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SCUOLA DI INGEGNERIA INDUSTRIALE
E DELL'INFORMAZIONE

Asset Management - Assignment

Computational Finance Course
Mathematical Engineering - Ingegneria Matematica

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Contents

1 | Part A

In Part A, portfolio allocation strategies are analyzed using prices from **01/01/2023** to **31/12/2023**. The analysis focuses on constructing efficient frontiers under a range of constraints and computing optimal portfolios, including those enhanced by resampling and the Black-Litterman model, to assess risk-return characteristics and performance metrics.

1.1. Question 1

1.1.1. Efficient frontier:

The analysis begins with the construction of the Efficient Frontier (Figure 1.1), which represents the set of portfolios achieving optimal risk-return combinations under specified foundational constraints. Modern Portfolio Theory, combined with numerical optimization techniques, is employed to ensure that portfolio weights $\mathbf{w_i}$ satisfy the following standard conditions:

$$\sum_{i=1}^{N} \mathbf{w_i} = 1, \quad 0 \leq \mathbf{w_i} \leq 1, \quad \forall i \in [1, \ldots, N].$$

Within this optimal frontier, two portfolios are selected according to distinct criteria:

- **Portfolio A**, which minimizes variance to achieve the lowest possible risk level.
- **Portfolio B**, which maximizes the Sharpe ratio, improving the return per unit of risk.

Both portfolios are subject to a no-short-sale restriction, ensuring non-negative asset weights. This initial construction of the Efficient Frontier serves as a foundational benchmark, facilitating systematic portfolio analysis under progressively complex constraints.

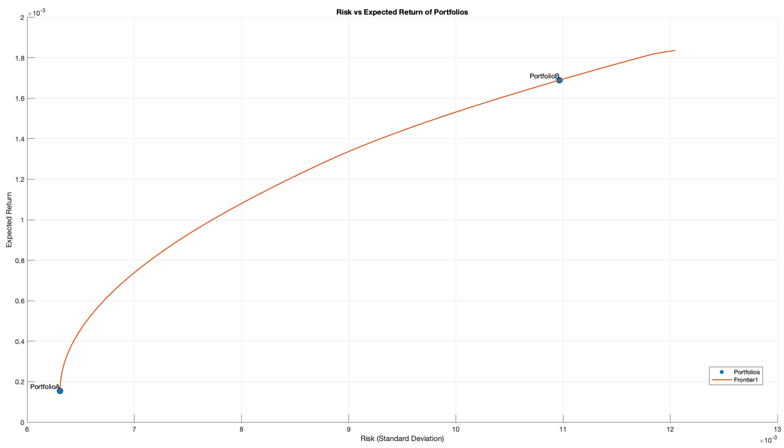

Figure 1.1: Efficient frontier in *red*

1.1.2. Portfolio A :

Portfolio A corresponds to the minimum-variance portfolio (MVP) on the Efficient Frontier (EF). The MVP weights, $\mathbf{w}_{\text{MVP}}$, are defined by:

$$\mathbf{w}_{\text{MVP}} = \arg \min_{\mathbf{w} \in EF} \left\{ \sigma_{\mathbf{w}}^2 \right\}$$

where $\mathbf{w}$ denotes the weight vector of the portfolio and $\sigma_{\mathbf{w}}^2$ represents the variance of the portfolio. This formulation identifies the portfolio with the lowest achievable risk within the constraints of the Efficient Frontier.

1.1.3. Portfolio B :

Portfolio B corresponds to the portfolio on the Efficient Frontier (EF) that maximizes the Sharpe ratio (SR). The weights for the maximum Sharpe ratio portfolio, $\mathbf{w}_{\text{MSR}}$, are given by:

$$\mathbf{w}_{\text{MSR}} = \arg \max_{\mathbf{w} \in EF} \{ SR(\mathbf{w}) \}$$

where $\mathbf{w}$ denotes the portfolio weight vector, and $SR(\mathbf{w})$ represents the Sharpe Ratio as a function of $\mathbf{w}$. This formulation identifies the portfolio with the highest risk-adjusted return achievable under the constraints of the Efficient Frontier.

The following presents the sector allocations for investment portfolios A and B, as illustrated in Figure 1.2.

