CS-174A Discussion 1C, Week 3

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@ Discussion 1C Github: https://github.com/NoctisZ/CS174A-1C-2020Fall (https://github.com/NoctisZ/CS174A-1C-2020Fall)

Outline

- Recap of Lecture Content
- Q&A about Assignment 2
- Assginment 3
- Team Project

Review of Lecture Content

Basic Transformations

- Translation
- Scaling
- Rotation
- Shearing

- Translation

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

- Scaling

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

- Rotation
  Rotate about $z-$axis:

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} \cos(\theta) & -\sin(\theta) & 0 & 0 \\ \sin(\theta) & \cos(\theta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

- E.g. Rotation when $\theta = \pi/4$,

$$R = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- Combination of transformation:
  Eg. Scale(2) first, then Translation(x = 1, y = 2, z = 3)

$$M_{translate} \cdot M_{scale} \cdot P_{original}$$

$$= \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 0 & 0 & 1 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 2 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} 2x+1 \\ 2y+2 \\ 2z+3 \\ 1 \end{pmatrix}$$

Shear

$$x' = x + \mathrm{Sh}_x^y y + \mathrm{Sh}_x^z z$$

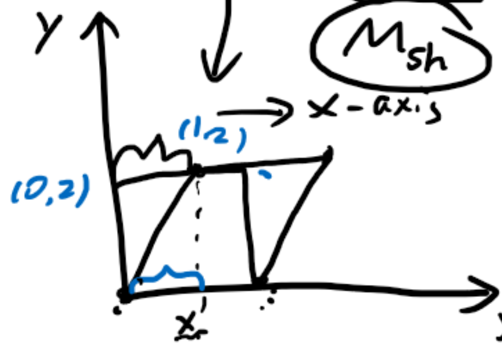
$$y' = \mathrm{Sh}_y^x x + y + \mathrm{Sh}_y^z z$$

$$z' = \mathrm{Sh}_z^x x + \mathrm{Sh}_z^y y + z$$

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & \mathrm{Sh}_x^y & \mathrm{Sh}_x^z & 0 \\ \mathrm{Sh}_y^x & 1 & \mathrm{Sh}_y^z & 0 \\ \mathrm{Sh}_z^x & \mathrm{Sh}_z^y & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & sh_x^y & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{M_{sh}} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\begin{cases} x' = x + sh_x^y \cdot y \\ y' = y \end{cases}$$



$$x' = x + \underbrace{sh_x^y}_{\frac{\Delta x}{y}} y$$

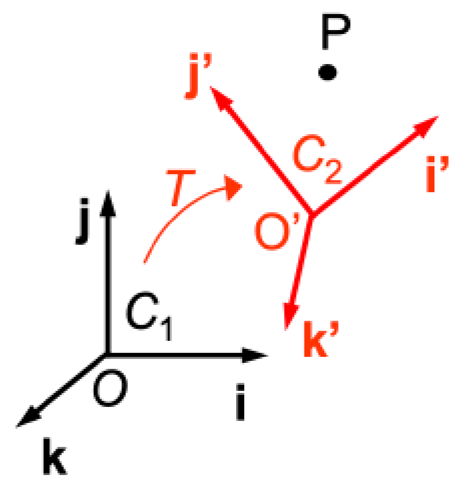
$$sh_x^y = \frac{1}{2}$$

Transformations are NOT commutative.

- Matrix multiplication is not commutative.
  - When performing multiple rotation transformations.
  - The order of transformation types matters.
  - i.e. $q = TRSTp$ is not the same as $q = TSRTp$

Change of Basis

- We want to find the coordinates of P in the new coordinate system C_2



$$\left\{\begin{aligned}
&C_1:\ P = x\overset{\begin{bmatrix}1\\0\\0\\0\end{bmatrix}}{i} + y\overset{\begin{bmatrix}0\\1\\0\\0\end{bmatrix}}{j} + z\overset{\begin{bmatrix}0\\0\\1\\0\end{bmatrix}}{k} + \overset{\begin{bmatrix}0\\0\\0\\1\end{bmatrix}}{O} = \underbrace{[i\ j\ k\ O]}_{\text{matrix}}\begin{bmatrix}x\\y\\z\\1\end{bmatrix}\\
&C_2:\ P = x'i' + y'j' + z'k' + O' = \underbrace{[i'\ j'\ k'\ O']}_{\text{matrix}}\begin{bmatrix}x'\\y'\\z'\\1\end{bmatrix}
\end{aligned}\right.$$

