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Optimization-based distribution grid hosting capacity calculations

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HIGHLIGHTS

- An optimization-based hosting capacity method is developed.
- The distribution grid power flow is linearized.
- The spatial interdependency of DG deployments is considered.
- Load variations are accounted for based on robust optimization.

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ABSTRACT

The distribution grid hosting capacity is defined as the amount of new production or consumption that can be added to the grid without adversely impacting the reliability or voltage quality for other customers. In this paper, an optimization-based method for determining the hosting capacity in distribution grids is proposed. The proposed method is developed based on a set of linear power flow equations that enable linear programming formulation of the hosting capacity model. Linearization further helps with determining a near-optimal solution in a short amount of time. The proposed method is examined on a test radial distribution grid to show its effectiveness and acceptable performance. Performance is further measured against existing iterative hosting capacity calculation methods. Results demonstrate that the proposed method outperforms traditional methods in terms of computation time while offering comparable results.

1. Introduction

Distributed Generators (DGs) are small units of generation that are directly connected to the distribution grid and are in close proximity to consumers. There is a growing proliferation of DGs in distribution grids, conceivably due to the falling cost of the technology as well as the promising benefits for end-use electricity customers such as payment reduction and potential load-point reliability improvement [1–3]. Once installed, the associated customers will be regarded as “prosumers”, meaning that they are consumers that also have the ability to produce electricity. Among available DG technologies, solar photovoltaic (PV) and small-scale wind turbines perceived to be the most adopted DG technologies for prosumers. At the end of 2016, the grid-connected solar PV installation in the United States reached a total capacity of 36 GW [4], up from 25.6 GW in 2015 and 18.3 GW in 2014.

Although interesting options for end-use customers and viable solutions for system operators to shift the generation from large-scale power plants to small-scale distributed resources, the DG installation could cause several negative impacts on distribution grids. Most notably, growing DG installations may put the grid at risk of having inefficient and/or low-reliability supply, as some of operational quantities can potentially hit their limits and result in power quality or reliability concerns at the system and customer levels [5,6]. In this case, a variety of factors, such as the rise/drop in nodal voltages and power flow in distribution branches (i.e., lines and transformers), needs to be considered when adding DGs. To determine the maximum amount of DG that a distribution grid can accommodate the concept of hosting capacity is introduced. The hosting capacity is defined as the amount of new production or consumption which can be connected to the grid without adversely impacting the reliability or voltage quality for other customers [7]. The operational performance is measured using various factors, from voltage magnitudes to feeder power flows to power quality issues [8]. Protection can also be considered as a critical performance measure as the DG deployment will potentially result in a reverse power flow in distribution feeders. The hosting capacity calculation sheds light on the role and impacts of DGs within the distribution grids. It can further provide grid planners with the required insight on how to build and upgrade the grid in a cheaper, greener, and

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Nomenclature

Sets

C_m	set of points of interconnection connected to bus *m*
L	set of lines
L_m	set of lines connected to bus *m*
B	set of buses
Λ	set of primal variables
U	set of uncertain parameters

Indices

c	index for points of interconnection
m, n	index for buses
∧	index for calculated variables

Parameters

b_{mn}	Susceptance of line *mn*
g_{mn}	conductance of line *mn*
r_{mn}	resistance of line *mn*
$P_m^{D,\max}$	upper limit of active load at bus *m*
$P_m^{D,\min}$	lower limit of active load at bus *m*
$P_c^{M,\max}$	maximum active power exchange with upstream grid at point of interconnection *c*
$PL_{mn}^{\max}$	maximum active power flow of line *mn*
$Q_m^{D,\max}$	upper limit of reactive load at bus *m*
$Q_m^{D,\min}$	lower limit of reactive load at bus *m*
$Q_c^{M,\max}$	maximum reactive power exchange with upstream grid at point of interconnection *c*
$QL_{mn}^{\max}$	maximum reactive power flow of line *mn*
x_{mn}	reactance of line *mn*
$\Delta V_m^{\min}$	lower limit of voltage magnitude deviation in bus *m*
$\Delta V_m^{\max}$	upper limit of voltage magnitude deviation in bus *m*

Variables

P_m^D	active load at bus *m*
P_m^G	active power of distributed generation at bus *m*
P_c^M	active power exchange with upstream grid at point of interconnection *c*
PL_{mn}	active power flow at line *mn*
Q_m^D	reactive load at bus *m*
Q_m^G	reactive power of distributed generation at bus *m*
Q_c^M	reactive power exchange with upstream grid at point of interconnection *c*
QL_{mn}	reactive power flow at line *mn*
V_m	voltage magnitude at bus *m*
θ_m	voltage angle at bus *m*
ΔV_m	voltage magnitude deviation in bus *m*
$\Delta\theta_m$	voltage angle deviation in bus *m*

more sustainable way. Hosting capacity calculations can also determine the maximum amount of DG that can be deployed to support reducing peak demand and postponing required grid upgrades.

The hosting capacity studies in the literature can be categorized into two main groups: (i) studies that propose hosting capacity calculation methods based on a variety of grid performance measures and system characteristics, and (ii) studies that focus on grid upgrades or operational practices to increase grid hosting capacity. Former studies further investigate the impact of DGs on selected operational performance measures as elaborated in [9,10]. These performance measures can be bus overvoltage, line overload, or power quality. The locational sensitivity analysis method of distribution feeders introduced in [11] estimates the grid hosting capacity by demonstrating the effect of DG distance on voltage deviations at feeder nodes. Similar studies are performed in [12] but with a focus on PV integration into distribution grids. Authors conclude that analyzing each feeder individually is faster than a simultaneous analysis of all feeders. However, the individual analysis method would not guarantee optimal, and in many cases accurate, solutions. Power quality as a performance measure for hosting capacity calculations, commonly studied in terms of harmonic distortion, is investigated in [13,14]. The model proposed in [13] explores the effects of harmonic distortion limits on hosting capacity under various active network management schemes, and authors in [14] investigate the impact of nondispatchable DGs on the harmonic distortion, and accordingly on grid hosting capacity. Optimal installation of DGs is derived in this work while preventing accumulated *h* order harmonic current from driving the harmonic voltage past acceptable limits.

