- You can also price a double barrier option.
  - ↳ $x_{min} \longrightarrow$ given by D } The boundaries become
    $x_{max} \longrightarrow$ given by U } the barriers.
  - ↳ Update the payoff: ... $x(S_0 \exp(x) > D).*(S_0 \exp(x) < U)$
  - ↳ The BCs are all equal to 0 (when you have a barrier $\longrightarrow$ BCs = 0).
  - ↳ ⚠ The Lévy Integral also changes: BCs are 0 $\longrightarrow$ fct is 0 outside domain.

$\longrightarrow$ Modify the code with EE, I.E, θ-Method.

So, all this was made splitting the integral. But we can decide not to split the integral. We can maintain the integral on its own and discretize it explicitely.

$\longrightarrow$ cf "... FullintegRal.m".

↳ α & λ are not defined anymore.

↳ so A, B, C are exactly B&S matrizes.

↳ But now we need to change the Levy_Integral.m to "Levy_Integral_Full.m". Now:

$$\underbrace{\left(V(x+y) - V(x) - (\exp(y)-1)V'(x)\right) k(y)\,dy}_{\text{Full integrand we have to consider}}.$$

↳ so, to adapt the code, we need 2 things:
- remove α & λ (from A, B, C & their def);
- Put everything inside J (the integral).

$\underbrace{\phantom{\text{Put everything inside J (the integral)}}}_{\text{Because we don't split } \int \text{ anymore.}}$

Why is it good? Because we can now use it for an ∞-activity Lévy!

⟶ So we can now create a code with an IA-Lévy, like extended VG for ex.
cf: "EU_CALL_ImplicitEuler_FullIntegral_extVG.m"

PARAMETERS:

└→ sigmaGBM, thetaVG, sigmaVG, kVG.

Given pdf during exam: { └→ to simulate (Ext) VG: cf VG_NIG.pdf where we have $\nu(.)$!

To compare our PIDE price we could use CM, so we also have to modify it for one model: cf FFT_CM_Call_VG.m.

You see, when you have a Carr Madan code or a PIDE code that works for any Lévy, things are easy. Just change the input model and then it works. These codes are very general. LEARN HOW TO QUICKLY MODIFY THEM TO SWITCH FROM ONE MODEL TO ANOTHER & ONE OPTION TO ANOTHER.

How can we extend the 1st code we wrote (the one with α & λ) to I.A?

→ Let's go back to the original code & modify it: "EU_CALL_ImplicitEuler_extVG.m".

Goal: to price a European Call Option under

Extended-VG process, with the logprice IDE, using Finite Difference with Implicit Euler.

↳ We use "Asmussen Rosinsky Truncation"

↳ Exactly as before, we have to define parameters + density for our model.
We need another trunc. parameter: epsilon ($=0.1$): all the jumps between -0.1 and $+0.1$ will be deleted.

↳ We create a "Lévy_Trunc_AR(k, N, epsilon)" function: "cf Lévy_Trunc_AR.m".

↳ The truncation does not change, but what about α and λ:

$$\begin{cases} \alpha = \alpha_{(y_{min}, -epsilon)} + \alpha_{(epsilon, y_{max})} \\ \lambda = \lambda_{(y_{min}, -epsilon)} + \lambda_{(epsilon, y_{max})} \end{cases}$$

↳ Then we have to compute sigma12_eps:

⚠ N has to be even.

y = linspace(-epsilon, epsilon, N)

sigma2 = trapz(y, (y.^2).*k(y)).

↳ we have also to update A, B, C:
sigma = sqrt(sigmaGBM^2 + sigma2_eps)

↑ The one we use in A, B, C.

↳ in the computation of the integral: if "Levy_Integral_eps.m", we have also to input epsilon since we have to compute the integral not on (y_{min}, y_{max}) but on

$(y_{min}, -epsilon) \cup (epsilon, y_{max})$.

→ ⚠ If we decrease $\varepsilon \to 0$, because of the problem of stability, the price is BAD.

↳ look at λ: it starts to explode.

Theoretically, it should CVerge, but numerically, it won't. → So don't try to put $\varepsilon \to 0$.

2D Finite Difference :

To price a Barrier option under B&S

Knock out Barrier Call on a Basket of 2 underlying assets.

S_1 : $dS_1 = rS_1\,dt + \sigma_1 S_1\,dW_t^1$

S_2 : $dS_2 = rS_2\,dt + \sigma_2 S_2\,dW_t^2$

Correlation between the 2 BM : $dW_t^1\,dW_t^2 = \rho\,dt$.

We are looking for the value of the derivative :

$$V(t, S_1, S_2)$$

It depends on S_1 & S_2 !

We have barrier :

$$\begin{cases} L_1 \leq S_1 \leq U_1 \\ L_2 \leq S_2 \leq U_2 \end{cases}$$

And we have : Basket of S_1 & S_2 weighted by w_1, w_2.

$$V(T, S_1, S_2) = (w_1 S_1 + w_2 S_2 - K)^+ \underbrace{\mathbb{1}_{\{L_1 \leq S_1(t) \leq U_1 \ \forall t \in [0,T]\}} \mathbb{1}_{\{L_2 \leq S_2(t) \leq U_2 \ \forall t \in [0,T]\}}}_{\text{Barriers !}}$$