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MILANO 1863

SCUOLA DI INGEGNERIA INDUSTRIALE
E DELL'INFORMAZIONE

Commodity - Assignment

Computational Finance Course
Mathematical Engineering - Ingegneria Matematica

Author: **Jacopo Raffaeli - Gabriele Radaelli - Alexis Martin - Noé Debrois - Jan Bartolini**

Student ID: 10794677 - 10678299 - 10987844 - 10949145 - 10768838
Advisor: Luca Russo
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1 | Introduction

We are tasked with calibrating an HJM model on French electricity swaps and pricing structured pay-off options using Monte Carlo simulation. The `DATA_FREEX.xlsx` file contains liquid maturities for French power futures and implied volatility quotes for options on the 2026 and 2028 futures as of 4th November 2024.

The HJM model is used to describe the evolution of forward prices. According to **Benth 2008 (6.9)** the forward price dynamics are given by:

$$
\begin{aligned}
F(t,\tau_1,\tau_2) = F(0,\tau_1,\tau_2)\exp\Bigg(\int_0^t A(u,\tau_1,\tau_2)\,du \\
+\sum_{k=1}^{p}\int_0^t \Sigma_k(u,\tau_1,\tau_2)\,dW_k(u) \\
+\sum_{j=1}^{n}\int_0^t \Upsilon_j(u,\tau_1,\tau_2)\,dJ_j(u)\Bigg)
\end{aligned}
$$

where:

- $A(u,\tau_1,\tau_2)$: Drift term.
- $\Sigma_k(u,\tau_1,\tau_2)$: Volatility of the Wiener processes $W_k(u)$.
- $\Upsilon_j(u,\tau_1,\tau_2)$: Coefficients of the jump processes $J_j(u)$.
- $0 \le t \le \tau_1 \le \tau_2 \le \tau$

For the given problem, according to the assignment requests, we simplify the model by setting $p = 0$, $n = 1$, and assuming a constant $\Upsilon(t,\tau_1,\tau_2) = \Upsilon$. The simplified dynamics become:

$$
F(t,\tau_1,\tau_2) = F(0,\tau_1,\tau_2)\exp\left(\int_0^t A(u,\tau_1,\tau_2)\,du + \Upsilon J_t\right)
$$

where $J_t = X_{S_t} = \theta S_t + \sigma W_{S_t}$ is a Normal Inverse Gaussian process of parameters (σ, θ, κ), obtained by the subordination of a drifted Brownian Motion $X_t = \theta t + \sigma W_t$ with an Inverse Gaussian subordinator S_t.

The characteristic exponent of J_t is $\psi_{J_t}(u) = \frac{1}{\kappa}(1 - \sqrt{1 + u^2\sigma^2\kappa - 2i\theta u\kappa})$

1.1. Admissible Range for Model Parameters

The admissible parameters for the HJM NIG model, ensuring arbitrage-free pricing and well-defined dynamics, are:

- **NIG process**:

$$\begin{aligned} \sigma &> 0 \\ \theta &\in \mathbb{R} \\ \kappa &> 0 \end{aligned}$$

- **Scaling Coefficient** (Υ): $\Upsilon \in \mathbb{R}$.
- **Drift Term** ($A(t, \tau_1, \tau_2)$): Determined by the martingality condition.
- **Initial value** ($F(0, \tau_1, \tau_2)$): $F(0, \tau_1, \tau_2) > 0$.

1.2. Martingale Condition

To ensure arbitrage-free pricing, the forward price $F(t, \tau_1, \tau_2)$ must be a martingale under the risk-neutral measure Q. The martingale condition requires:

$$\mathbb{E}^Q[F(t, \tau_1, \tau_2) \mid \mathcal{F}_s] = F(s, \tau_1, \tau_2), \quad \forall s \leq t.$$

Alternatively, considering the characteristic exponent of J_t, we impose the following condition on the drift:

$$\int_0^t A(u, \tau_1, \tau_2)\, du = -\psi_{J_t}(-i\Upsilon)t = -\psi_{\tilde{J}_t}(-i)t = -\frac{1}{\kappa}(1 - \sqrt{1 - \Upsilon^2\sigma^2\kappa - 2i\theta\Upsilon\kappa})t$$

Where $\tilde{J}_t = \Upsilon J_t$ is a NIG process of parameters $(\Upsilon\sigma, \Upsilon\theta, \kappa)$. In the calibration process, in the Carr-Madan formula for EU Call options we used $\tilde{J}_t$ and its characteristic exponent $\psi_{J_t}(u)$ for simplicity, keeping the estimation of Υ separated from σ and θ respectively.