Among the methods proposed to increase grid hosting capacity, active power management, power curtailment, and voltage control can be pointed out. A profit maximization strategy is developed in [15] for distribution utilities specializing in providing network access for third party DGs. The strategy informs infrastructure investment decisions by optimizing the profit from the acceptable hosting capacity. In addition, the active/reactive power curtailment strategy, specifically for voltage rise mitigation, has been demonstrated to produce beneficial results in the hosting capacity optimization problems in [16]. In [17], an active and reactive power control of the solar PV inverter to increase overall hosting capacity is explored. The studies in this work, however, are limited to only a few snapshots of demand and generation rather than a longer time horizon analysis. The impact of solar PV reactive power absorption on excessive voltage rise is inspected in [18] to assess DG performance. Multiple feed-in management strategies in order to increase the hosting capacity in a synthetic distribution system is studied in [19], benefiting from Monte Carlo simulations to derive general trends and to analyze specific grid, load, and DG architectures. A decentralized power control strategy is used in [20] to optimize grid hosting capacity by regulating the feeder voltage profiles. In a related study in [21], a hosting capacity optimization method is proposed to determine the optimal size and location of DGs using on-load tap changers (OLTC) and static Var compensators (SVC). The volt/Var control problem based for maximizing hosting capacity is modeled as a single-objective optimization problem in [22,23]. This model is extended to a multi-objective optimization problem in [24], in which a cuckoo search method is used to improve voltage profiles and reduce losses by optimizing DG allocation. The authors indicate two indices to measure quality improvement: voltage deviations from a reference value (which should be minimized) and voltage differences before and after DG integration (which should be maximized). The cuckoo search method is reported to outperform competing algorithms in efficiency in this particular problem. In [25], the impact of increasing solar PV units in residential neighborhoods is investigated and the hosting capacity is obtained in systems ranging from low voltage to medium voltage through a stochastic analysis framework. A C-type passive filter is used to optimize the hosting capacity while reducing harmonic distortions from DGs in [26]. In a related study, a variety of PV inverters are tested in [27] to find out how efficient the use of active and reactive power control strategies would be in increasing hosting capacity. However, it is concluded that the slow response time and switching restrictions of typical compensators prevent a fast and reliable control, which accordingly underscores the need for efficient voltage and reactive power control to achieve acceptable results when solving this problem. An optimization strategy for stabilizing nodal voltages and reducing system losses is employed in [28]. Bifurcation analysis is used to rank the nodal voltage stability. It is discussed that poorly-ranked buses benefit more from voltage stabilization via DG power injection, therefore the associated locations are weighed as preferred candidates for DG installation. It is further shown that DG reactive power limits directly impact the optimization results, highlighting their importance in voltage stability. Network reconfiguration is perceived as another viable method in increasing grid hosting capacity as explored in [29–31]. In [30], static and dynamic reconfigurations are used to determine the optimal hosting capacity, further benefiting from a multi-period optimal power flow approach. In [31], soft open points (SOPs) are used to reconfigure radial distribution systems. Hosting capacity is increased by optimizing the size and location of SOPs while maintaining the network radiality. The study does not consider the optimal DG sizes and locations; instead uses a scenario generation method to find DGs operational characteristics. The authors report that SOPs can significantly reduce the operating costs of active distribution grids.

In addition to system-oriented approaches in increasing grid hosting capacity, there exists some methods with a focus on technology-oriented approaches, i.e., to increase hosting capacity by integrating other distribution grid-integrated technologies. A decentralized approach based on multi-agent analysis is proposed in [32] to increase distribution grid hosting capacity for expected electric vehicle loads. A combination method based on storage system incorporation and day-ahead projection is used in [33] to optimize hosting capacity. The addition of a battery storage system increases the grid hosting capacity while also improving voltage profiles. Study in [34] uses hosting capacity calculation methods to compare the improvements from different types of DGs and energy storage systems on operating costs during power outages. However, this paper does not consider the impact of these technologies during normal operation. In [35], the authors optimize the system configuration while incorporating DG and energy storage systems. Their proposed multi-stage optimization has a fivefold minimization of costs as the objective function, including the investment, maintenance, energy, unserved power, and system emission costs. Results show a marginal impact of network switching on DG integration and grid hosting capacity.

There are extensive discussions on the hosting capacity problem in the literature. However, a closer look at the aforementioned studies reveals that the majority of the existing methods, both on hosting capacity calculation and maximization, rely on an iterative approach to determine the distribution grid hosting capacity. That is, one initial value for DG capacity at a specific bus in the system is considered and that capacity is incrementally increased to the point that the desired performance measure leaves the acceptable region. This common practice has two major drawbacks: first, the spatial interdependency of DG deployments is ignored, i.e., these methods do not offer the capability to study and analyze the impact of DG deployment in one bus to other buses, nor the impact of simultaneous DG deployments in several buses to the grid hosting capacity results. This is a major shortcoming as it can potentially prevent finding the optimal, or even a near-optimal, hosting capacity solution. Second, these methods are time-consuming as increasing the DG capacity after each increment should be followed by solving a complete power flow problem. In some cases, it may cause thousands of iterations to find the grid hosting capacity, which is proven to be ineffective for large distribution grids. The computation time and solution accuracy in this case further depend on the DG size increments. If large increments are considered, the model will find the solution faster but at the expense of losing accuracy. If small increments are considered, the solution will be potentially accurate but it will need a long computation time to reach the final solution. Large and small increments of course are relative terms in these cases, depending on the distribution grid size.

The aim of this paper is to address these two shortcomings by proposing an optimization-based hosting capacity calculation method. The objective will be to maximize allowable DG capacity that can be hosted in the distribution grid without negatively impacting grid performance. Two performance measures, namely bus overvoltage and line overload, are used for this purpose. Unlike the existing methods, the proposed method can effectively consider the spatial interdependencies and also find the solution in one instance instead of using many iterations. The proposed method uses a linear model for power flow analysis and formulates the problem based on linear programming. This would allow for dynamic changes to the model to account for installed generation and to update the hosting capacity results as new DG is integrated to the grid. The major contributions of the paper are listed as follows:

- The distribution grid power flow is linearized based on a few approximations obtained from practical assumptions. Unlike traditional nonlinear power flow models, the linearized model does not require iterations to find the final feasible solution and can be efficiently integrated into an optimization framework.
- An optimization-based hosting capacity calculation method is developed based on the linear power flow model. This method is capable of finding a near-optimal hosting capacity solution in a short amount of time, eliminating the need for extensive iterations as in traditional methods.
- The spatial interdependency of DG deployments is effectively considered within the developed method, in which all buses in the distribution grid are simultaneously analyzed for their hosting capacity (both individual and in aggregate).
- Load variations are accounted for based on robust optimization. A worst-case solution is obtained which encompasses all possible realizations of loads in all buses. Accordingly, the seasonal load variations are captured in hosting capacity calculations, removing the need for repeating studies when load values are changed.

The rest of the paper is organized as follows. The linear power flow model is presented in Section 2. The proposed grid hosting capacity calculation method is presented in Section 3. The simulation results on a test distribution grid are provided in Section 4. Finally, the conclusion is provided in Section 5.

2. Linear power flow

As the amount of nodal generation in the distribution grid changes, as a result of DG integration, the network power flow will accordingly change. It is important in this case to closely monitor grid performance to ensure that it is not negatively impacted. To study this impact, a full AC power flow should be solved to determine changes in line flows and bus voltage magnitudes and angles. A majority of existing distribution power flow methods are nonlinear and should be solved in an iterative manner (either through successive linearization around the current operating point, or through successive update of network quantities based on calculated increments). There exists some linear models as well [36–38], however, these models are mostly based on ZIP load models which are not useful in modeling DG generation. To address this issue, a linear power flow model is developed. Let's start with generic line flow equations. Eqs. (1) and (2) represent the active and reactive flow of line *mn,* which is assumed to be between buses *m* and *n*, respectively:

$$PL_{mn} = g_{mn}V_m^2 - g_{mn}V_mV_n\cos(\theta_m-\theta_n) - b_{mn}V_mV_n\sin(\theta_m-\theta_n) \quad \forall\, mn \in \mathrm{L} \tag{1}$$

$$QL_{mn} = -b_{mn}V_m^2 - b_{mn}V_mV_n\cos(\theta_m-\theta_n) - g_{mn}V_mV_n\sin(\theta_m-\theta_n) \quad \forall\, mn \in \mathrm{L} \tag{2}$$