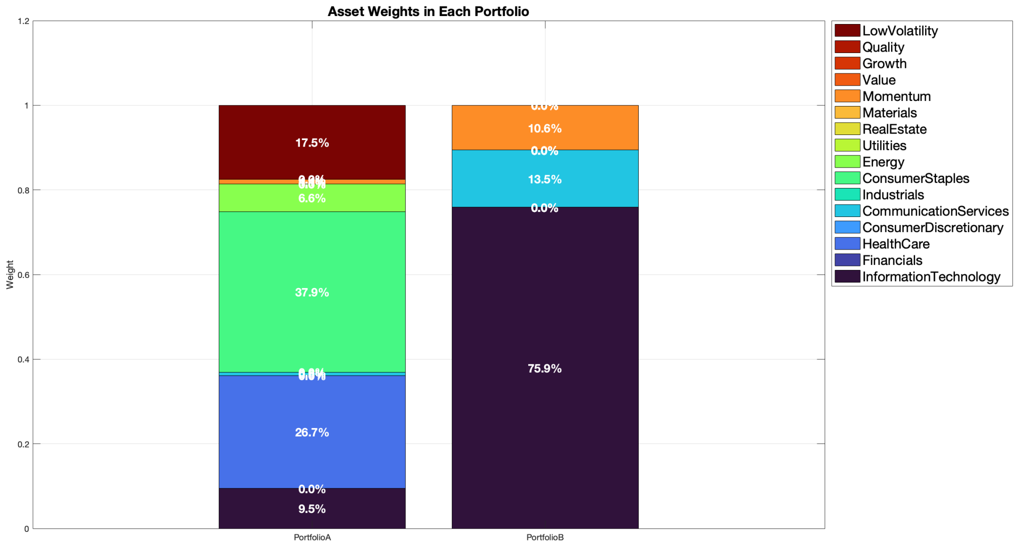

Figure 1.2: Weights of portfolios A & B

1.2. Question 2

The Efficient Frontier (EF) is recalculated under a comprehensive set of additional constraints, applied simultaneously as follows:

1. **Standard Constraints**: The portfolio weights must satisfy the basic conditions $\sum_{i=1}^{N} \mathbf{w_i} = 1$ and $0 \leq \mathbf{w_i} \leq 1$ for all assets $i \in [1, \dots, N]$.
2. **Sectoral Exposure Constraints**:
   - The total exposure to sensitive sectors is restricted to be less than 50%.
   - The defensive sector must represent at least 30% of the portfolio.
3. **Volatility Constraints on High-Risk Sectors**: Exposure to the most volatile sectors, specifically Communication Services and Energy, is restricted within the bounds $0.05 \leq \sigma_{\mathbf{i}} \leq 0.1$.
4. **Balance Between Cyclical and Defensive Sectors**: The total allocation to the cyclical sector is required to be equal to the allocation to defensive sector.

With these constraints in place, the Efficient Frontier yields two optimized portfolios that meet the specified criteria:

- **Portfolio C**: The portfolio on this constrained frontier that minimizes variance, representing the new Minimum-Variance Portfolio (MVP).
- **Portfolio D**: The portfolio that maximizes the Sharpe Ratio, representing the Maximum Sharpe Ratio Portfolio (MSR) within the additional sectoral and volatility constraints.

These portfolios, computed under the specified constraints, provide enhanced control over sectoral exposure and risk, ensuring adherence to predefined sectoral and volatility thresholds.

Sector allocations for investment portfolios C and D are illustrated in Figure 1.3.

