Case Study: Using Lion to Solve a Convex Optimization Problem

1. Chosen Problem: Image Classification

Image classification is a fundamental problem in computer vision with numerous applications, such as healthcare, autonomous driving, and security. The objective of image classification is to assign a label to an image based on its content.

2. Dataset Used

Overview

Dataset	Fashion MNIST
Description	A dataset of grayscale images of clothing items designed to replace the classic MNIST database.
Total Images	70,000
Image Shape	28x28 grayscale
Number of Categories	10
Training Set	60,000 images
Test Set	10,000 images
Categories	T-shirt/top, Trouser, Pullover, Dress, Coat, Sandal, Shirt, Sneaker, Bag, Ankle boot
Purpose	Image classification
Creator	Zalando Research Lab

Data Sample

You can find a sample in this repository.

Preprocessing

This preprocessing code snippet is commonly used in preparing image data for training a machine learning model, particularly in deep learning with neural networks. Let's break it down step by step:

1. **Normalization**:

   ```
   self.train_images = self.train_images / 255.0
   self.test_images = self.test_images / 255.0
   ```

   - **Purpose**: Normalizing pixel values to be between 0 and 1.

2. **Adding a Channel Dimension**:

   ```
   self.train_images = np.expand_dims(self.train_images, -1)
   self.test_images = np.expand_dims(self.test_images, -1)
   ```

   - **Purpose**: Adding a new dimension to the image arrays to match the shape expected by the model.

3. Model Description

The chosen model is a Convolutional Neural Network (CNN). This model includes three convolutional layers, each followed by a max-pooling layer. Dropout layers are included to prevent overfitting. The model ends with two dense layers, the last one being the output layer with a shape of 10, corresponding to the number of classes in the dataset.

Model Architecture

Layer (type)	Output Shape	Param #	Activation	Regularization
conv2d (Conv2D)	(None, 26, 26,	320	relu	None

Layer (type)	Output Shape	Param #	Activation	Regularization
	32)			
max_pooling2d (MaxPooling2D)	(None, 13, 13, 32)	0	None	None
dropout (Dropout)	(None, 13, 13, 32)	0	None	Dropout(0.25)
conv2d_1 (Conv2D)	(None, 11, 11, 64)	18,496	relu	None
max_pooling2d_1 (MaxPooling2D)	(None, 5, 5, 64)	0	None	None
dropout_1 (Dropout)	(None, 5, 5, 64)	0	None	Dropout(0.25)
conv2d_2 (Conv2D)	(None, 3, 3, 64)	36,928	relu	None
flatten (Flatten)	(None, 576)	0	None	None
dense (Dense)	(None, 64)	36,928	relu	l2(0.001)
dropout_2 (Dropout)	(None, 64)	0	None	Dropout(0.5)
dense_1 (Dense)	(None, 10)	650	softmax	None

Total params: **93,322 (364.54 KB)**
Trainable params: **93,322 (364.54 KB)**
Non-trainable params: **0 (0.00 B)**

4. Training

We want to compare the capabilities of the Lion optimizer with other classic optimizers. The chosen competitors are:

- Adam
- AdamW
- Nadam
- RMSprop
- SGD
- Adagrad

Training Parameters

40 Epochs

Parameter	Value
Optimizers	Adam, AdamW, Nadam, RMSprop, SGD, Adagrad
Epochs	40
Batch Size	128
Validation Split	0.2
Early Stopping Patience	20
Learning Rates	0.0001 to 0.001 (step 0.0001), 0.002 to 0.01 (step 0.001), 0.02 to 0.1 (step 0.01)

80 Epochs

Parameter	Value
Optimizers	Adam, AdamW, Nadam, RMSprop, SGD, Adagrad
Epochs	80
Batch Size	128
Validation Split	0.2
Patience	20

Parameter	Value
Learning Rates	0.01 to 0.09 (step 0.01), 0.1 to 1.0 (step 0.1)

160 Epochs Parameters

Parameter	Value
Optimizers	Adam, AdamW, Nadam, RMSprop, SGD, Adagrad
Epochs	160
Batch Size	128
Validation Split	0.2
Patience	20
Learning Rates	Logarithmic scale from 10^{-5} to 10^{-1}

We use a logarithmic scale for learning rates because it is a common practice in deep learning to search for the optimal learning rate on a logarithmic scale.

5. Training

Loss Function

In this classification problem, we use the sparse categorical cross-entropy loss function, suitable for multi-class classification problems where the target labels are integers.

The sparse categorical cross-entropy loss function is defined as:

$$\text{Loss} = -\frac{1}{N} \sum_{i=1}^{N} \log P(y_i|x_i)$$

where:

- N is the number of samples in the dataset.
- y_i is the true label of the i-th sample.