C_2 is transformed from C_1 using M:

$$\underline{M[i\ j\ k\ O]} = [i'\ j'\ k'\ O'] \quad \cdots (1)$$

$$\Rightarrow \cancel{[i\ j\ k\ O]}\begin{bmatrix}x\\y\\z\\1\end{bmatrix} = M\cancel{[i\ j\ k\ O]}\begin{bmatrix}x'\\y'\\z'\\1\end{bmatrix} \Rightarrow \underline{P_{c_1} = M\cdot P_{c_2}}$$

$$\underline{P_{c_2} = M^{-1}P_{c_1}}$$

From eq. (1): $M = [i' \;\; j' \;\; k' \;\; O']$

$$= \begin{bmatrix} i'_x & j'_x & k'_x & O'_x \\ i'_y & j'_y & k'_y & O'_y \\ i'_z & j'_z & k'_z & O'_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P_{C_2} = M^{-1} P_{C_1} \qquad M^{-1} = M^T$$

$$= M^T P_{C_1}$$

$$P_{C_2} = M_{CoB} P_{C_1}$$

$$= \begin{bmatrix} - & i' & - \\ - & j' & - \\ - & k' & - \\ - & O' & - \end{bmatrix} P_{C_1}$$

- $P_{C_2} = M^{-1} P_{C_1}$ where P_{C_2} is the representation of point P in new coordiante system C_2
- where

$$M_1 = TR = \begin{bmatrix} i'_x & j'_x & k'_x & O'_x \\ i'_y & j'_y & k'_y & O'_y \\ i'_z & j'_z & k'_z & O'_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

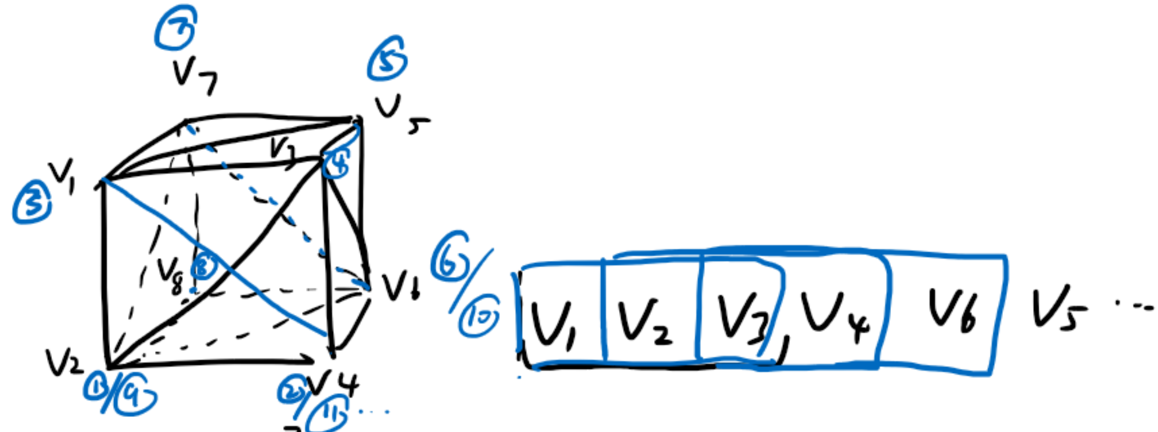
and

P's posotion in C_2, $[x', y', z', 1]^T$, can be calculated with the inverse of the transformations

$$P_{C_2} = M_1^{-1} P_{C_1} = R^{-1} T^{-1} P_{C_1} = \begin{bmatrix} i'_x & j'_x & k'_x & 0 \\ i'_y & j'_y & k'_y & 0 \\ i'_z & j'_z & k'_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^T \begin{bmatrix} 1 & 0 & 0 & -O'_x \\ 0 & 1 & 0 & -O'_y \\ 0 & 0 & 1 & -O'_z \\ 0 & 0 & 0 & 1 \end{bmatrix} P_{C_1}$$

Q&A about Assignment 2

- Due on Oct. 25 at 11:59 PM
- Carefully design your triangle strip and transformation for Extra Credit parts



