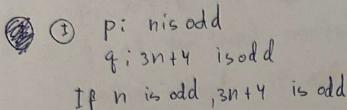
1. (15 points) Assuming that n is a positive integer, prove that n is odd if and only if 3n + 4 is odd.



For odd positive integer: 2ktl, so n = 2ktl 3 (akt) +4 = 6kt 3t4 = 6k+7 = 6k+6+1 = = 2(3k+3)+1 r=3k+3:. 2 (3k+3) +1= 2r+1, so by the definition of odd number, 3n +4 should be odd

1 If 3n+Y is odd, then nis odd.

Assume that 3n+4 is odd, and n is even.

- · 3n+4-4=3h 3h is also odd because odd-even should give odd number.
  - · 3n-n= 2n an should be odd by the definition we used above, but it is, in fact, wen. Because number multiplied by 2 gives even number which contradicts out point.

50, by proof of contradiction, If 3n+4 is odd, then n is odd.

Conclusion: Both directions are proven, so p => q.

2. (16 points) Prove that if x, y, and z are integers and x + y + z is odd, then at least one of x, y, and z is odd.

By proof by controposition,: P->q is true

- 3. (10 points) Prove that you can start with 10,000, subtract a power of 2, then divide by a power of 2, then divide by a power of 3, and end with 13. You can use any means necessary. What type of proof did you use?
  - . Start with 10000

Constructive proof is used

4. (14 points; 2 points per line) The following is a proof that if  $A \subseteq B$  and  $B \subseteq A$ , then A = B. For each line in the proof, fill in the name of the definition being used, the name of the set identity (page 136), the name of the logical equivalence (page 29), or the rule of inference (pages 76 and 80) used. If there is no specific name in the book, then cite the page number (and table number if applicable) that shows the equivalence or rule.

Step

Identity, rule, law, or definition used

1. ∀x[x∈A→x∈B] Definition of subset A ⊆ B

2. ∀x[x∈B→x∈A] Definition of subset B ⊆ A

3. cEA→cEB Universal Instantiation (

4. CEB→CEA Universal Instantiation (2)

5. (c∈A → c∈B) ∧ (c∈B → c∈A) Logical conjunction

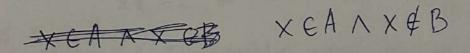
6. cEA + cEB Definition of Riconditional (5)

7. ∀x[x ∈ A ↔ x ∈ B] <u>Univer Sal</u> Generalization 6

- 5. Express the following statements using x to represent a set element, and the symbols ∈ and ∉ to represent membership. For example, to specify the union of sets A and B, you would use x ∈ A ∨ x ∈ B. You may need to use quantifiers. For example, to express that set S is the empty set, you could state ¬∃x[x∈S], or ∀x[x∉S]. (Hint: One or more of these requires a quantifier, but not all of these require a quantifier; one or more of these requires implication, but not all require implication).
  - a. (5 points)  $A \subseteq B$

Vx(xEA -> xEB)

b. (5 points)  $A \cap \overline{B}$ 



c. (5 points)  $A \cap \overline{B} = \emptyset$ 

-Jx(xEA ∧ X &B)

Answer: Neither injective nor surjective c. (5 points) f(x) = x3

Imjective?  $f(a) = f(b) \implies a = b$   $for f(x) = x^3 \qquad a^3 = b^3$  a = b

It is injective.

Surjective?  $f(x) = x^3 = 7$  x = 357Since  $f(x) = x^3 \text{ is not surjective}$ Answer: It is injective but not surjective.

- 7. A relation R is defined on {1, 10, 100} by: R = { (1,1), (1,10), (10, 1), (10,10), (10,100), (100, 10)}
  - a. (5 points) Prove that R is not reflexive.

For R to be reflexive, we should have: (1.1), (10,10), (100,100) [1,1] and (10,10) are in R, but (100,100) is not in R. So R is not reflexive

b. (5 points) Prove that R is symmetric.

So we ha to there is (x,y), (y,x) should also

for (1,1) and (10,10), (1,1) and (10,10) exist respectively.

We have (1,10), (10,1) also exists.

We have (10,100), (100,10) also exists

For (10,1) and (100,10), (1,10) and (10,100) exist

respectively

So R is symmetric

c. (5 points) Prove that R is not transitive.

(1,10) and (10,100) in R imply that (1.100) but (1,100) is not in R. This means R is not transitive.