Considering the martingality condition we have the following forward price dynamics:

$$\begin{aligned} F(t,\tau_1,\tau_2) &= F(0,\tau_1,\tau_2)\exp(-\psi_{J_t}(-i\Upsilon)t + \Upsilon J_t) \\ &= F(0,\tau_1,\tau_2)\exp(-\frac{1}{\kappa}(1-\sqrt{1-\Upsilon^2\sigma^2\kappa - 2i\theta\Upsilon\kappa})t + \Upsilon J_t) \end{aligned}$$

1.3. Model Calibration on 2026 prices

The calibration was conducted using the entire volatility surface of 2026 French option prices. Market prices were derived from the implied volatility using the Black-Scholes formula, which served as a benchmark for the calibration. The set of calibrated parameters was $[\sigma,\ \theta,\ \kappa,\ \Upsilon]$.

The calibration was performed with `lsqnonlin.m` on the differences between market and model prices computed by `price_err.m`, several trials were made in order to asses the ideal search space for the parameter set. To evaluate the quality of the calibrations made `MAE (Mean Absolute Error)` metric was used.

During the development of the model we noticed that it was struggling to correctly fit the market data, in particular the second half of the price surface with respect to strikes was not correctly reproduced as showed in 1.1. Looking at the implied volatility surfaces in 1.4 we observed a substantially flat behavior for model implied volatility. We concluded that possibly the dynamic of the model is way too simple to capture the correct behavior of market prices.

σ	θ	κ	Υ	MAE
0.2475	0.4066	3.0462	0.3921	15.10$

Table 1.1: Calibrated parameters for HJM NIG model on 2026 data

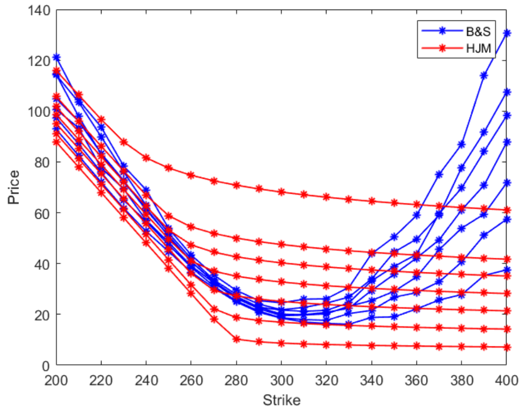

Figure 1.1: Price vs Strike

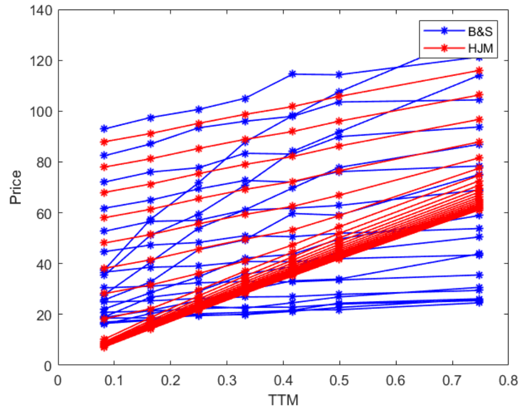

Figure 1.2: Price vs TTM

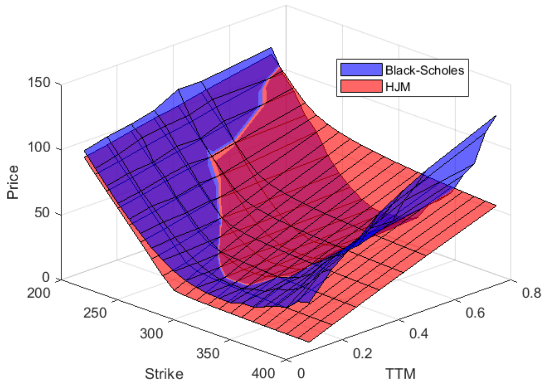

Figure 1.3: Price surface

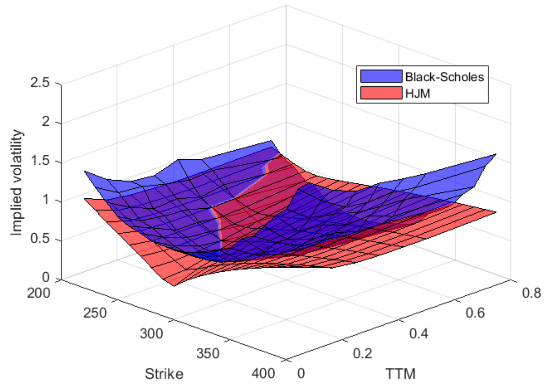

Figure 1.4: Volatility surface

To assess this issue we propose two possible solutions.

We assigned different combination of weights and losses to the `price_error.m` function, in order to take into consideration specific features of the prices surface, however we do not investigate deeply this approach since it only leaded to minimum local improvements at the price of loss of generality for the model.