- x_i is the i-th input sample.
- $P(y_i|x_i)$ is the predicted probability of the true label y_i given the input x_i.

Metrics

We choose accuracy as the metric to evaluate the performance of the model. The accuracy is defined as the ratio of the number of correct predictions to the total number of predictions made by the model.

$$\text{Accuracy} = \frac{\text{Number of Correct Predictions}}{\text{Total Number of Predictions}}$$

6. Technical Implementation

Libraries Used

- **TensorFlow GPU/Keras** for model construction and training (1 second per epoch on a NVIDIA GeForce RTX 4070).
- **NumPy** for data processing.
- **Matplotlib** for result visualization.

You can find the open source code here.

Example Implementation with the Lion Optimizer

```
optimizer = keras.optimizers.Lion(learning_rate=lr)

self.model.compile(optimizer=optimizer,
    loss='sparse_categorical_crossentropy',
    metrics=['accuracy'])
history = self.model.fit(self.train_images,
    self.train_labels,
    epochs=epochs,
    batch_size=batch_size,
    validation_split=0.2)
```

7. Results

Visualizing Methodology

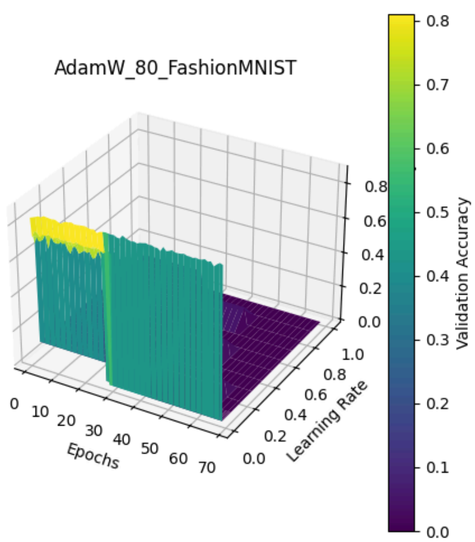
We use a 3D projection to visualize the results of the training. We use the following axes:

- **x-axis**: Learning Rate Range
- **y-axis**: Epochs (0-40; 0-80; 0-160)
- **z-axis**: Accuracy Value

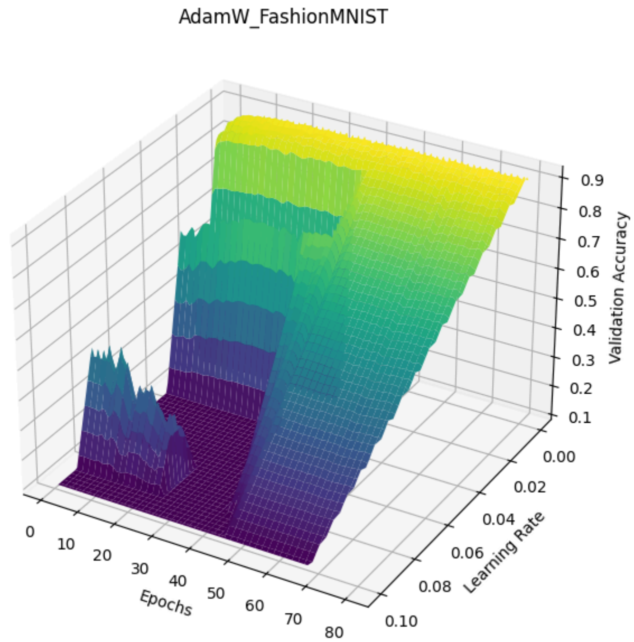
We use a color gradient to represent the accuracy value. The color gradient goes from purple (low accuracy) to green (high accuracy).

The plots are made with matplotlib and the 3D projection. (see the code here)