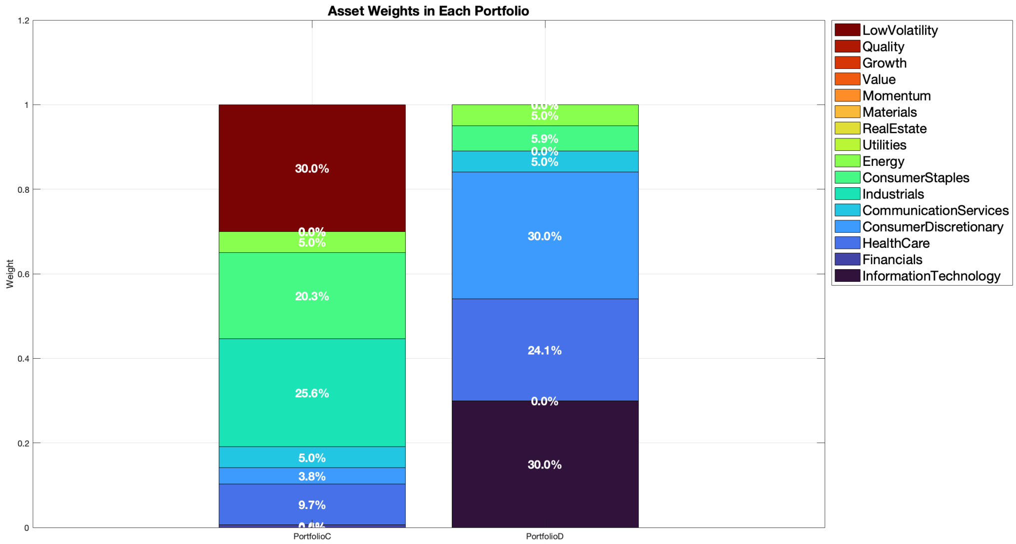

Figure 1.3: Weights of portfolios C & D

1.3. Question 3

The Efficient Frontiers from steps 1 and 2 are recalculated using the resampling method to enhance robustness. This approach involves repeatedly simulating the optimization process by generating multiple sets of expected returns and covariance matrices based on the original data. For each simulation, an optimal portfolio is calculated, and the final frontier is derived as the mean of these individual frontiers. This process yields two resampled frontiers, each providing improved stability in portfolio allocation by reducing sensitivity to estimation errors.

From each resampled frontier, two key portfolios are saved:

- **Portfolio E** and **Portfolio F**: Representing the Minimum Variance Portfolios (MVPs) for each resampled frontier.
- **Portfolio G** and **Portfolio H**: Representing the Maximum Sharpe Ratio Portfolios (MSRs) for each resampled frontier.

These robust portfolios are designed to mitigate estimation error and improve the resilience of optimal allocations.

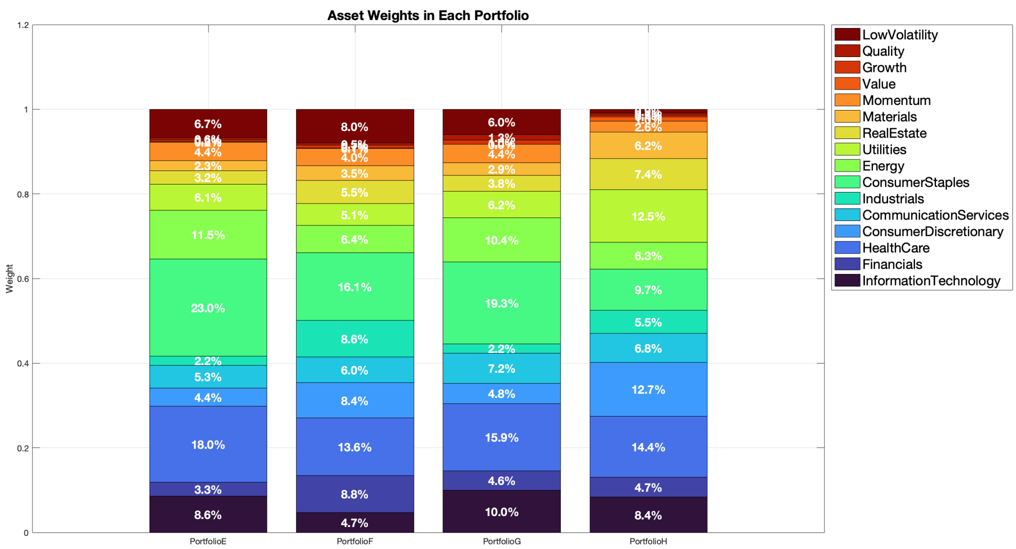

Figure 1.4: Weights of portfolios E, F, G & H

1.4. Question 4

The portfolio frontier is further recalculated using the Black-Litterman model, incorporating specific views on market performance. This model allows the integration of subjective market expectations with historical data to adjust the expected returns of the assets. The following views are applied:

1. **View on Performance of Energy Sector**: In a scenario of rising commodity prices, the Energy sector is assumed to have better prospects relative to sectors that are more exposed to commodity costs. This view is incorporated by assuming an annual performance of 3% for the Energy sector.

2. **View on the Momentum vs. Quality factor**: During periods of economic instability, it is expected that the Quality factor will outperform the Momentum factor, as investors tend to favor companies with strong fundamentals. This view assumes an annual overperformance of 1% for the Quality factor relative to Momentum.

Under these views, the Black-Litterman model is applied to adjust the expected returns, and the portfolio frontier is recalculated. From this adjusted frontier, two portfolios are identified:

- **Portfolio I**: The Minimum Variance Portfolio (MVP) under the Black-Litterman adjusted returns.

- **Portfolio L**: The Maximum Sharpe Ratio Portfolio (MSR) under the Black-Litterman adjusted returns.

