and the jump intensity is $\lambda = \nu(\mathbb{R})$. A trajectory of this process can be simulated exactly on the interval $[0, T]$ using the following simple algorithm (which uses the fact that waiting times between the jumps are independent exponentially distributed random variables with parameter λ):

ALGORITHM 6.1 *Simulation of compound Poisson process*

Initialize $k := 0$

REPEAT while $\sum_{i=1}^{k} T_i < T$

Set $k := k + 1$

Simulate $T_k \sim \exp(\lambda)$

Simulate Y_k *from the distribution* $\mu = \nu/\lambda$

The trajectory is given by

$$X(t) = \gamma b + \sum_{i=1}^{N(t)} Y_i \quad \textit{where} \quad N(t) = \sup\{k : \sum_{i=1}^{k} T_i \leq t\}.$$

We will now improve this algorithm using two following observations

- The number of jumps $N(T)$ of a compound Poisson process on the interval $[0, T]$ is a Poisson random variable with parameter λT.
- Conditionally on $N(T)$, the exact moments of jumps on this interval have the same distribution as $N(T)$ independent random numbers, uniformly distributed on this interval, rearranged in increasing order (see Proposition 2.9).

ALGORITHM 6.2 *Improved algorithm for compound Poisson process*

- *Simulate a random variable* N *from Poisson distribution with parameter* λT. N *gives the total number of jumps on the interval* $[0, T]$.
- *Simulate* N *independent r.v.,* U_i*, uniformly distributed on the interval* $[0, T]$. *These variables correspond to the jump times.*
- *Simulate jump sizes:* N *independent r.v.* Y_i *with law* $\frac{\nu(dx)}{\lambda}$.

  The trajectory is given by

$$X(t) = bt + \sum_{i=1}^{N} 1_{U_i < t} Y_i.$$

Figure 6.1 depicts a typical trajectory of compound Poisson process, simulated using Algorithm 6.2.

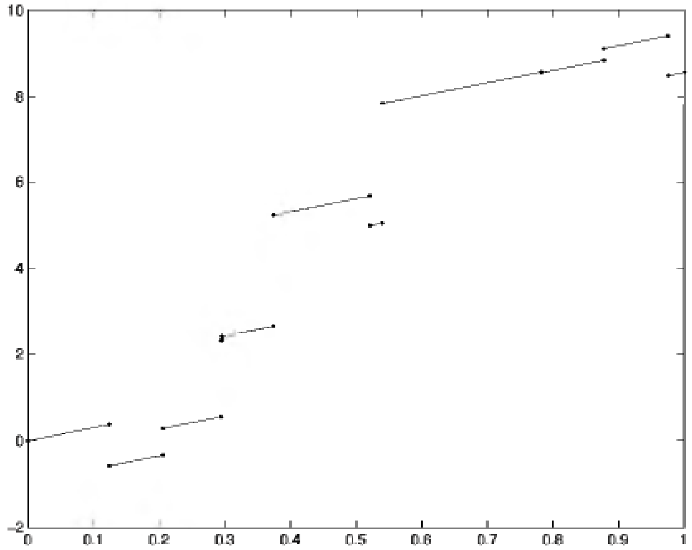

FIGURE 6.1: Typical trajectory of a compound Poisson process. Here jump size distribution is standard normal, the jump intensity is equal to 10 and the drift parameter is equal to 3.

When the Lévy process has a Gaussian component and a jump component of compound Poisson type (in this book, such a process is called a *jump-diffusion*), one can simulate the two independent components separately. The following algorithm gives a discretized trajectory for a process of this type with characteristic triplet (σ^2, ν, b).

ALGORITHM 6.3 Simulating jump-diffusions on a fixed time grid

Simulation of $(X_1, \ldots, X_n)$ *for* n *fixed times* $t_1, \ldots, t_n$.

- *Simulate* n *independent centered Gaussian random variables* G_i *with variances* $\mathrm{Var}(G_i) = (t_i - t_{i-1})\sigma^2$ *where* $t_0 = 0$. *A simple algorithm for simulating Gaussian random variables is described in Example 6.2.*
- *Simulate the compound Poisson part as described in the Algorithm 6.2.*

The discretized trajectory is given by

$$X(t_i) = bt_i + \sum_{k=1}^{i} G_k + \sum_{j=1}^{N} 1_{U_j < t_i} Y_j.$$

A typical trajectory of process simulated by Algorithm 6.3 is shown in Figure 6.2. In Section 6.4, we will see that many infinite activity Lévy processes