Example Plot



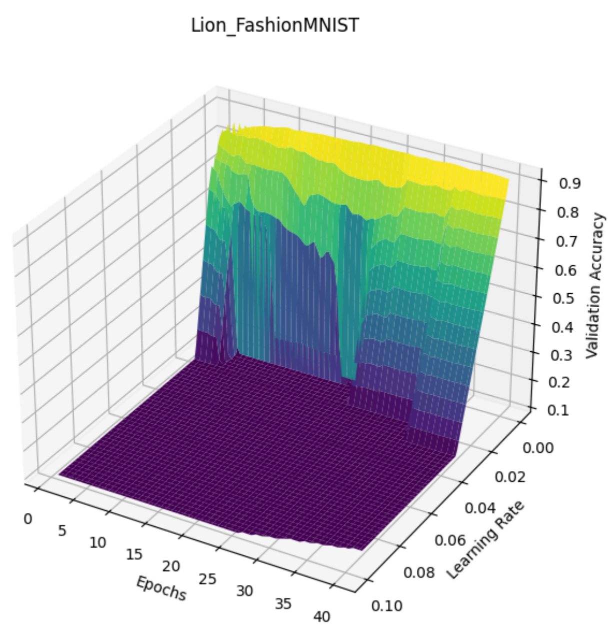
Wrong size & orientation :(

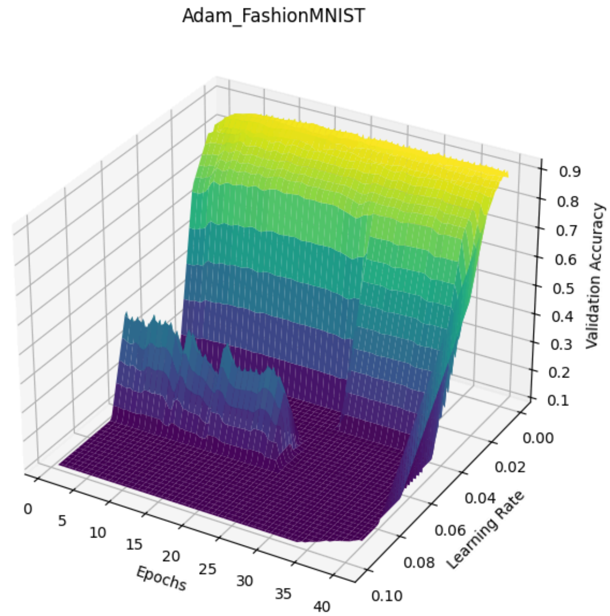


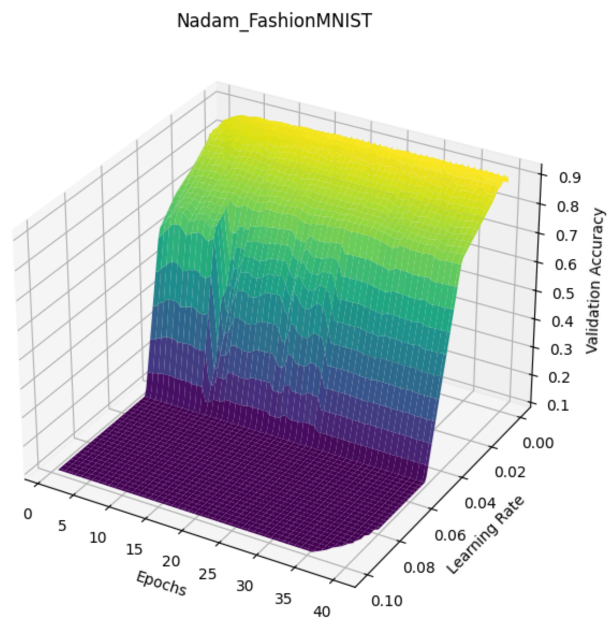
It's better (with a little bit of interpolation) :)

