Example: on MATLAB ...

- For Merton, for the time of the jumps:

We can generate NT & NTAV OR to take the same jumptimes & change only sizes.

```
u = rand(Nsim,1);
NT = icdf('Poisson', u, ...);
NTAV = icdf('Poisson', 1-u, ...);
```

⚠ Simulate u before! Don't put "1-..." directly inside.

OR, for the size of the jumps:

Here we change jump sizes.

```
Z = randn(1,N);
X(j,i) = X(j,i-1) + mu*dt + sigma*sqrt(dt)*Z(i-1);
XAV(j,i) = XAV(j,i-1) + mu*dt + sigma*sqrt(dt)*(-Z(i-1));
```

- For Kou: we can modify the continuous part:

```
XAV(j,i) = XAV(j,i-1) + mu*dt - sigma*sqrt(dt)*
                                          Z(i-1);
```

⚠

Then, for the ~~jumps~~: when one has positive jump, YAV gets ~~a negative~~ jump; & vice versa.

↳ No, that's an error, because then X and XAV don't have the same distribution anymore.

So here is what we can do: on MATLAB...

**1st Method:**

```
% pos jump:

u = rand;
Y = icdf('Exponential', u, 1/lambdap);
YAV = icdf('Exponential', 1-u, 1/lambdap);
```

negative correlation.

```
% neg jump

u = rand;
Y = -icdf('Exponential', u, 1/lambdam);
YAV = -icdf('Exponential', 1-u, 1/lambdam);

[...]

X(j,i) = X(j,i) + Y;

XAV(j,i) = XAV(j,i) + YAV;
```

**2nd Method:** Just negatively correlate the BM (G° part) & same jumps.

→ Then you can use all that in our MC to perform "Antithetic Variable MC": you compute the two "disc_payoff" and then average the two.

In profile report you can check that this is really ZERO COST : we get better precision with practically no cost.

---

## $2^{nd}$ technique: control variable:

Let's start with $Z$ which is one source of randomness, & $\theta = \mathbb{E}[g(Z)]$.

Example: $Z \sim N(0,1)$ & $g(Z) = e^{-rT}\max\left(S_0 e^{(r-\frac{\sigma^2}{2})T+\sigma\sqrt{T}Z} - K, 0\right)$ (EU call option under B&S).

Assume we have $f$ a fct and we can compute $f(Z)$. And assume that $\mathbb{E}[f(Z)]$ is known : $\mathbb{E}[f(Z)] = \underset{\text{KNOWN}}{\theta_f}$.

- What is the idea? May I exploit the knowledge of $\theta_f$ to improve the computation of $\theta$? YES: instead of studying $g(Z)$ I will study $g(Z) + \alpha\left(f(Z) - \theta_f\right)$.

$f(z) - \theta_f$ is the distance between $f(z)$ & its expected value. <u>Our MC estimator:</u>

$$\hat{\theta} = \frac{1}{n} \sum_{i=1}^{n} \left[ g(z_i) + \alpha \left( f(z_i) - \theta_f \right) \right]$$

If $\alpha = 0$, we have a classical MC.

<u>On the other hand:</u> $\forall \alpha$,

$$\mathbb{E}\left[ g(z) + \alpha \left( f(z) - \theta_f \right) \right] = \mathbb{E}\left[g(z)\right] + \alpha\left(\theta_f - \theta_f\right) = \mathbb{E}\left[g(z)\right] = \theta.$$

So: $\forall \alpha \in \mathbb{R}, \ \hat{\theta} \xrightarrow[n \to +\infty]{} \theta = \mathbb{E}\left[g(z)\right]$.