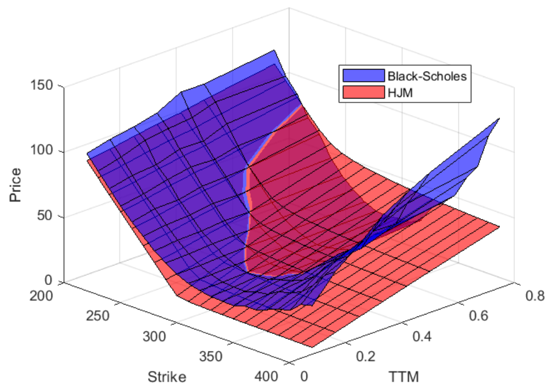

Figure 1.5: Price surface - Weights inverse proportional to market implied volatility - $MAE = 14.87\$$

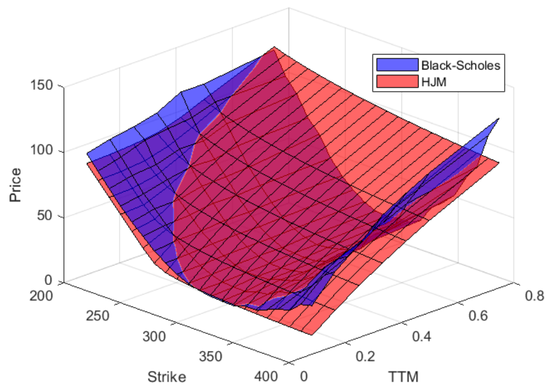

Figure 1.6: Price surface - Weights proportional to distance from ATM - $MAE = 15.4\$6$

We also tried to increase the model complexity (similarly to what is discussed in **Benth 2008 6.4.1**) allowing Υ to vary with tenors, so that each maturity can be calibrated with its own factor $\{\Upsilon_i\}_{i=1}^{7}$. This approach lead to more significant improvements, specifically observed in the volatility surface. The problem is that, allowing Υ to vary in this way, introduce a different risk neutral condition on the drift for each maturity considered, leading to important inconsistency in the model formulation. Moreover the increasing complexity (From 4 to 10 parameters) was too big to justify such improvements. Because of this we decided to avoid using this results too.

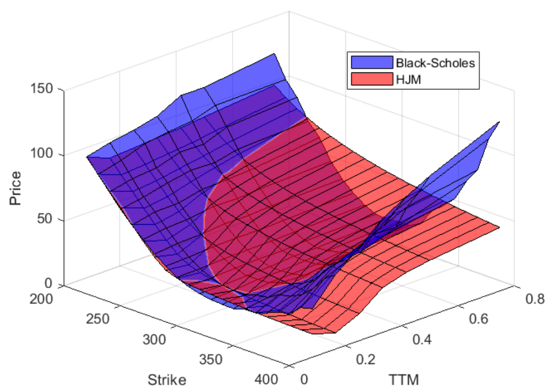

Figure 1.7: Price surface

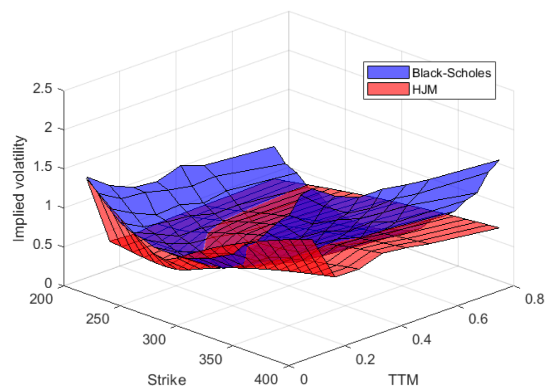

Figure 1.8: Volatility surface

1.4. Option Pricing 1

In order to price an option with payoff $[(\max_{t\in[0,T]} F(t,\tau_1,\tau_2)) - K]^+$ $T = 1,$ $K = 300,$ $\tau_1 = 1^{st}$of January of 2026, $\tau_2 = 31^{th}$of December 2026, we have simulated the NIG process J_t as showed in `option_26.m` both using a Monte Carlo and an Antithetic Variable approach. The results are showed in 1.2, however, possibly because of the poor calibration precision, the process $F(t,\tau_1,\tau_2)$ does not present a perfect martingale behavior and for this reason the price are affected by a certain degree of error. Moreover since it does not exist a closed formula for the option under J_t we do not have a benchmark to evaluate the accuracy of our prices.

	Price	CI
MC	50.64$	±9.45$
AV	42.74$	±3.12$

Table 1.2: MC and AV prices for the 2026 option

1.5. Model Calibration on 2026 & 2028 prices

The calibration on both the volatility surfaces of 2026 and 2028 at once presented the same criticalities as the one described in question 1.3, exacerbated by the presence of more data distributed among two different delivery periods.