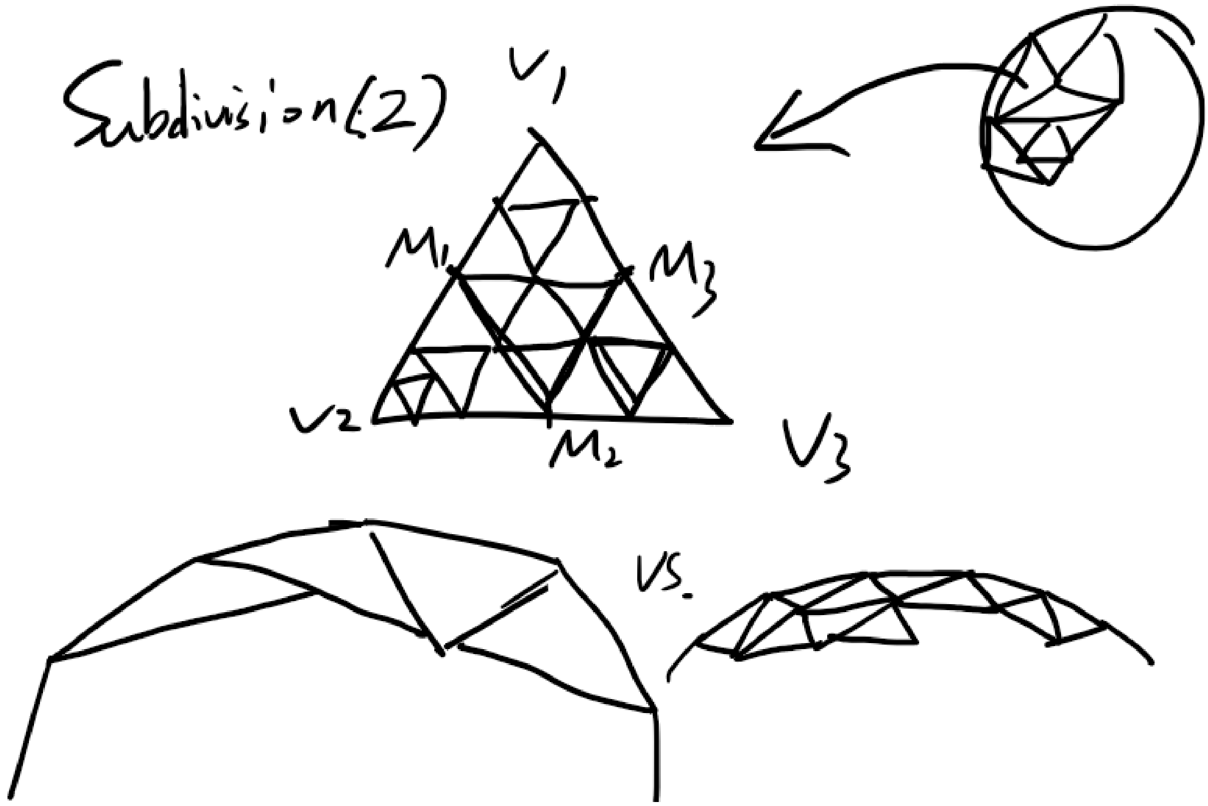
[V1 , V2 V3]

[V2 , V4, V3]

[V4 , V6, V5]

Assignment 3

- Create GitHub repo from: [https://classroom.github.com/a/sHEKI20M (https://classroom.github.com/a/sHEKI20M)](https://classroom.github.com/a/sHEKI20M)
- Some introduction to shading: [https://github.com/NoctisZ/CS174A-1C-2020Fall/blob/master/week3/Intro%20to%20Shader.pdf (https://github.com/NoctisZ/CS174A-1C-2020Fall/blob/master/week3/Intro%20to%20Shader.pdf)](https://github.com/NoctisZ/CS174A-1C-2020Fall/blob/master/week3/Intro%20to%20Shader.pdf)



Team Project

- Proposal due: 11/15/2020 11:59pm.
- Team size: 2-4 people
- Need to implement 1 advanced feauture (listed in specs) for groups with 2-3 people, 2 advanced features for group with 4 people
- Some ideas: https://piazza.com/class/kfirp13mgg86zk?cid=46 (https://piazza.com/class/kfirp13mgg86zk?cid=46)

Midterm Time

- On 11/5 7:00 - 8:30 PM PST
- Please have your camera ready