Plots Comparison

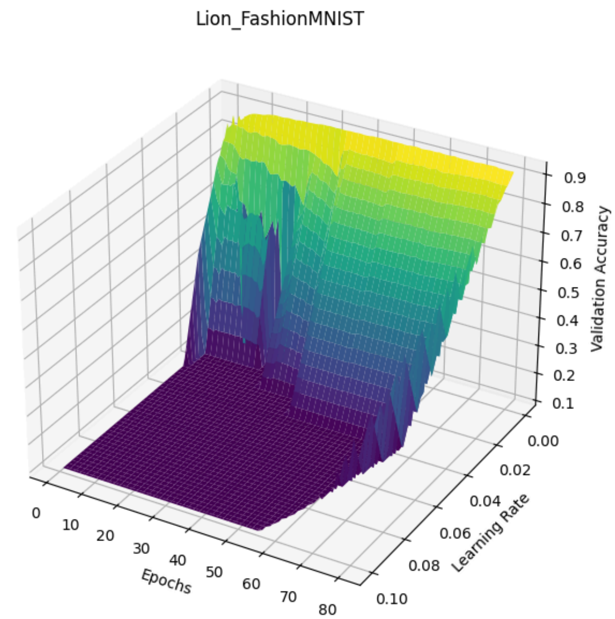
40 Epochs

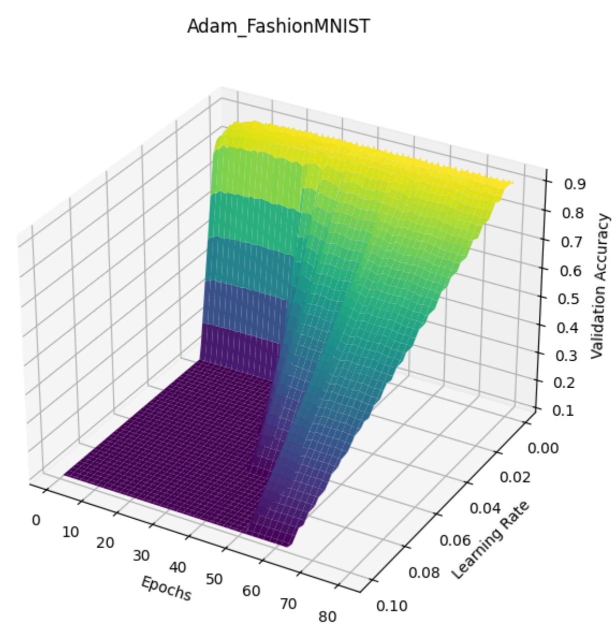
Lion 40	
	

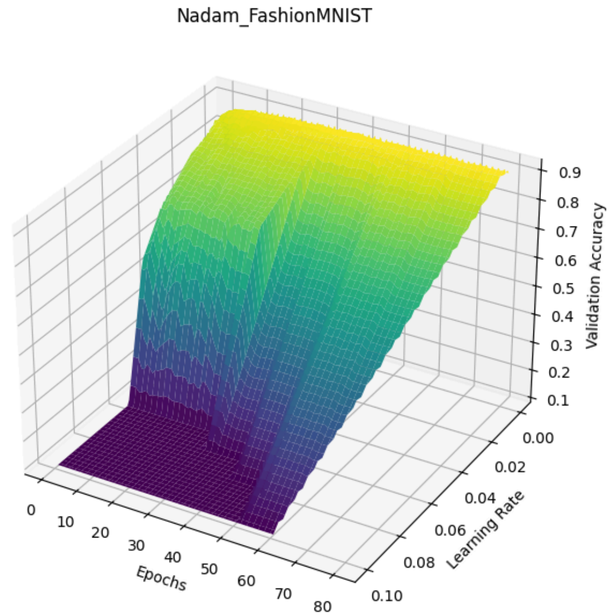
Adam 40	
 	

Nadam 40	
 	

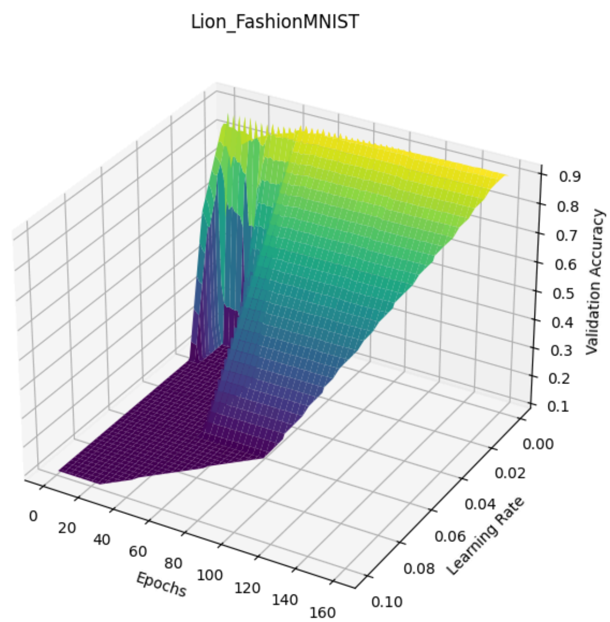
80 Epochs

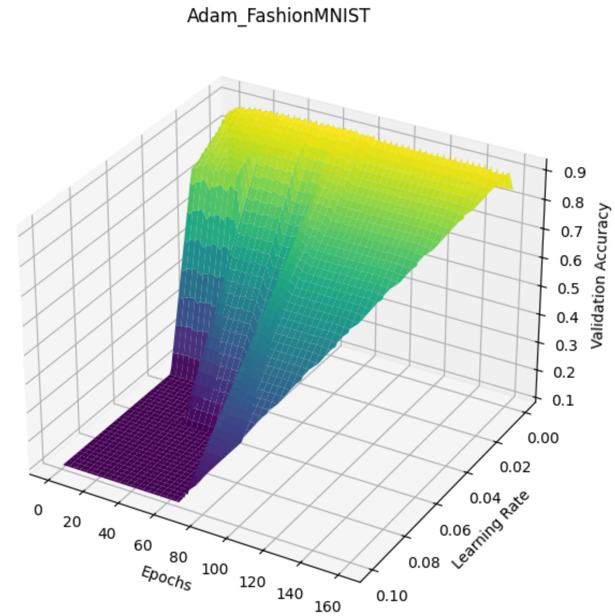
Lion 80	
	

Adam 80	
	

Nadam 80	
	

160 Epochs

Lion 160	
	

Adam 160	
	

RMSprop 160