Let's simulate some Lévy processes :

- Keep in mind that $X_t = log(S_t/S_0)$.

I) Let's start with jump-diffusion :

We simulate here "jump diffusion" processes, which can be written as :

$X_t = \mu t + \sigma W_t + \sum_{s\in[0,t)} \Delta X_s = \mu t + \sigma W_t + \sum_{i=1}^{N(t)} Y_i$, N being a Poisson Process of intensity λ and $(Y_i)_i$ i.i.d random variables.

I-1) Merton Model :

- $Y_i \sim N(\mu_J, \delta_J^2)$
- Defined by $(\sigma, \lambda, \mu_J, \delta_J)$

Process parameters :

```
S0 = 1;          % Initial value
mu = 0.05;       % Drift (as in GBM)
sigma = 0.4;     % Volatility (as in GBM)
lambda = 2;      % Poisson intensity/rate
muJ = 0.01;      % Jumpsize mean
deltaJ = 0.2;    % Jumpsize std
```

Simulation parameters :

```
T = 2;      % Maturity
M = 100;    % Number of steps in time
dt = T/M;   % Time step
```

Conditional simulation of jump times (the easiest way) :

(cf ALGORITHM 6.2 from Simulate_Jump_Diffusion.pdf)

- The number of jumps N(T) of a CPP on the interval [0, T] is a Poisson random variable with parameter λT;
- Conditionally on N(T), the exact moments of jumps on this interval have the same distribution as N(T) independent random numbers, uniformly distributed on this interval, rearranged in increasing order.

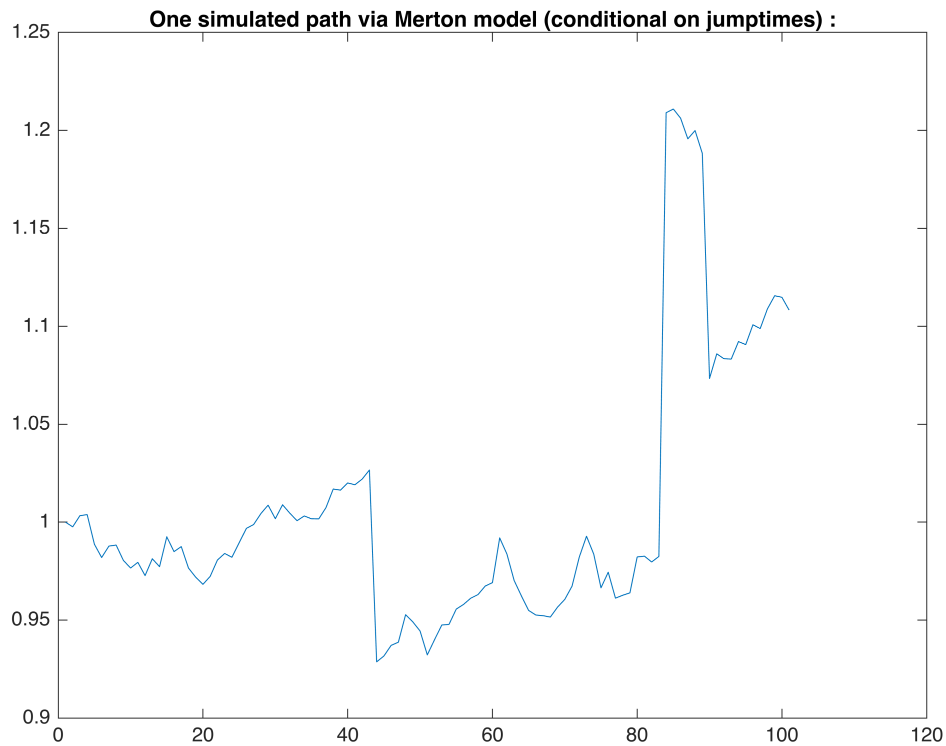
```
X = zeros(M + 1, 1); % X = (X0, X1, ..., XM)
Z = randn(M, 1);     % Z = (Z1, Z2, ..., ZM) [BM]

NT = poissrnd(lambda*T); % Nb of jumps in [0,T]

% Conditional to NT, we simulate when jumps occur
% via an Uniform Distribution :
jumpT = sort(rand(1,NT)*T);

% In Merton, jumpSize ~ Normal(muJ, deltaJ^2) :
```

```
jumpSize = muJ + deltaJ * randn(NT,1);
for i=1:M
    % Continuous part of the process :
    X(i+1) = X(i) + mu * dt + sigma * dt * Z(i);
    % Jump part :
    for j=1:NT
        % Did a jump occur in ](i-1)dt, idt] ?
        if jumpT(j) > (i-1)*dt && jumpT(j) <= i*dt
            X(i+1) = X(i+1) + jumpSize(j);
        end
    end
end
figure; % Plot
plot(S0 * exp(X));
title("One simulated path via Merton model (conditional on jumptimes) :")
```