These line flow equations are nonlinear, as they include second order terms, the multiplication of variables, and trigonometric terms. The conductance g_{mn} and the susceptance b_{mn} represent the real and imaginary components of the line admittance, respectively. They are calculated as follow:

$$g_{mn} = r_{mn}^2/(r_{mn}^2 + x_{mn}^2) \quad \forall\, mn \in \mathrm{L} \tag{3}$$

$$b_{mn} = -x_{mn}^2/(r_{mn}^2 + x_{mn}^2) \quad \forall\, mn \in \mathrm{L} \tag{4}$$

When performing a steady state analysis of the distribution grid, it can be assumed that the voltage magnitude and angle at the point of interconnection (POI), i.e., where the distribution grid is connected to the upstream subtransmission system, are known and fixed. This is a valid assumption as the upstream grid acts as an infinite bus with a constant voltage. Assuming that the voltage at POI is $1\angle 0^{\circ}$ p.u., all downstream bus voltages and angles can be represented as deviations from this value as in (5) and (6). In other words, the voltage magnitude in each bus is defined as 1.0 p.u. plus the deviation from the POI voltage magnitude, and the phase angle of each bus is defined as 0° plus the deviation from the POI voltage angle.

$$V_m = 1 + \Delta V_m \quad \forall\, m \in \mathrm{B} \tag{5}$$

$$\theta_m = 0 + \Delta\theta_m \quad \forall\, m \in \mathrm{B} \tag{6}$$

It is important to note that (5) and (6) add no approximations to line flow equations; rather, they simply redefine V_m and θ_m using the POI as a reference. Any other constant values can be considered for reference voltage magnitude and angle at the POI without loss of generality. After this initial change in problem variables, two assumptions are made to simplify line flow equations:

(i) The difference in voltage angles of adjacent buses m and n is considered to be small, thus trigonometric terms can be approximated as follows:

$$\sin(\theta_m - \theta_n) \approx \theta_m - \theta_n = \Delta\theta_m - \Delta\theta_n \quad \forall\, mn \in \mathrm{L} \tag{7}$$

$$\cos(\theta_m - \theta_n) \approx 1 \quad \forall\, mn \in \mathrm{L} \tag{8}$$

By using (5)–(8), the line flow equations can be reformulated as:

$$PL_{mn} = g_{mn}(1 + \Delta V_m)^2 - g_{mn}(1 + \Delta V_m)(1 + \Delta V_n) - b_{mn}(1 + \Delta V_m)(1 + \Delta V_n)(\Delta\theta_m - \Delta\theta_n) \quad \forall\, mn \in \mathrm{L} \tag{9}$$

$$QL_{mn} = -b_{mn}(1 + \Delta V_m)^2 - b_{mn}(1 + \Delta V_m)(1 + \Delta V_n) - g_{mn}(1 + \Delta V_m)(1 + \Delta V_n)(\Delta\theta_m - \Delta\theta_n) \quad \forall\, mn \in \mathrm{L} \tag{10}$$

(ii) Terms including the multiplication of ΔV and $\Delta\theta$ are very small and can be ignored. In other words, it is assumed that $\Delta V_m \Delta\theta_m = \Delta V_m \Delta\theta_n = \Delta V_n \Delta\theta_m = \Delta V_n \Delta\theta_n \approx 0$. This a reasonable assumption because both voltage magnitude and angle deviations from the POI values are small. Based on this assumption, the real and reactive line flows in (9) and (10) can be simplified, and then by rearranging the terms, can be reformulated as in (11) and (12), respectively.

$$PL_{mn} = g_{mn}(\Delta V_m - \Delta V_n) + g_{mn}\Delta V_m(\Delta V_m - \Delta V_n) - b_{mn}(\Delta\theta_m - \Delta\theta_n) \quad \forall\, mn \in \mathrm{L} \tag{11}$$

$$QL_{mn} = -b_{mn}(\Delta V_m - \Delta V_n) - b_{mn}\Delta V_m(\Delta V_m - \Delta V_n) - g_{mn}(\Delta\theta_m - \Delta\theta_n) \quad \forall\, mn \in \mathrm{L} \tag{12}$$

These two equations represent real and reactive line flows, not based on actual bus voltage magnitudes and angles but based on voltage magnitude and angle deviations from the POI values. In both equations, the first and third terms are linear, however the second terms are nonlinear.

This nonlinearity can be taken care of in two easy successive steps: In the first step, the nonlinear terms are simply removed from the equations and the resultant linear line flow equations are used to find the power flow solution. The power flow solution in this case will ensure that $PL_{mn} + PL_{nm} = 0$ and $QL_{mn} + QL_{nm} = 0$ (can be seen from the equations), so line losses would be zero, hence it can be called a "lossless power flow". In the second step, ΔV_m values obtained from the lossless power flow solution can be considered as constants in the nonlinear terms in line flow equations, i.e., $\Delta\hat{V}_m(\Delta V_m - \Delta V_n)$, where ˆ represents the already-calculated variable obtained from the lossless power flow solution. The nonlinear terms are now converted into linear ones, further ensuring that the approximation is much smaller than the lossless power flow. In this case, $PL_{mn} + PL_{nm} \neq 0$ and $QL_{mn} + QL_{nm} \neq 0$, so these equations consider line losses as well.

It can be discussed that if ΔV_m value is calculated again and plugged back into the line flow equations, a more accurate solution will be achieved. This is a valid discussion and the results will definitely improve. However, it can be shown with simple calculations that the amount of change in voltage magnitudes and angles after the second step will be minimal, thus eliminating the need to perform additional steps beyond the second step.

3. Hosting capacity calculation

The objective of the hosting capacity calculation is to maximize the total amount of DG capacity that can be installed in the distribution grid (13). The total installed DG capacity is considered as the summation of installed DG capacity in all buses.

$$\min_{U}\max_{\Lambda} \sum_{m\in B} P_m^G \tag{13}$$

The objective function is maximized over the set of "primal variables" shown with Λ, and is further minimized over the set of "uncertain parameters" denoted by U. Primal variables include DG capacities, i.e., the primary variable to be determined via this problem, along with bus voltage magnitudes and angles, real and reactive line flows, and real and reactive power exchange with the upstream grid. The uncertain parameters include real and reactive loads in each bus. The distribution grid hosting capacity is highly dependent on bus load values. If load values change, in one or more buses, the hosting capacity solution may accordingly change. Therefore, either all possible load variations should be considered when calculating the grid hosting capacity and then the minimum obtained solution can be considered as the final solution, or a worst-case analysis should be performed using robust optimization. The latter is employed in this paper, in which the maximum hosting capacity value is minimized over a set of uncertain parameters, here loads. Loads are further assumed to change within a polyhedral uncertainty set. This way, the worst-case solution is obtained without the need for considering all possible load variation scenarios. This solution is robust against all realizations of load variations, i.e., if loads obtain any other values within their identified bounds, the hosting capacity solution will not change.

