Questions:

1. (15 points) Assuming that n is a positive integer, prove that n is odd if and only if 3n + 4 is odd.

① p: n is odd
q: 3n+4 is odd

If n is odd, 3n+4 is odd

for odd positive integer: $2k+1$, so $n = 2k+1$

$3(2k+1)+4 = 6k+3+4 = 6k+7 = 6k+6+1 =$

$= 2(3k+3)+1$ $r = 3k+3$

$\therefore 2(3k+3)+1 = 2r+1$, so by ~~dfi~~ definition of odd number, 3n+4 should be odd

② If 3n+4 is odd, then n is odd.

Assume that 3n+4 is odd, and n is even.

- $3n+4-4 = 3n$ 3n is also odd because odd - even should give odd number.
- $3n-n = 2n$ 2n should be odd by the definition we used above, but it is, in fact, even. Because number multiplied by 2 gives even number which contradicts out point.

So, by proof of contradiction, If 3n+4 is odd, then n is odd.

Conclusion: Both directions are proven, so $p \leftrightarrow q$.

2. (16 points) Prove that if x, y, and z are integers and x + y + z is odd, then at least one of x, y, and z is odd.

p: $x+y+z$ is odd $\quad p \to q$

q: at ~~bas~~ least one is odd

Assume that if all x, y, z are even, then $x+y+z$ is even. In this case: $\neg q \to \neg p$

By definition of even number, $x = 2m$, $y = 2c$, $z = 2n$

$x+y+z = 2m + 2c + 2h = 2(m+c+h)$

$\therefore 2(m+c+n) = 2r$ where $r = m+c+h$

So, by definition of even number $x+y+z$ is even and $\neg q \to \neg p$ is correct.

By proof by contraposition, $\therefore p \to q$ is true

3. (10 points) Prove that you can start with 10,000, subtract a power of 2, then divide by a power of 2, then divide by a power of 3, and end with 13. You can use any means necessary. What type of proof did you use?

- start with 10000
- subtract a power of 2: $1000 - 2^4 = 9984$
- divide by a power of 2: $9984 / 2^8 = 39$
- divide by a power of 3; $39 / 3^1 = 13$

$\therefore 13 = 13$

Constructive proof is used

4. (14 points; 2 points per line) The following is a proof that if $A \subseteq B$ and $B \subseteq A$, then $A = B$. For each line in the proof, fill in the name of the definition being used, the name of the set identity (page 136), the name of the logical equivalence (page 29), or the rule of inference (pages 76 and 80) used. If there is no specific name in the book, then cite the page number (and table number if applicable) that shows the equivalence or rule.

Step	Identity, rule, law, or definition used
1. $\forall x\,[x \in A \rightarrow x \in B]$	Definition of subset $A \subseteq B$
2. $\forall x\,[x \in B \rightarrow x \in A]$	Definition of subset $B \subseteq A$
3. $c \in A \rightarrow c \in B$	Universal Instantiation 1
4. $c \in B \rightarrow c \in A$	Universal Instantiation 2
5. $(c \in A \rightarrow c \in B) \wedge (c \in B \rightarrow c \in A)$	Logical conjunction
6. $c \in A \leftrightarrow c \in B$	Definition of Biconditional 5
7. $\forall x\,[x \in A \leftrightarrow x \in B]$	Universal Generalization 6

5. Express the following statements using x to represent a set element, and the symbols ∈ and ∉ to represent membership. For example, to specify the union of sets A and B, you would use x ∈ A ∨ x ∈ B. You may need to use quantifiers. For example, to express that set S is the empty set, you could state ¬∃x[x∈S], or ∀x[x∉S]. (Hint: One or more of these requires a quantifier, but not all of these require a quantifier; one or more of these requires implication, but not all require implication).

   a. (5 points) A ⊆ B

   $\forall x (x \in A \rightarrow x \in B)$

   b. (5 points) $A \cap \overline{B}$

   ~~$x \in A \wedge x \notin B$~~ $x \in A \wedge x \notin B$

   c. (5 points) $A \cap \overline{B} = \emptyset$

   $\neg \exists x (x \in A \wedge x \notin B)$

6. Determine whether each of these functions from **Z** to **Z** is one-to-one, onto, both (a bijection), or neither. Explain your reasoning (if onto, why? If not onto, why not? If one-to-one, why? If not one-to-one, why not?).

a. (5 points) f(x) = x + 100

$2 + \cancel{100} = 2 + \cancel{100}$

$2 = 2$

(I) Injective?

$f(a) = f(b) \Rightarrow a = b$

For ~~$\$$~~ $f(x) = x + 100$, lets say $f(a) = f(b)$ $a + \cancel{100} = b + \cancel{100}$

$a = b$

Yes, it is injective

(II) Surjective?

say $f(x) = y$

$y = x + 100 \Rightarrow x = y - 100$ so every integer y is covered,

Yes, it is surjective

Answer: It is bijection

b. (5 points) $f(x) = x^2$

(I) One-to-one?

$f(2) = 2^2 = 4$ $f(-2) = (-2)^2 = 4$

$2 \neq -2$, so there is two values of x fo y meaning it is not injective

(II) Surjective?

$f(x) = -1$ is ~~p~~ possible since square of x can never be negative, meaning it is not surjetive

Answer: Neither injective nor surjective

c. (5 points) $f(x) = x^3$

Injective?

$f(a) = f(b) \Rightarrow a = b$

For $f(x) = x^3$ $a^3 = b^3$

$a = b$

It is injective.

surjective?

$f(x) = x^3 = 7$

$x = \sqrt[3]{7}$

since $\sqrt[3]{7}$ does not belong to Z, $f(x) = x^3$ is not surjective

Answer: It is injective but not surjective.

7. A relation R is defined on {1, 10, 100} by:
R = { (1,1), (1,10), (10, 1), (10,10), (10,100), (100, 10)}

a. (5 points) Prove that R is not reflexive.

For R to be reflexive, we should have : (1,1), (10,10), (100,100)

(1,1) and (10,10) are in R, but (100,100) is not in R.

So R is not reflexive

b. (5 points) Prove that R is symmetric.

~~So we ha~~ If there is (x,y), (y,x) should also exist.

For (1,1) and (10,10), (1,1) and (10,10) exist respectively.

We have (1,10), (10,1) also exists.

we have (10,100), (100,10) also exists

For (10,1) and (100,10), (1,10) and (10,100) exist respectively

So R is symmetric

c. (5 points) Prove that R is not transitive.

(1,10) and (10,100) in R imply ~~that~~ (1,100)

but (1,100) is not in R. This means

R is not transitive.