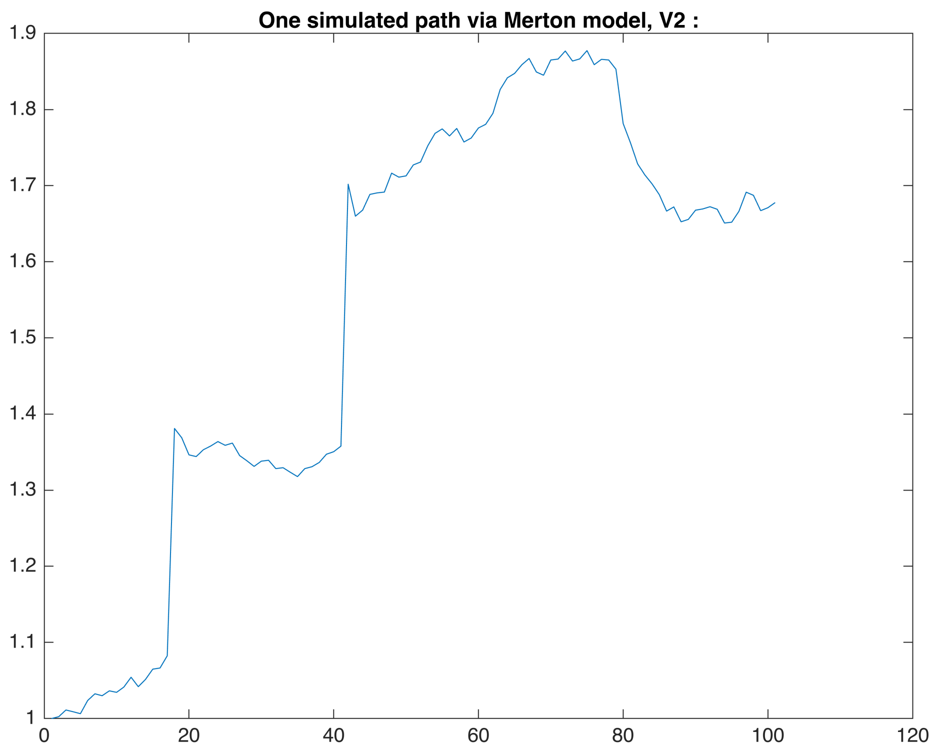


Another simulation of jump times : (cf ALGO 6.1 from Simulate_Jump_Diffusion.pdf) we simulate jumps inside the loop, at each time step.

```
X = zeros(M + 1, 1); % X = (X0, X1, ..., XM)
Z = randn(M, 1);     % Z = (Z1, Z2, ..., ZM) [BM]

for i=1:M
    % Continuous part of the process :
    X(i+1) = X(i) + mu * dt + sigma * dt * Z(i);
    % Jump part :
    Ndt = poissrnd(lambda * dt);
```

```
    if Ndt == 0
        J = 0; % No jump !
    else
        J = sum(muJ + deltaJ * randn(Ndt,1)); % Sum of the jump(s)
        % that happened during this time step.
    end
    X(i+1) = X(i+1) + J; % @ each time step : add the jump(s).
end
figure; % Plot
plot(S0 * exp(X));
title("One simulated path via Merton model, V2 :")
```



I-2) Kou Model :

- Positive jumpsize $\sim exp(\lambda^+)$;
- Negative jumpsize $\sim exp(\lambda^-)$;

Process parameters :

```
S0 = 1;           % Initial value
mu = 0.05;        % Drift (as in GBM)
sigma = 0.4;      % Volatility (as in GBM)
lambda = 2;       % Poisson intensity/rate
p = 0.6;          % Probability of a positive jump
lambda_plus = 10; % Parameter of Exp() for POSITIVE jumps
```

```
lambda_minus = 3; % Parameter of Exp() for NEGATIVE jumps
```