This objective is subject to operational constraints (14)–(26):

$$\sum_{c\in C_m} P_c^M + \sum_{n\in L_m} PL_{mn} + P_m^G = P_m^D \quad \forall\, m \in \mathrm{B} \tag{14}$$

$$\sum_{c\in C_m} Q_c^M + \sum_{n\in L_m} QL_{mn} + Q_m^G = Q_m^D \quad \forall\, m \in \mathrm{B} \tag{15}$$

$$PL_{mn} = g_{mn}(1 + \Delta\hat{V}_m)(\Delta V_m - \Delta V_n) - b_{mn}(\Delta\theta_m - \Delta\theta_n) \quad \forall\, mn \in \mathrm{L} \tag{16}$$

$$QL_{mn} = -b_{mn}(1 + \Delta\hat{V}_m)(\Delta V_m - \Delta V_n) - g_{mn}(\Delta\theta_m - \Delta\theta_n) \quad \forall\, mn \in \mathrm{L} \tag{17}$$

$$-P_c^{M,\max} \leqslant P_c^M \leqslant P_c^{M,\max} \quad \forall\, c \in \mathrm{C_m} \tag{18}$$

$$-Q_c^{M,\max} \leqslant Q_c^M \leqslant Q_c^{M,\max} \quad \forall\, c \in \mathrm{C_m} \tag{19}$$

$$P_m^{D,\min} \leqslant P_m^D \leqslant P_m^{D,\max} \quad \forall\, m \in \mathrm{B} \tag{20}$$

$$Q_m^{D,\min} \leqslant Q_m^D \leqslant Q_m^{D,\max} \quad \forall\, m \in \mathrm{B} \tag{21}$$

$$-PL_{mn}^{\max} \leqslant PL_{mn} \leqslant PL_{mn}^{\max} \quad \forall\, mn \in \mathrm{L} \tag{22}$$

$$-QL_{mn}^{\max} \leqslant QL_{mn} \leqslant QL_{mn}^{\max} \quad \forall\, mn \in \mathrm{L} \tag{23}$$

$$\Delta V_m^{\min} \leqslant \Delta V_m \leqslant \Delta V_m^{\max} \quad \forall\ m \in \mathrm{B} \tag{24}$$

$$\Delta V_m^{\min} = V_m^{\min} - 1 \quad \forall\ m \in \mathrm{B} \tag{25}$$

$$\Delta V_m^{\max} = V_m^{\max} - 1 \quad \forall\ m \in \mathrm{B} \tag{26}$$

The active power balance (14) ensures that the generation from local installed DGs plus the line flows in each bus m will be equal to the load at that bus. The installed DG generation is considered a free positive variable in all buses. If that bus is the POI, the utility power exchange is further considered in the load balance equation. In a similar manner, the reactive power balance Eq. (15) ensures that a balance is met for the reactive power at each bus. Eqs. (16) and (17) represent the active and reactive line flows as developed in Section 2. Constraints (18) and (19) impose limits on the active and reactive power exchange with the upstream grid. It should be noted that this power exchange is another free variable that can be positive (importing power from the upstream grid) or negative (exporting power to the upstream grid), or zero (no power exchange). Constraints (20) and (21) show nodal load variations which are limited by a lower bound and an upper bound. These bounds can be obtained based on historical load data. Loads can freely change within their associated bounds, and at the end would select the values that will result the worst-case maximum hosting capacity solution under load variations. Performance measures are considered as line overload and bus overvoltage. To prevent such violations, real and reactive line flows are constrained by (22) and (23), respectively, and bus voltage magnitude is limited by (24). With these three constraints, it is ensured that DG injections do not cause a deterioration in grid performance measures by violating associated operational limits. The lower and upper bounds on the voltage magnitude deviation from the POI at each bus are defined by (25) and (26), respectively.

The objective and all the constraints in the formulated problem are linear, except line flow Eqs. (16) and (17). To convert this problem to a linear problem, and accordingly enable a faster and better solution, the two-step process explained for the linear power flow will be used. Fig. 1 depicts the flowchart of this optimization-based hosting capacity calculation method. The method starts by identifying grid topology and characteristics, as well as a set of selected grid performance measures. In the first step, the grid hosting capacity is calculated by ignoring losses, i.e., based on the lossless power flow model. In the second step, the full power flow equations are solved by using the results for ΔV_m obtained from the lossless model as a constant introduced to linearize the nonlinear terms.

With the proposed model, it becomes possible to obtain hosting capacity solution quickly, efficiently, and with access to active and reactive power flow information at the solution operating point. The solution is further robust against all realizations of load variations. One last issue to consider is how to solve the min-max problem. The complex objective function is not tractable in its current form which makes it challenging to solve the problem. To address this issue, the dual problem of the inner maximization problem is obtained and then combined with the outer minimization problem. This is doable as the problem is linear. The final form will be min-min problem which can be written in terms of a single-objective minimization problem. This solution method is generic and applicable to any radial distribution grid.

4. Numerical studies

The IEEE 33-bus distribution test system is studied to show the performance of the proposed optimization-based hosting capacity method. This system contains 33 buses, 32 branches, and no existing DG as shown in Fig. 2. The detailed data of the system is given in [39]. Bus 1 is considered as the POI where no DG can be installed. During the analysis, all loads are initially set to be a constant value, called the base load. When accounting for the inherent uncertainty of loads within the system, an uncertainty range, i.e., lower and upper bounds, is defined. For each candidate DG, the maximum power output is assumed to be equal to its installed capacity and the minimum power output is assumed to be zero. The voltage at POI is assumed to be 1 p.u. with an angle of 0°. Considering respective minimum and maximum bus voltage limits of 0.9 p.u. and 1.1 p.u., the lower and upper voltage deviation limits are obtained as -0.1 p.u. and 0.1 p.u., respectively. Active power exchanged with the upstream grid is capped at 4.6 MW. The problem is formulated as a linear programming problem and solved using CPLEX 12.4. The following cases are studied:

Case 0: Validating the accuracy of the linear power flow solution.
Case 1: Grid-level hosting capacity calculation for base loads.
Case 2: Grid-level hosting capacity calculation with uncertain loads.
Case 3: Sensitivity analysis with respect to line flow limits.
Case 4: Sensitivity analysis with respect to voltage limits.
Case 5: Comparison with the traditional method.

Case 0 validates the linear power flow solution by providing comparisons with solutions from nonlinear full AC power flow. This comparison is needed to show the accuracy of the developed model and furthermore allow integration with the hosting capacity calculation method. Cases 1 and 2 use the developed optimization-based method to calculate hosting capacity at grid level, i.e., considering all buses at the same time. Case 1 focuses on base load, i.e., one single load snapshot, while Case 2 captures load uncertainty. The comparison of results between these two cases will show a tradeoff that will occur when uncertainties are considered. Cases 3 and 4 further elaborate results of Case 2 by analyzing the sensitivity of the hosting capacity results on performance measures, here line overload and bus overvoltage, respectively. Case 5 provides comparisons with a traditional iteration-based hosting capacity calculation method to show the superior performance of the proposed method. Cases 1, 2, and 3 are studied under three scenarios:

Scenario 1: All buses are considered for DG installation.
Scenario 2: DG installation is allowed at buses 2 and 3 only, as these two buses are directly connected to the highest-capacity lines in the system.
Scenario 3: DG installation is allowed at end buses only (buses 18, 22, 25, and 33).

