip to the following pages until told to by your proctor.

**1**: _____ **6 (a)**: _____

**2**: _____ **6 (b)**: _____

**3**: _____ **6 (c)**: _____

**4**: _____ **6 (d)**: _____

**5 (a)**: _____ **7 (a)**: _____

**5 (b)**: _____ **7 (b)**: _____

**5 (c)**: _____ **7 (c)**: _____

**5 (d)**: _____ **7 (d)**: _____

For graders only:

| Question: | 1 | 2 | 3 | 4 | 5 | 6 | 7 | Total |
|---|---|---|---|---|---|---|---|---|
| Points: | 5 | 5 | 5 | 5 | 20 | 20 | 20 | 80 |
| Score: | | | | | | | | |

1. (5 points) The day after tomorrow is my birthday. The day after yesterday is Friday. What day is my birthday on?

2. (5 points) How many pairs of prime numbers $p$ and $q$ exist such that $p < q < 100$ and $q - p = 41$?

3. (5 points) Vanessa rides her bikes to school. Today, she left for school 4 minutes early. So, she has exactly the right amount of time to take the scenic route, which makes her trip take 25% longer. How many minutes does it normally take Vanessa to bike to school?

4. (5 points) The following proof can't be true!

   Step 1: Let $x$ be a solution to the equation $x^2 + x + 1 = 0$.
   Step 2: $x^2 = -x - 1$
   Step 3: Note that $x = 0$ isn't a solution to the equation in Step 1. Then $x \neq 0$.
   Step 4: Because $x \neq 0$, we can divide by $x$ and get: $x = -1 - \frac{1}{x}$.
   Step 5: Substituting this into the equation from Step 1, $x^2 + (-1 - \frac{1}{x}) + 1 = 0$.
   Step 6: $x^2 - \frac{1}{x} = 0$.
   Step 7: $x^3 = 1$.
   Step 8: $x = 1$.
   Step 9: Substituting this into the equation from Step 1, $1^2 + 1 + 1 = 0$.
   Step 10: $3 = 0$.

   Which step contains the error?

5. Two weighted coins lie on a table. One of the coins is head-weighted and lands on heads $\frac{3}{4}$ths of the time. The other coin lands is tail-weighted and on tails $\frac{3}{4}$ths of the time. You can't tell the two coins apart. You randomly take one of the two coins.

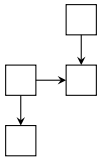
   (a) (5 points) You flip the coin once. What is the probability that it lands on heads? Express your answer as a fraction.

   (b) (5 points) You flip the coin a second time. What is the probability that it lands heads both times? Express your answer as a fraction.

(c) (5 points) The coin landed heads on the first coin flip. What is the probability that the coin you took is the head-weighted coin? Express your answer as a fraction.

(d) (5 points) The coin landed heads on the second coin flip, in addition to the first. What is the probability that the coin you took is the head-weighted coin? Express your answer as a fraction.

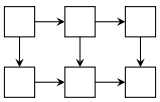
6. (a) (5 points) Consider the following grid.



How many ways are there to fill in the above grid with numbers 1 to 4 such that:

- Each number is used exactly once.
- If an arrow points from Box A to Box B, then Box B contains a larger number.

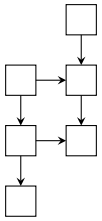
(b) (5 points) Consider the following grid.



Just as in part (a), how many ways are there to fill in the above grid with numbers 1 to 6 such that:

- Each number is used exactly once.
- If an arrow points from Box A to Box B, then Box B contains a larger number.

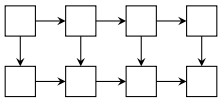
(c) (5 points) Consider the following grid.



Just as in part (a) and (b), how many ways are there to fill in the above grid with numbers 1 to 6 such that:

- Each number is used exactly once.
- If an arrow points from Box A to Box B, then Box B contains a larger number.

(d) (5 points) Consider the following grid.



Just as in part (a), (b), and (c), how many ways are there to fill in the above grid with numbers 1 to 8 such that:

- Each number is used exactly once.
- If an arrow points from Box A to Box B, then Box B contains a larger number.

7. There are 10 boxes in a line in front of you. They are labeled Box A, B, C, ... J. Each box contains a certain number of a chocolates, as given in the following table:

| Box | A | B | C | D | E | F | G | H | I | J |
|---|---|---|---|---|---|---|---|---|---|---|
| Number of Chocolates | 4 | 8 | 6 | 12 | 9 | 8 | 10 | 4 | 10 | 3 |

You are allowed to open and eat all the chocolates from any combination of boxes, so long as you don't take from two consecutive boxes. For example, if you open Box C, then you can't open Box B or D. If you open Box A, then you can't open Box B.

(a) (5 points) What is the maximum amount of chocolates you can take?

(b) (5 points) Now suppose you have to open exactly 5 of the 10 boxes. What is the maximum amount of chocolates you can take?

(c) (5 points) How many ways are there to open exactly 5 of the 10 boxes? You cannot open consecutive boxes.

(d) (5 points) How many ways are there to open a combination of the 10 boxes? Include the case where you open none of the boxes. You cannot open consecutive boxes.