These portfolios, constructed with modified expected returns, provide an alternative perspective on optimal allocation, taking into account specific market views.

The sector allocations for investment portfolios I and L are illustrated in Figure 1.5.

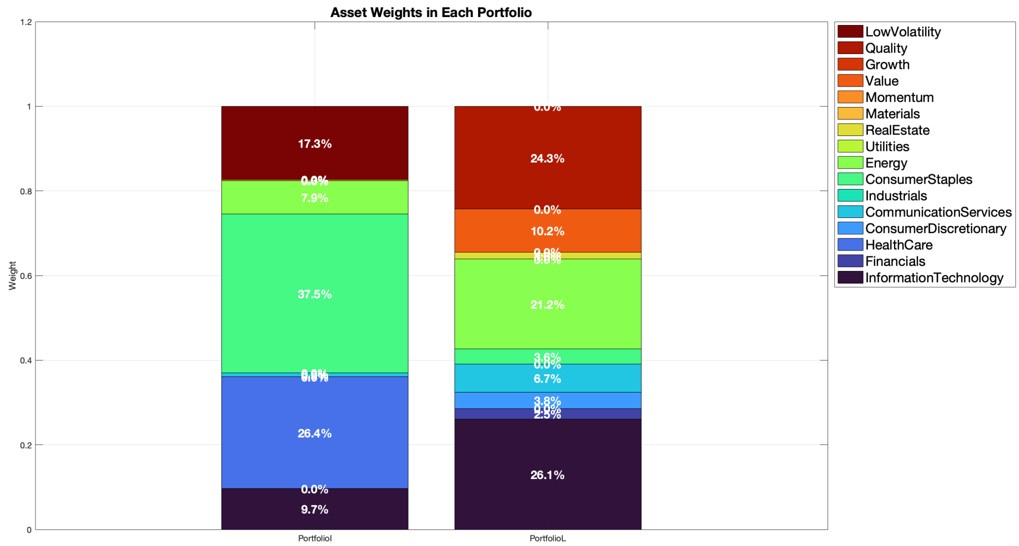

Figure 1.5: Weights of portfolios I & L

1.5. Question 5

This section addresses the construction of two portfolios: the Maximum Diversified Portfolio (Portfolio M) and the Maximum Entropy (in Risk Contributions) Portfolio (Portfolio N). Both portfolios are optimized under specific constraints using the benchmark Capitalization Weighted Portfolio (PortfolioCap) as a reference.

The following constraints are applied to both Portfolio M and Portfolio N:

- **Standard constraints:** The sum of portfolio weights equals 1, and individual weights are bounded between 0 and 1.
- **Defensive sector exposure:** The total weight of assets in the defensive sector is limited to less than 20%.
- **Deviation from benchmark:** The sum of the absolute differences between the weights of PortfolioCap and the optimized portfolio is constrained to exactly 30%.

Portfolio M aims to maximize the diversification ratio, which is defined as the ratio of the weighted sum of individual asset volatilities to the portfolio volatility. Portfolio N, on the other hand, seeks to maximize the entropy of risk contributions, promoting an even distribution of risk among portfolio assets.

PortfolioCap serves as the benchmark, achieving the highest annual return (22.71%)

but the lowest diversification ratio (1.17). This portfolio is heavily weighted by market capitalization, resulting in a concentration of risk.

PortfolioM achieves a diversification ratio of 1.30, the highest among the three portfolios. While its return (18.76%) and volatility (12.07%) are slightly lower than PortfolioCap, it offers better risk dispersion. The optimization process ensures that PortfolioM satisfies the benchmark deviation constraint of 30% while limiting defensive sector exposure.

PortfolioN prioritizes a balanced distribution of risk, achieving the highest entropy of risk contributions (2.60). It delivers a competitive return (18.99%) and a slightly higher volatility (12.16%) compared to PortfolioM. This portfolio satisfies all constraints, demonstrating its suitability for investors seeking even risk contributions.