Simulation parameters :

```
T = 2;      % Maturity
M = 100;    % Number of steps in time
dt = T/M;   % Time step
```

Conditional simulation of jumptimes : (cf ALGO 6.2)

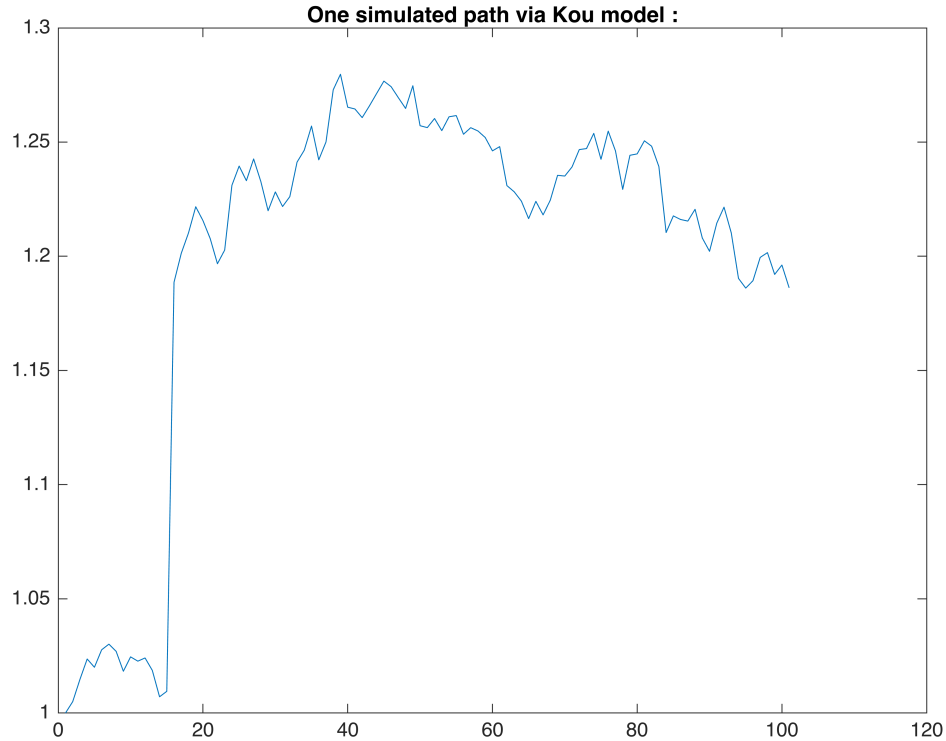
- The number of jumps N(T) of a CPP on the interval [0, T] is a Poisson random variable with parameter λT;
- Conditionally on N(T), the exact moments of jumps on this interval have the same distribution as N(T) independent random numbers, uniformly distributed on this interval, rearranged in increasing order.

```
X = zeros(M + 1, 1); % X = (X0, X1, ..., XM)
Z = randn(M, 1);     % Z = (Z1, Z2, ..., ZM) [BM]

NT = poissrnd(lambda*T); % Nb of jumps in [0,T]

% Conditional to NT, we simulate when jumps occur
% via an Uniform Distribution :
jumpT = sort(rand(1,NT)*T);

% In Kou, jumpSize ~ Exp(lambda^+ or lambda^-) :
for i=1:M
    % Continuous part of the process :
    X(i+1) = X(i) + mu * dt + sigma * dt * Z(i);
    % Jump part :
    for j=1:NT
        % Did a jump occur in ](i-1)dt, idt] ?
        if jumpT(j) > (i-1)*dt && jumpT(j) <= i*dt
            % RV ~ U((0,1)) gives type of jump (+ or -) :
            u = rand;
            if u < p % >0 jump
                jumpSize = exprnd(1/lambda_plus);
            else     % <0 jump
                jumpSize = -exprnd(1/lambda_minus);
            end
            % Add the simulated jumpsize :
            X(i+1) = X(i+1) + jumpSize;
        end
    end
end
figure; % Plot
plot(S0 * exp(X));
title("One simulated path via Kou model :")
```



One simulated path via Kou model :
1.3
1.25
1.2
1.15
1.1
1.05
1
0
20
40
60
80
100
120