Individual Round

Absolute Value

Spring 2023 Tournament

Solutions

Full name: ______________________________

Email: ______________________________

Put all answers in the following spaces. Guessing is not penalized. Work or answers written on other pages will not be graded. Submit only this page. Do not flip to the following pages until told to by your proctor.

1: ____________________ **6**: ____________________

2: ____________________ **7**: ____________________

3: ____________________ **8**: ____________________

4: ____________________ **9**: ____________________

5: ____________________ **10**: ____________________

Tie-breaker value: ______________________________

The following should only be filled out by graders:

Question:	1	2	3	4	5	6	7	8	9	10	Total
Points:	10	10	10	10	10	10	10	10	10	10	100
Score:											

Tie-Breaker

To break ties, please write the largest *permutable prime* you can in the "Tie-breaker value" section of your solution sheet. A permutable prime is a number such that every ordering of its digits is prime. For example, 13 is a permutable prime because both 13 and 31 are prime. 101 is not a permutable prime because 110 is not prime. It is recommended you spend time on this **after** solving all the math questions you can.

For specific details as to how the tie-breaker works:

1. The tie-breaker only determines ranking between those with the exact same score on the Individual round.
2. Those with a larger permutable prime will be placed higher.
3. Those whose tie-breaker value is not a permutable prime will be placed lower than all those whose tie-breaker value is a permutable prime.
4. Those who do not fill in the tie-breaker will be placed last.

Enjoy the problems!

1. (10 points) On average, each of the Absolute Value math competitions has 32 competitors. San Diego has x competitors, Yorba Linda has $3x$ competitors, and New Jersey has x^2 competitors. The average number of competitors at these three locations is the same as the national average. What is x?

2. (10 points) How many ways can we trace the word "Absolute"?

O	S	B	A	B	S	O
L	O	S	B	S	O	L
U	L	O	S	O	L	U
T	U	L	O	L	U	T
E	T	U	L	U	T	E

3. (10 points) A jar contains 9 red balls, 8 green balls, 13 blue balls, and 17 yellow balls. What is the minimum number of balls that you must draw, without replacement, to ensure that you have 10 balls of the same color?

4. (10 points) How many unique solutions does $(x-1)^2(x-1)+(x^2-x)(x-1)-(x-1)^2=0$ have?

5. (10 points) The earth completes a revolution around the sun roughly every 365.2422 days. To account for this, the Gregorian calendar uses leap years—years when we add February 29th to the calender. We generally add a day to the calendar every four years. However, the average-calendar year would then be $365 + \frac{1}{4}$ days, which isn't precise enough! So, years that occur at the end of the century that aren't divisible by 400 aren't leap years. While 2100, 2200, and 2300 aren't leap years, 2000 and 2400 both are leap years. This makes the average-calendar year $365 + \frac{1}{4} - \frac{3}{400} = 365.2425$ days, which is a better approximation. Determine the number of leap years between 2023 and 12023.

6. (10 points) Each of the following eight statements is either true or false:

   (a) All of the below statements are true.

   (b) All of the above statements are true.

   (c) No other answer choice is true.

   (d) All of the above statements are true.

   (e) Statement (c) is false.

   (f) Exactly two statements are true.

   (g) At least two statements are true.

   (h) Exactly three of the statements are true.

   Find the maximum number of statements that can be simultaneously true.

7. (10 points) Does there exist a triangle with sides of length 1, 2, and 0.5?

8. (10 points) We call n *triangular* if there exists a triangle with non-zero area and sides of length 1, 2, and n. There is exactly one triangular integer n. Determine the value of n.
The integers are the numbers: ...-2, -1, 0, 1, 2, ...

9. (10 points) We call n *pentular* if there exists a pentagon with non-zero area and sides of length 1, 2, 3, 4, and n. Find the number of pentular integers.

10. (10 points) In general, n is z-ular if there exists a z-sided polygon with non-zero area and sides of length 1, 2, 3, $\ldots$, $z-2$, $z-1$, and n. Find the number of 101-ular integers.