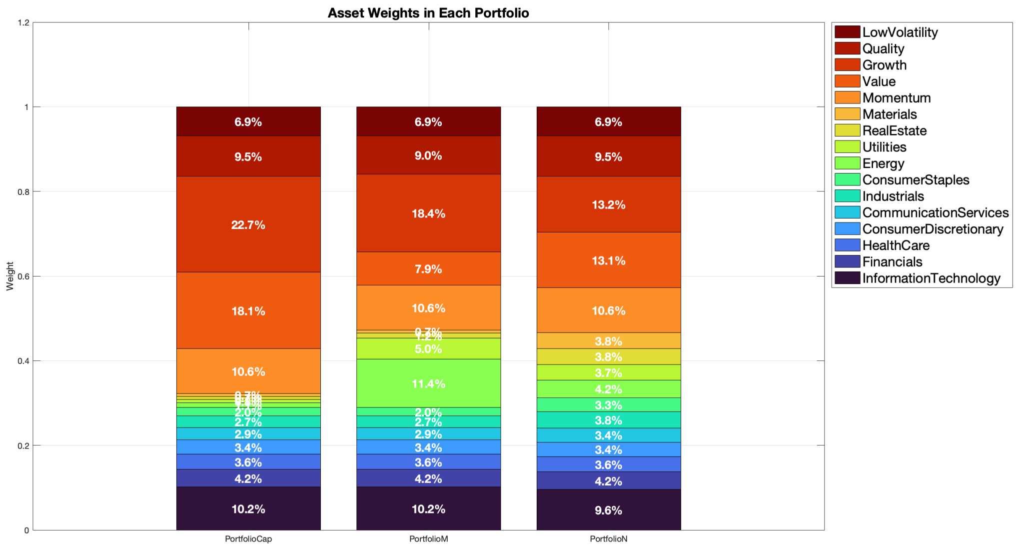

Figure 1.6: Weights of portfolios Cap, M & N

1.6. Question 6

This section focuses on constructing a portfolio (Portfolio P) using **Principal Component Analysis (PCA)** to maximize expected return under specific constraints. The following constraints were applied:

- **Standard constraints:** The sum of portfolio weights equals 1, and individual weights are bounded between 0 and 1.
- **Volatility constraint:** The portfolio volatility is restricted to a target value of $\sigma_{\text{tgt}} = 0.75$.

- **Dimensionality reduction:** Only the minimum number of factors explaining more than 95% of cumulative variance were considered.

PCA was applied to the standardized log returns. The eigenvalues of the covariance matrix were sorted in descending order, and the cumulative explained variance was computed. The number of principal components required to surpass the 95% threshold was determined equal to 9, explaining 96,20% of the variance. Subsequently, the log returns were reconstructed using these principal components, and the covariance matrix of the asset returns was derived.

Despite adjustments to the optimization process, issues with convergence were noted, particularly when applying PCA. Testing with 1000 iterations with random initial point suggested that the identified solution is a global minimum. Additionally, the volatility constraint appears to have limited impact, as the optimization naturally satisfies the threshold by selecting the asset with the highest expected return.

1.7. Question 7

In this section, we intend to compute the portfolio maximizing the Expected Shortfall-modified Sharpe Ratio (ESSR) using the historical method, **Portfolio Q**. This approach uses historical data to estimate the portfolio return distribution and the Expected Shortfall (ES), which replaces standard deviation as the risk measure in the Sharpe Ratio formula.

The ESSR is defined as:

$$\mathbf{ESSR} = \frac{\mathbf{R_p - R_f}}{\mathbf{ES}_\alpha}$$

where R_p is the portfolio's expected return, R_f is the risk-free rate, and ES_α is the Expected Shortfall at confidence level α. The ES is calculated as:

$$\mathbf{ES}_\alpha = -\mathbb{E}[\mathbf{R} \mid \mathbf{R} \leq \mathbf{VaR}_\alpha]$$

where VaR_α is the Value at Risk.

The historical method computes VaR_α by sorting historical returns and identifying the threshold corresponding to the $(1-\alpha)$-quantile, and ES_α as the average of returns below this threshold. This method is particularly advantageous as it does not assume a specific distribution for returns, making it robust and well-suited to real-world financial data.