σ	θ	κ	Υ	MAE
0.7745	-0.6288	5.0000	1.6101	21.65$

Table 1.3: Calibrated parameters for HJM NIG model on 2026 & 2028 data

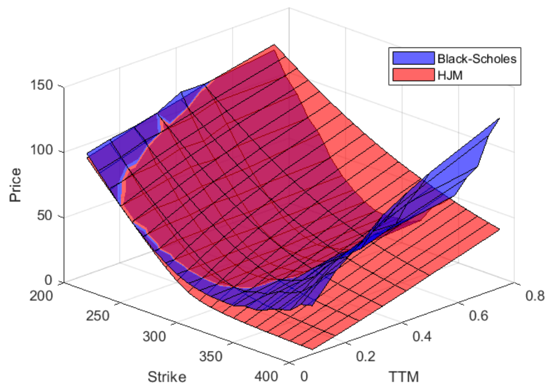

Figure 1.9: Price surface 2026

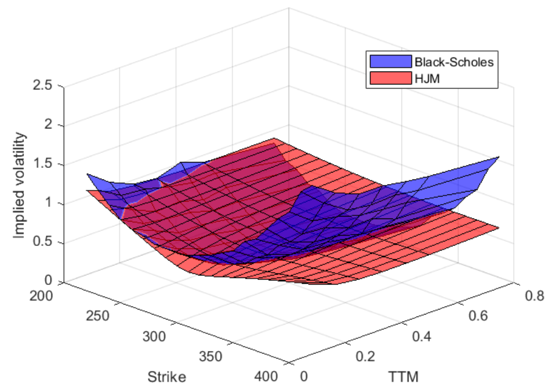

Figure 1.10: Volatility surface 2026

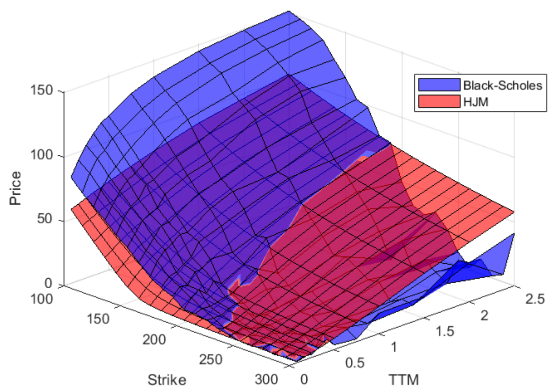

Figure 1.11: Price surface 2028

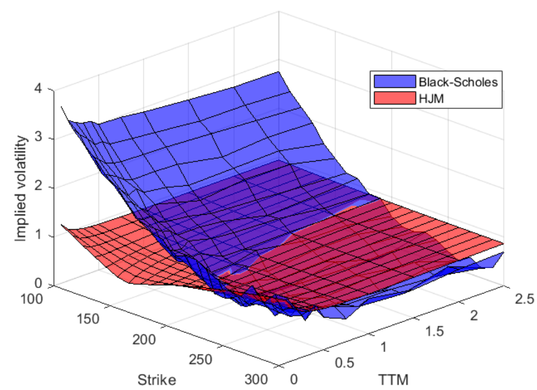

Figure 1.12: Volatility surface 2028

Also in this case the same approaches presented in point 3 were implemented to obtain a better calibration with similar results. In addition we also tried to let Υ vary with respect to the delivery period (τ_1, τ_2) without significant improvements.

In conclusion both calibrations present heavy criticalities, specifically for ITM options, it seem like the model is not able to capture the correct shape of the price surface. Possible improvements could be obtained by the mean of finite jump diffusion processes or by the addition of a Brownian Motion process (Extended NIG) driven by time dependent volatility parameter.

1.6. Option Pricing 2

In order to price an option with payoff $[\max(F(T,\tau_1,\tau_2), F(T,\tau_3,\tau_4)) - K]^+$ $T = 1,\ K = 300,\ \tau_3 = 1^{st}$of January of 2028, $\tau_4 = 31^{th}$of December 2028, we used an approach similar to question 1.4, described in `option_26_28.m`, also in this case a closed formula is not provided and the process $F(t, \tau_1, \tau_2)$ does not enjoy exact martingality.

	Price	CI
MC	106.53$	±2.57$
AV	109.59$	±1.13$

Table 1.4: MC and AV prices for the 2026 & 2028 option

Conclusion

Commodities market often shows counterintuitive and intricate dynamics that heavily differ from what we are usually observe on other financial markets. For this reason a NIG driven model like the one described in this report possibly does not offer enough complexity and sophistication to capture the electricity market features, while it could behave better on more 'classical' markets calibration and pricing tasks.