Case 0: The linear power flow is applied to the radial distribution test system shown in Fig. 2 to find power flow solution and compare it with those of nonlinear AC power flow analysis. Results obtained from the linearized method compared with the nonlinear method show an average percent error for voltage magnitudes, voltage

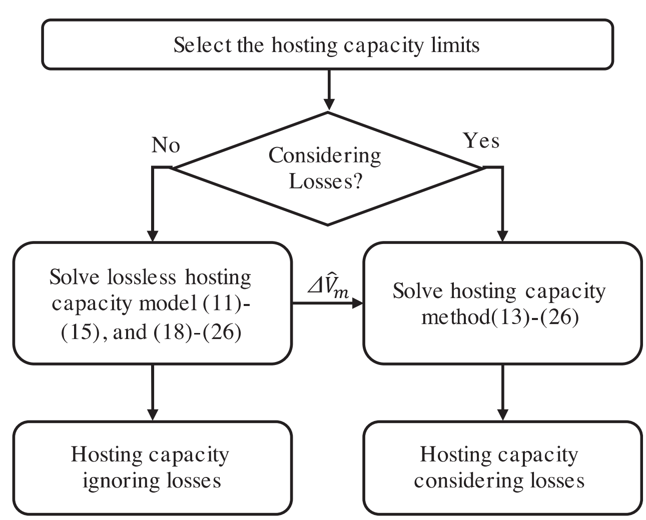

Fig. 1. Flowchart of the hosting capacity calculation method.

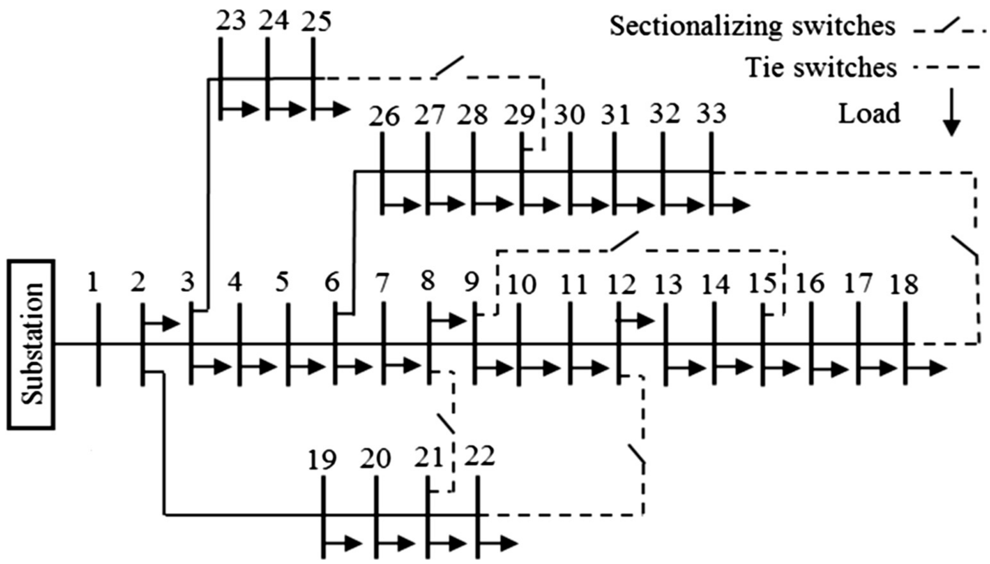

Fig. 2. Single-line diagram of the IEEE 33-bus distribution test system.

angles, line flows, and total line losses of 0.002%, 16.2%, 0.21%, and 9.4%, respectively. The results advocate a very high accuracy in determining voltage magnitude and line flows. This accuracy is less for voltage angles, however it should be considered that voltage angles are less important factors in distribution network power flow studies when compared to voltage magnitudes. Their impact on line flows can be clearly seen from line flow equations. The average values of the percent error are found by first calculating the percent error for each individual bus/line, and then averaging across all buses/lines. Fig. 3 displays voltage magnitude results in each bus for both methods. The results advocate solution accuracy of the proposed linear method compared to the nonlinear AC power flow method. As discussed, the results can be improved by incorporating additional steps in finding voltage magnitudes and angles, which is however not required as the obtained results are already close to actual values.

Case 1: In this case, the grid hosting capacity is determined using the base case load for the three considered scenarios. The grid hosting capacity in scenario 1 is when DGs are installed at buses 2, 19, and 20 with capacities of 7624 kW, 90 kW, and 770 kW, respectively, resulting in a grid hosting capacity of 8484 kW. This result is shown in Table 1. The hosting capacity is limited by the maximum acceptable active power flow through the lines connected to bus 1. The hosting capacity in scenario 2, for which DGs may be placed only at buses 2 and 3, is calculated at the same value of 8484 kW, with the difference that it is fully installed at bus 2. The point of this scenario is to explore the variation for which the influence of line capacity limits is the weakest (i.e. limiting the optimal placement of DGs). This highlights the bottlenecking role that

Table 1
Base load hosting capacity results (kW).

Bus #	Scenario 1	Scenario 2	Scenario 3
2	7624	8484	0
16	0	0	0
18	0	0	190
19	90	0	0
20	770	0	0
21	0	0	0
22	0	0	200
25	0	0	920
32	0	0	0
33	0	0	160
Total DG (kW)	8484	8484	1470
Total loss (kW)	175.12	175.15	114.43

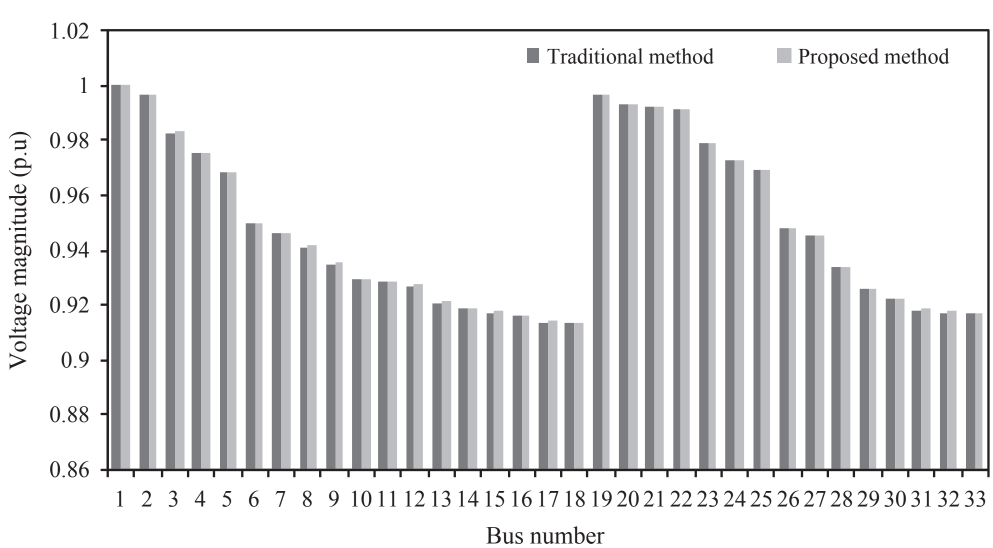

Fig. 3. Voltage magnitude comparison between linear and nonlinear power flow methods.

a line may play in the grid. In scenario 3, where DGs may only be installed at end buses, the grid hosting capacity results in installations at buses 18, 22, 25 and 33 with capacities of 190 kW, 200 kW, 920 kW, and 160 kW, respectively, for a total capacity of 1470 kW. Line losses are decreased by 34.7% in this case, but the overall hosting capacity is decreased by 82.7% when compared with the first two scenarios. This result could be foreseen as the end-buses are connected to lines that have considerably smaller capacities compared to other buses.