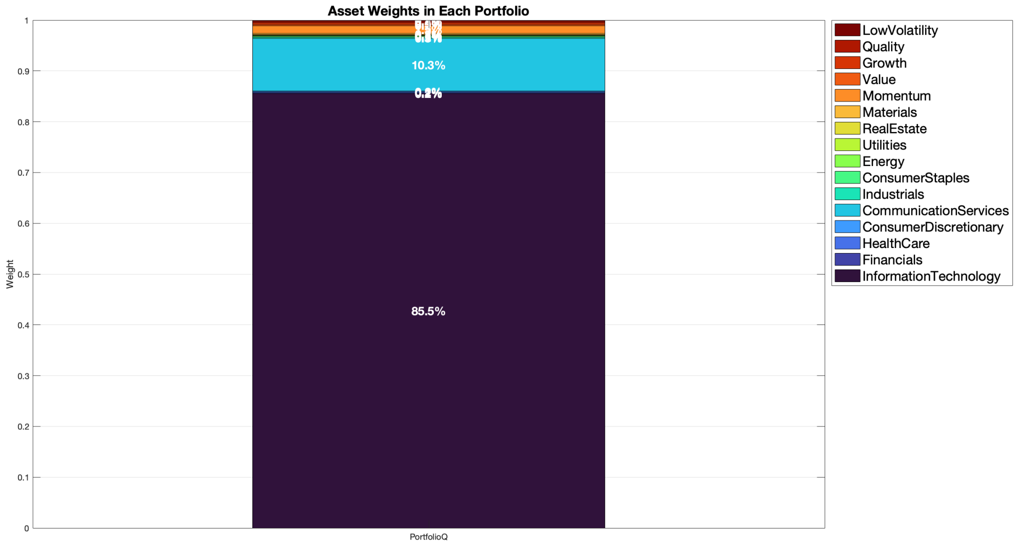

Figure 1.7: Weights of portfolio Q

1.8. Question 8

PTF	AnnRet	AnnVol	SR	MDD	Calmar	Entropy
A	0.0405	0.1001	0.4047	-0.0965	0.4199	1.5157
B	0.5300	0.1746	**3.0400**	-0.0936	**5.6624**	0.5575
C	0.0632	0.1077	0.5868	-0.1031	0.6132	1.7142
D	0.3047	0.1391	2.1908	-0.1006	3.0294	1.3445
E	0.0742	0.1067	0.6957	-0.0935	0.7940	2.4426
F	0.0981	0.1149	0.8543	-0.1029	0.9541	2.5440
G	0.1259	0.1190	1.0582	-0.0960	1.3119	2.4475
H	0.1306	0.1200	1.0881	-0.1013	1.2888	2.4894
I	0.0407	0.1002	0.4062	-0.0959	0.4244	1.5120
L	0.2647	0.1356	1.9529	**-0.0829**	3.1940	1.7868
M	0.1876	0.1207	1.5547	-0.0845	2.2193	2.4810
N	0.1900	0.1217	1.5615	-0.0930	2.0429	2.5974
P	**0.5820**	0.1912	3.0400	-0.1150	5.0600	0.0440
Q	**0.5582**	**0.1841**	3.0322	-0.1052	5.3041	0.4397
Cap	0.2271	0.1237	1.8358	-0.0919	2.4703	2.2722
EW	0.1536	0.1216	1.2632	-0.0981	1.5647	**2.7509**

Table 1.1: Performance, Risk, and Diversification Metrics for Portfolios

The portfolios constructed in steps 1–7 demonstrate a range of performance, risk, and diversification characteristics. This analysis highlights the trade-offs between return and risk, considering diversification as a key factor. In this sense, we include an **Equally Weighted Portfolio (EW)** (which maximizes the entropy) for comparison.

- **Performance**: The Maximum Sharpe Ratio portfolios (B, D, G, H, L, and Q) deliver the highest annual returns, with **Portfolio Q**, which maximizes the Expected Shortfall-modified Sharpe Ratio, achieving the highest return (55.82%), only beaten by **Portfolio P** (58.20%) . **Portfolio B** also has a strong return (53.00%). **EW**, with an annual return of 15.36%, underperforms compared to the top-performing portfolios but still offers a decent return relative to the Minimum Variance portfolios like **A** (4.05%) and **C** (6.32%). This is expected, as the Minimum Variance portfolios are primarily designed to minimize risk, which naturally comes at the cost of return. While **EW** outperforms the Minimum Variance Portfolios, it is significantly behind the portfolios with higher Sharpe Ratios, which are more aggressive in their return-seeking strategies.

- **Risk**: The risk metrics reveal that **Portfolio B** has the highest Sharpe Ratio (3.04) (**Portfolio Q** has a SR of 3.03), indicating that its return is well-compensated by its volatility (17.46%). This is in contrast to **Portfolio A**, which offers low volatility (10.01%) but with much lower return (4.05%). **EW**, with a volatility of 12.16%, sits between these extremes, balancing moderate risk with modest return. It has a lower Sharpe Ratio (1.26) than more aggressive portfolios like **B** and **Q**. In terms of **Maximum Drawdown (MDD)**, **EW** has a moderate MDD of -9.81%, which is comparable to **A** (-9.65%), but significantly better than more volatile portfolios like **D** (-10.06%) and **Q** (-10.52%) and **H** (-10.13%).