- <u>What about variance?</u> We would like to make it as small as possible (using $\alpha$ !?).

$$\operatorname{Var}\left( g(z) + \alpha\left(f(z) - \mathbb{E}\left[f(z)\right]\right)\right) = \operatorname{Var}(g(z)) + \alpha^2 \operatorname{Var}(f(z)) + 2\alpha \operatorname{cov}(g(z), f(z)).$$

We want to minimize that wrt $\alpha$ :

$$"\frac{\partial}{\partial \alpha}" : \cancel{2}\alpha \, Var(f(z)) + \cancel{2} cov(g(z), f(z)) = 0$$

$$\Leftrightarrow \boxed{\alpha = \frac{-cov(f(z), g(z))}{Var(f(z))}}$$

. The $\alpha$ we have to use to minimize the variance.

- **Question :** who is $f$ ? $f$ is the CONTROL VARIABLE.

$\longrightarrow$ $\mathbb{E}[f(z)]$ is known.

$\longrightarrow$ We need $cov(f(z), g(z)) \neq 0$.

**Example :** CALL B&S : (EU)

$$e^{-rT}\left(S_0 e^{(r-\frac{\sigma^2}{2})T + \sigma\sqrt{T} z} - K\right)^+ = g(z)$$

$$f(z) = S_0 e^{(r-\sigma^2/2)T + \sigma\sqrt{T} z}$$

So : $g(z) = e^{-rT}\left(f(z) - K\right)^+$ &

$$\mathbb{E}[f(z)] = S_0 e^{rT} \quad \text{(RISK NEUTRAL)}.$$

**Example :** CALL LÉVY : (EU)

If $g(z) = e^{-rT}\left(S_0 e^{rT + X} - K\right)^+$, $X \sim$ Lévy, under $\mathbb{Q}$ measure (RISK NEUTRAL MEASURE). Then, take : $f(z) = S_0 e^X$ : { Discounted price is a MG, under $\mathbb{Q}$.

$$g(z) = e^{-rT}\left(e^{rT} f(z) - K\right)^+ .$$

$$\mathbb{E}[f(z)] = S_0 \qquad (\text{under } \mathbb{Q}).$$

<u>Example:</u> $\boxed{\text{ASIAN OPTION, GENERALLY:}}$

$$g = \left(S(T) - \frac{1}{N}\sum_{i=1}^{N} S(t_i)\right)^+, \quad t_i = i\Delta t, \quad \Delta t = \frac{T}{N}.$$

↳ Call option, floating strike.

<u>Candidate for $f$:</u> $\quad f = S(T) - \frac{1}{N}\sum_{i=1}^{N} S(t_i)$.

So : $g = (f)^+$.

$$\mathbb{E}_{\mathbb{Q}}[f] = e^{rT} S_0 + \frac{1}{N}\sum_{i=1}^{N} S_0 e^{ri\Delta t}$$

Very general (whatever $t \mapsto S(t)$ !).

<u>Example:</u> ⚠ Even in B&S, we don't know $\mathbb{E}[\min_t S(t)]$ ...

1) $g = \left(S(T) - \min_i S(t_i)\right)^+$ $\boxed{\text{LOOKBACK CALL FLOATING STRIKE ON MIN:}}$

2) $g = \left(\max_i S(t_i) - K\right)^+$ $\boxed{\text{FIXED STRIKE LOOKBACK CALL OPTION ON THE MAX}}$

For 1) : $f = S(T)$

For 2) : $f = ??$ ⚠ WE DON'T KNOW.

Example: $\boxed{\text{BARRIER D\&O CALL:}}$

$g = (S(T) - K)^+ \mathbb{1}_{\{\min_i S(t_i) > D\}}$. Choose: $f = S(T)$.

⚠ NOT A GREAT CONTROL VARIABLE

- Last missing point: how to compute $\alpha$ ?

$$\alpha = - \frac{Cov(f(Z), g(Z))}{Var(f(Z))}.$$

We don't know exactly $\alpha$, so we will simulate $\alpha$ but in order not to be too computationally-intense, we'll do a small number of simulations. So usually we use a bad estimation of $\alpha$.

→ Now let's do the code on MATLAB :

see the "VR" .m files. } See Lecture 8 folder.

$\boxed{\text{END OF 30/09/2024 RECORDING}}$