Case 2: Using the uncertain load data, the grid hosting capacity is calculated for the same three scenarios used in Case 1 and the results are summarized in Table 2. The minimum/maximum load recorded over a year-long horizon is used as the lower/upper bound of the uncertain load in each bus. Since the worst-case solution will be obtained based on this uncertain load profile, the grid hosting capacity will never result in unacceptable performance for other load profiles. The total real and reactive base load in Case 1 is 3715 kW and 2300 kVAR, respectively, while in this case the real and reactive load can change in the range of [1490 kW, 3715 kW] and [922.5 kVAR, 2300 kVAR], respectively.

For scenario 1, the total hosting capacity is calculated as 6116 kW with DGs being placed in buses 2, 19, and 20. Similar to Case 1, scenario 2 has all the DGs placed at bus 2 to optimize the hosting capacity with a total of 6116 kW. As in Case 1, the capacity of the line connecting this system to the upstream grid is the limiting factor. The results here are considered more reliable, as they demonstrate the minimum expected hosting capacity when including load uncertainty into the model, i.e., the obtained result will still be valid for any other realizations of loads. Comparing the obtained solution in these two scenarios, the grid hosting capacity is reduced down to 63.15% of the hosting capacity in Case 1. For scenario 3, the grid hosting capacity is calculated as 1074 kW and the system losses are reduced by 63.4%. Power flow capacities at lines 17, 21, 24, and 32 are limiting the hosting capacity in this scenario. The obtained results in this case underscore that when handling the worst-case load profile, the system cannot accommodate more than 72% of the base load hosting capacity.

In this case, none of the scenarios have been limited by voltage magnitude constraints; rather, distribution line capacities limit the hosting capacity. Fig. 4 shows the voltage magnitudes for studied scenarios. Note that the voltage never dips below 0.96 p.u. and thus falls within the acceptable range of 0.90 p.u. to 1.1 p.u.

Case 3: In this case, the sensitivity of the hosting capacity results with respect to line capacities is studied. Line flow limits are changed to reflect capacity upgrades to specific lines instead of upgrading the entire system. Similar to Case 2, this case explores what the worst-case solutions for a given system parameter for the same scenarios are. This is reflected in the problem by increasing the line limits of the critical lines 1 and 2. Cases 1 and 2 highlighted the crucial role that these lines play in grid hosting capacity. The capacity limits of lines 1 and 2 are increased by 10% increments up to 40%. Fig. 5 shows the grid hosting capacity as a function of the line capacity limit variations. For the first two scenarios, the grid hosting capacity is increased by 7.5%, 15%, 22.5%, and 30% when the capacity limits of the lines are increased by 10%, 20%, 30%, and 40% respectively. However, the hosting capacity in the last scenario does not improve as the adjusted line limits are not hit, and thus are not involved, in hosting capacity calculations. Fig. 6 shows the change in total system losses due to increase in line capacity limits. As the grid hosting capacity is increased in the first two scenarios, the total losses also increase by 5.2%, 10.8%, 14.2%, and 22% for the line limit increases of 10%, 20%, 30%, and 40% respectively. However, there is no change in the total losses of the last scenario as the solution does not change in response to the increase in line limits. A clear pattern emerges, wherein hosting capacity in scenarios 1 and 2, which were demonstrated to be limited by line 1 capacity limits, are positively affected by the increased capacity. The hosting capacity results in scenario 3, however, remain unaffected as their limitations are due to line capacities in multiple smaller lines elsewhere in the grid. Fig. 7 re-expresses the data as the percent change in the grid hosting capacity and total losses as the critical line limits are changed. This figure highlights that in some cases local upgrades can increase the grid hosting capacity.

Case 4: In this case, the change in hosting capacity with respect to voltage magnitude limits is considered by reducing voltage deviations limits to ± 0.05 p.u. The grid hosting capacity results in this case are compared to those of Case 2 and shown in Fig. 8 using the same uncertain load data. The comparison shows that there is no change in the grid hosting capacity results. However, the hosting capacity location of scenario 1 is changed to bus 2 but with the same amount. This is because in this case some of the downstream buses reach their voltage limit, so the DG installation is moved to bus 2. Table 3 compares the solution for the two considered voltage deviation limits, i.e., ± 0.1 p.u. and ± 0.05 p.u. The tighter voltage limits in the base load analysis lead to a reduction in the grid hosting capacity results. The result is decreased from 8484 kW to 8400 kW, as the voltage magnitude at buses 15–18, 32, and 33 has reached the limit of 0.95 p.u. (Fig. 9). This is expected, since a reduction in the allowed voltage fluctuations means that a smaller DG capacity can be accepted.

Comparing the uncertain load before and after implementing the reduction of the change in voltage deviation limits in Table 3 shows that the grid hosting capacity of the uncertain load remains unchanged.

Case 5: In this case, the performance of the proposed method is compared with that of a traditional iterative method. The performance is checked in terms of the solution accuracy and the computation time. In the traditional method, the power flow for all possible DG site/size combinations must be solved, which requires an extensive computation effort. This is especially true if a large search space is considered, as this requires many solutions of the power flow for each DG site/size combination. Due to the simplicity of the linear programming, many of the drawbacks noted in the traditional method are avoided. Thus, it is expected that a lower computation time is observed in the proposed method as a result.

For the first comparative study, individual hosting capacities are determined for each bus assuming there will be no DG installation at other buses (i.e., ignoring spatial interdependency). Each bus's individual hosting capacity is optimized for the uncertain load profile. Fig. 10 represents the grid hosting capacity results for each individual bus using the proposed method and the traditional method. As can be seen, the results of the two methods are very similar. The time required to solve the problem based on the proposed method is 2 s, while the

Table 2
Uncertain load hosting capacity results (kW).

Bus #	Scenario 1	Scenario 2	Scenario 3
2	5471	6116	0
16	0	0	0
18	0	0	136
19	37	0	0
20	608	0	0
22	0	0	146
25	0	0	668
32	0	0	0
33	0	0	124
Total DG (kW)	6116	6116	1074
Total loss (kW)	35.12	34.1	12.87