- **Diversification and Entropy**: **EW** has the highest Entropy value (2.75), reflecting its most diversified allocation, with an equal distribution across all assets. This contrasts with the more concentrated portfolios such as **Portfolio B**, which has a much lower entropy (0.56), indicating a more concentrated risk allocation. **Portfolio N**, which emphasizes diversification (since it was built by maximizing entropy at question 5), has a high entropy of 2.60, showing a balanced risk distribution similar to **EW** but with a higher return. **Portfolio G**, with 2.45 entropy, strikes a balance between strong diversification and performance.

In conclusion, **EW** serves as a well-diversified, relatively low-risk benchmark, providing solid diversification benefits with a moderate return. However, it underperforms in terms of return and risk-adjusted return (Sharpe and Calmar Ratios) compared to more focused portfolios like **B** and **Q**. The **Maximum Sharpe Ratio portfolios B and Q** offer the highest returns but come with increased volatility. Meanwhile, the **Minimum Variance portfolios A, C, E, F and I** offer lower return, lower risk with more diversification, making them more appealing for risk-averse investors. The **Maximum Diversified Portfolio (N)** also offers better performance than **EW**, for practically the same entropy (**EW** achieves the global entropy maximum, by construction). Now that all those portfolios are built, one can choose the one he prefers based on his/her goals, i.e the amount of risk he/she is willing to take to receive a certain return.

2 | Part B

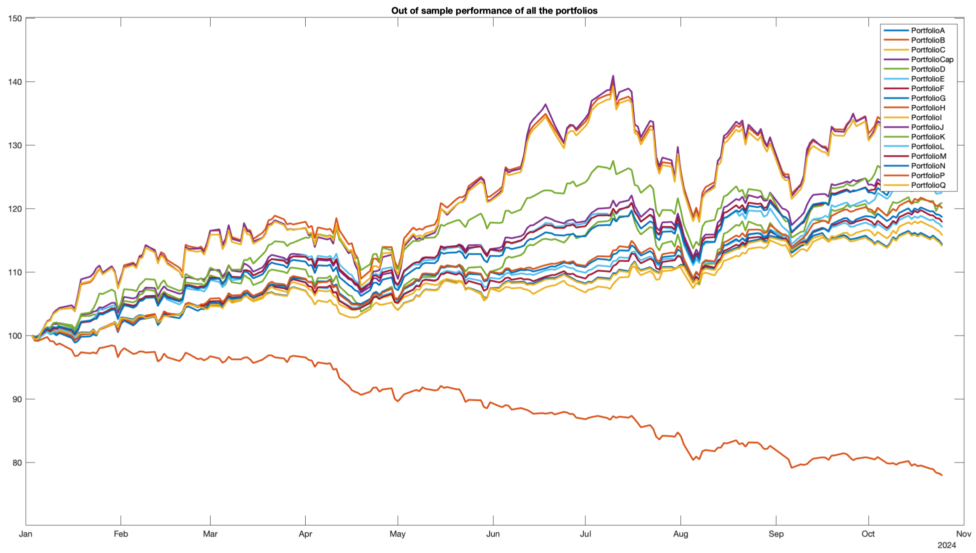

Figure 2.1: Out of sample performance of the portfolios

Figure 2.1 shows the trend of the **Portfolios** analyzed from **question 1** to **question 7**. We can notice how all portfolios, except for **PCA** one follow similar trends.

PTF	AnnRet	AnnVol	SR	MDD	Calmar
A	0.1776	0.0811	2.1882	-0.0445	3.9858
B	**0.4823**	0.2104	2.2927	-0.1520	3.1728
C	0.1959	0.0865	2.2649	**-0.0404**	4.8470
D	0.2613	0.1352	1.9321	-0.0967	2.7024
E	0.2121	0.0862	2.4612	-0.0411	**5.1597**
F	0.2224	0.0920	2.4166	-0.0434	5.1293
G	0.2322	0.0958	2.4239	-0.0465	4.9911
H	0.2503	0.0994	2.5165	-0.0514	4.8696
I	0.1743	0.0807	2.1588	-0.0440	3,9614
L	0.2815	0.1228	2.2925	-0.0815	3.4517
M	0.3063	0.1163	2.6330	-0.0751	4.0753
N	0.2982	0.1131	**2.6358**	-0.0691	4.3152
P	0.3814	0.1536	2.48	-0.0810	4.71
Q	0.4724	**0.2139**	2.2088	-0.1537	3.0740

Table 2.1: Metrics for Portfolios under latest market conditions

By comparing the values in table 2.1 against those in table 1.1, we can notice how portfolios **B** and **Q** performances worsen: They are the least diversified, as we can see from their entropy metric, in fact they cannot maintain their effectiveness under mutated market conditions. In particular, they both are highly exposed to **Information Technology** sector, with weights respectively 0.7591 and 0.8552. They do follow patterns similar to the other portfolio's, but their oscillations are much more significant (2.1). In particular, portfolio **B** has the highest Sharpe Ratio in the sample, whereas in the latest observations it is outperformed by several other portfolios, in terms of this metric. Calmar ratio dramatically drops for **B** and **Q**, showing the occurrence of relevant down-drops after the sample time interval.