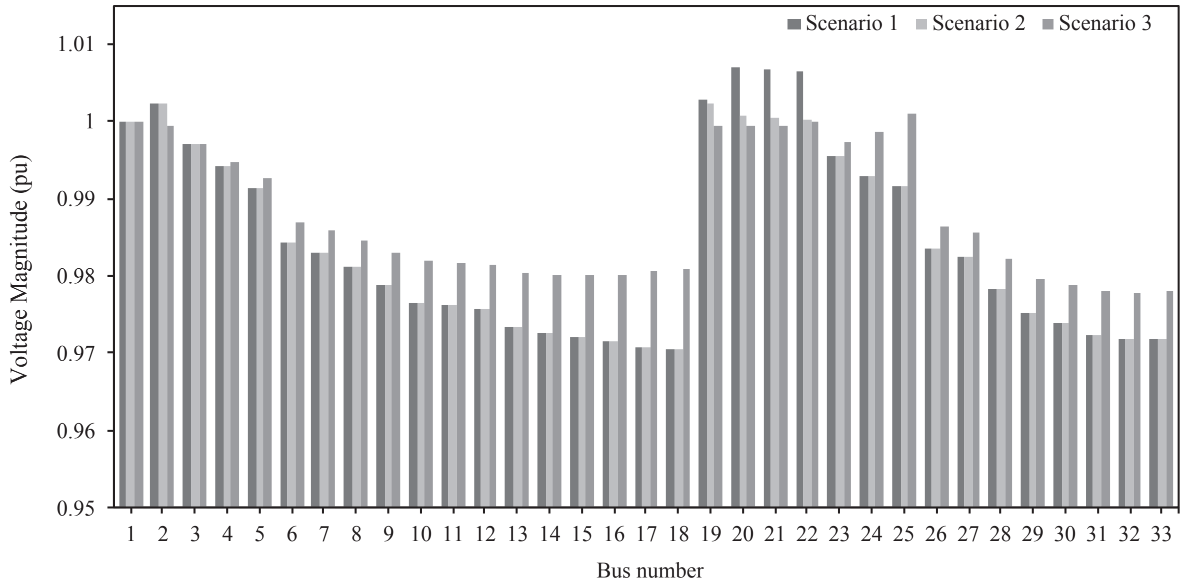

Fig. 4. Voltage magnitude profile in Case 2.

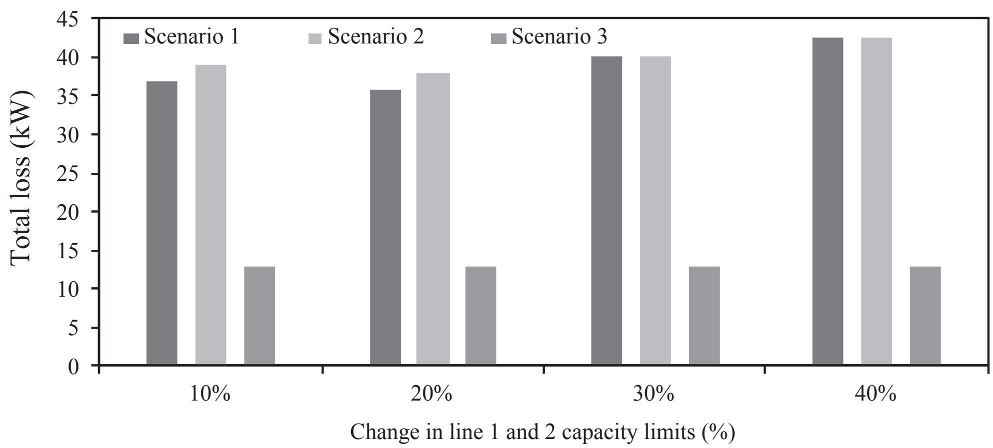

Fig. 5. Hosting capacity results based on the change in capacity limits of lines 1 and 2.

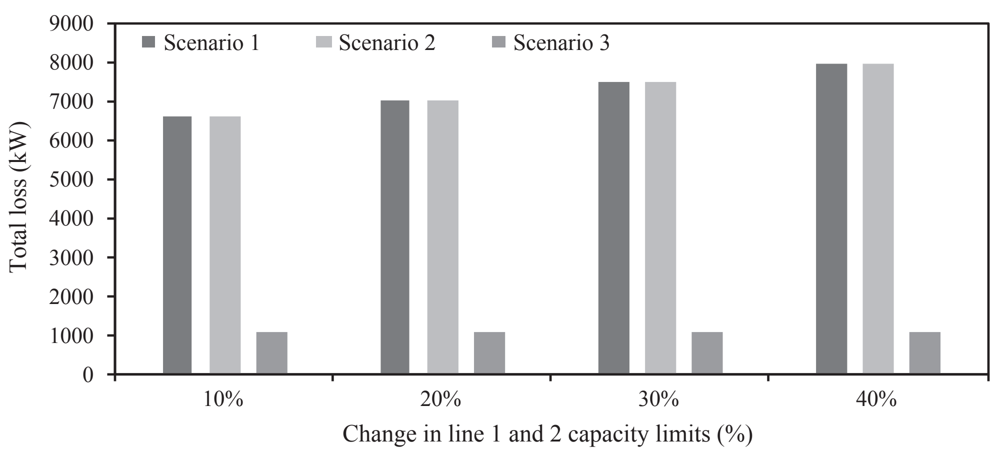

Fig. 6. Line loss based on the change in capacity limits of lines 1 and 2.

computation time in traditional method is 359 s. The average percent error of the proposed method is 1.08% compared to the traditional method. These results demonstrate the significantly-improved computation speed and the acceptable accuracy that the proposed method provides over the traditional hosting capacity method when analyzing single-bus hosting capacities.

In the second comparative study, the proposed method and the traditional method are used to compute the grid-level hosting capacity when all buses are considered. The traditional hosting capacity method takes 84 h to find the final solution, while the proposed method finds the solution in 21s. The average percent difference in the final solution is 1.2%. The obtained results advocate that the proposed method is accurate and extremely fast.

These cases show that the hosting capacity in a given system can be

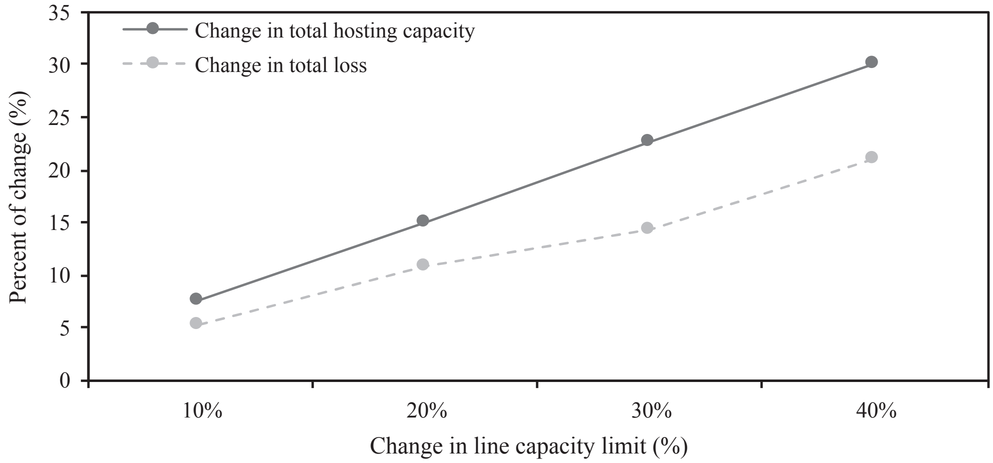

Fig. 7. Change in gird hosting capacity and losses due to the change in capacity limits of lines 1 and 2.

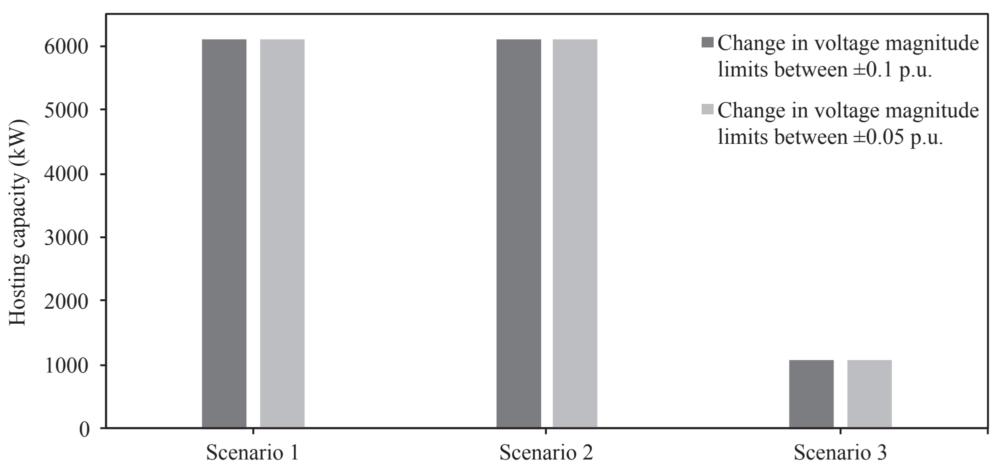

Fig. 8. Uncertain load hosting capacity results.

Table 3
Grid hosting capacity results (kW) based on the change in voltage deviation limits.

Bus #	Base load hosting capacity		Uncertain load hosting capacity	
	± 0.1 p.u voltage deviation limit	± 0.05 p.u voltage deviation limit	± 0.1 p.u voltage deviation limit	± 0.05 p.u voltage deviation limit
2	7624	7541	5471	6116
15	0	71	0	0
16	0	38	0	0
17	0	108	37	0
18	0	121	608	0
19	90	0	0	0
20	770	0	0	0
32	0	481	0	0
33	0	40	0	0
Total DG (kW)	8484	8400	6116	6116
Total loss (kW)	175.12	93.5	35.12	34.09

achieved through the proposed optimization-based method, and DGs could be incorporated while guaranteeing no detrimental impact on grid performance. Adding DGs to the grid has also been demonstrated to reduce system losses when properly placed, again possible to study through the proposed method. In addition, variety of scenarios, system setups, and objectives are possible to investigate via this method.

5. Conclusion

A method for finding the hosting capacity in a radial distribution grid was proposed in this paper. The method was developed based on an optimization-based mathematical model for calculating the operating conditions of a distribution grid given the size and the location of potential DG deployments. The method benefited from a linear power flow model which enabled a linear programming formulation of the developed method and alleviated the need for performing iterations. Results showed that the proposed method can outperform traditional hosting capacity methods in terms of computation time while obtaining an almost similar hosting capacity solution.

This method can be further extended considering its extensive potential applications. An immediate follow up research is to use this method in finding effective approaches in maximizing the grid hosting capacity. Considering additional performance measures, such as harmonics and protection, in calculating a more accurate hosting capacity

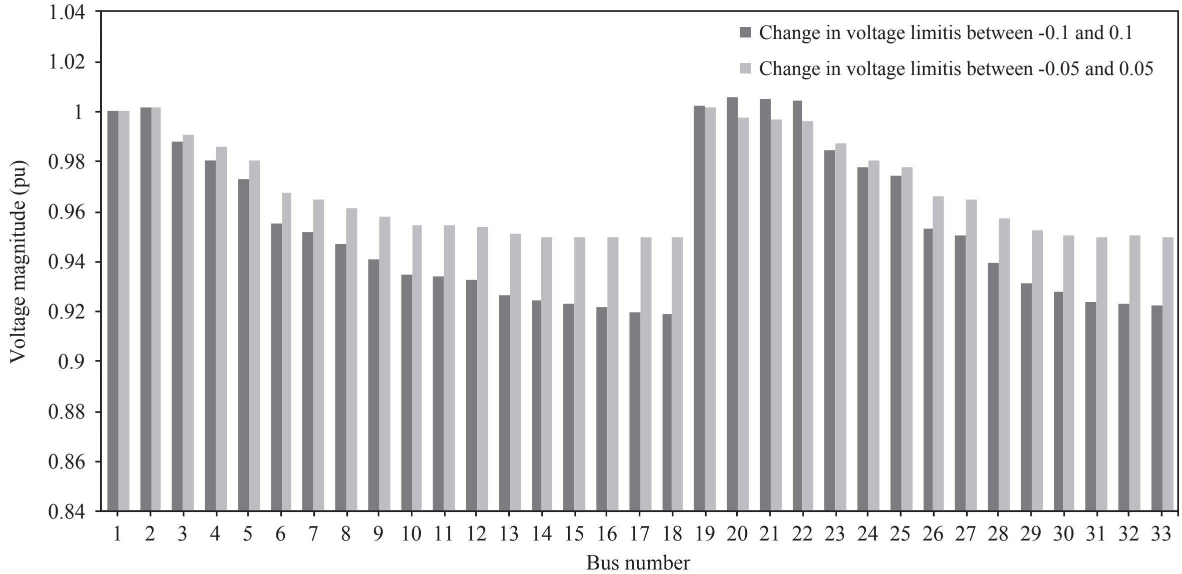

Fig. 9. Voltage magnitudes based on hosting capacity results under base load conditions.

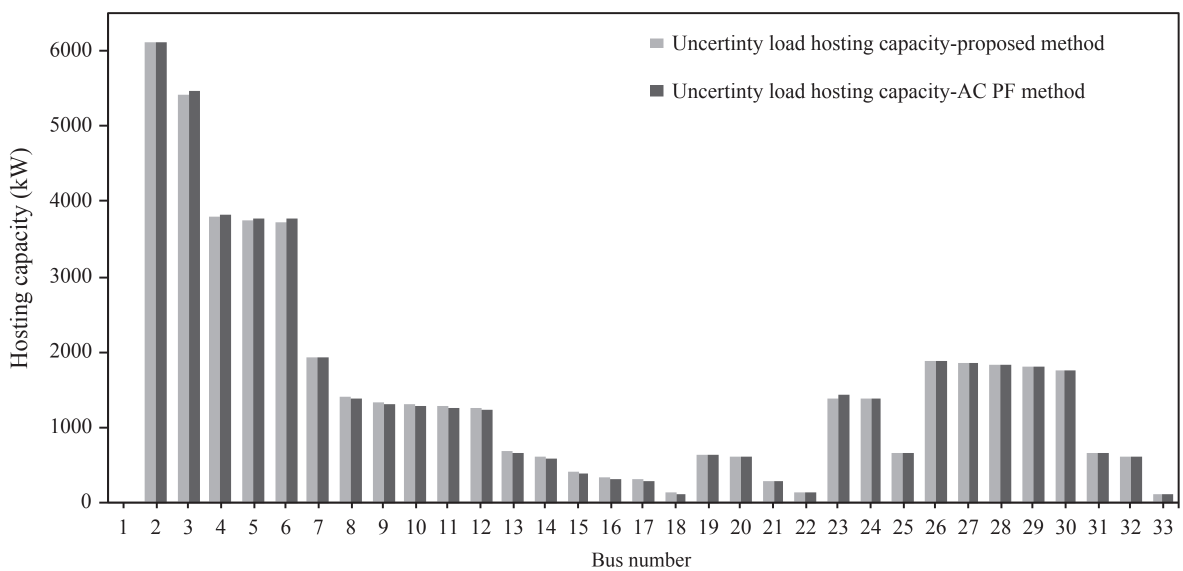

Fig. 10. Hosting capacity for each individual bus (accuracy comparison).

is also another area of extension which will be discussed in future work.

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