Portfolios **D** and **L** show very similar values of the metrics in the two time intervals. They both maximize the Sharpe Ratio, like **B**, but under exposure constraints and Black Litterman model, respectively. They result more diversificated and less exposed to the most volatile sectors.

All other Portfolios present a better performance, as we can see in table 2.1, their trend is overall positive, with moderate oscillations. Annual returns grow, whereas annual volatilities slightly decrease. Sharpe ratios and Calmar ratios improve remarkably.

Table 2.2: Recap of Portfolio Weights by Asset Class

Asset Class	A	B	C	D	E	F	G	H	I	L	M	N	P	Q	Cap
InformationTechnology	9.5%	75.9%	0.0%	30.0%	6.8%	4.8%	9.9%	6.6%	9.7%	26.1%	10.2%	9.6%	99.1%	76.8%	10.2%
Financials	0.0%	0.0%	0.6%	0.0%	3.8%	5.2%	4.2%	3.8%	0.0%	2.5%	4.2%	4.2%	0.0%	0.7%	4.2%
HealthCare	26.7%	0.0%	9.7%	24.1%	17.4%	13.9%	8.6%	10.5%	26.4%	0.0%	3.6%	3.6%	0.0%	0.6%	3.6%
ConsumerDiscretionary	0.0%	0.0%	3.8%	30.0%	4.4%	9.4%	6.7%	10.7%	0.0%	3.8%	3.4%	3.4%	0.1%	0.8%	3.4%
CommunicationServices	0.8%	13.5%	5.0%	5.0%	7.6%	6.0%	11.9%	6.3%	0.9%	6.7%	2.9%	3.4%	0.4%	9.2%	2.9%
Industrials	0.0%	0.0%	25.6%	0.0%	3.0%	6.2%	3.0%	5.8%	0.0%	0.0%	2.7%	3.8%	0.0%	0.9%	2.7%
ConsumerStaples	37.9%	0.0%	20.3%	5.9%	24.1%	13.7%	10.7%	11.4%	37.5%	3.6%	2.0%	3.3%	0.0%	0.8%	2.0%
Energy	6.6%	0.0%	5.0%	5.0%	10.6%	6.7%	15.6%	6.8%	7.9%	21.2%	11.5%	4.2%	0.0%	0.6%	1.1%
Utilities	0.0%	0.0%	0.0%	0.0%	6.2%	6.5%	7.4%	14.3%	0.0%	0.0%	4.9%	3.7%	0.0%	0.4%	0.8%
RealEstate	0.0%	0.0%	0.0%	0.0%	3.7%	6.0%	11.4%	9.6%	0.0%	1.6%	1.2%	3.7%	0.0%	0.5%	0.7%
Materials	0.0%	0.0%	0.0%	0.0%	2.0%	7.3%	6.4%	6.4%	0.0%	0.0%	0.7%	3.8%	0.0%	0.7%	0.7%
Momentum	1.1%	10.6%	0.0%	0.0%	3.1%	5.0%	2.7%	4.7%	0.3%	0.0%	10.6%	10.6%	0.0%	3.7%	10.6%
Value	0.0%	0.0%	0.0%	0.0%	0.2%	0.3%	0.0%	0.1%	0.0%	10.2%	8.2%	13.1%	0.0%	0.9%	18.1%
Growth	0.0%	0.0%	0.0%	0.0%	0.2%	0.7%	0.0%	0.2%	0.0%	0.0%	19.2%	13.2%	0.1%	1.3%	22.7%
Quality	0.0%	0.0%	0.0%	0.0%	1.9%	0.8%	0.0%	0.3%	0.0%	24.3%	7.8%	9.5%	0.0%	1.4%	9.5%
LowVolatility	17.5%	0.0%	30.0%	0.0%	5.0%	7.4%	1.5%	2.7%	17.3%	0.0%	6.9%	6.9%	0.0%	0.8%	6.9%