Cheatsheet for simulation :

Compound Poisson Process : A trajectory of this process can be simulated exactly on the interval $[0,T]$ using the following simple algorithm (which uses the fact that waiting times between the jumps are independent exponentially distributed random variables with parameter λ):

ALGORITHM 6.1 Simulation of compound Poisson process

Initialize $k := 0$

REPEAT while $\sum_{i=1}^{k} T_i < T$

Set $k := k+1$

Simulate $T_k \sim \exp(\lambda)$ % interjump time $\sim Exp(\lambda)$.

Simulate Y_k *from the distribution* $\mu = \nu/\lambda$ % once there is a jump,
% we should simulate
% its size.

The trajectory is given by

$$X(t) = \gamma b + \sum_{i=1}^{N(t)} Y_i \quad where \quad N(t) = \sup\{k : \sum_{i=1}^{k} T_i \le t\}.$$

We will now improve this algorithm using two following observations

In one step : $N(T) \sim Poiss(\lambda T)$.

- The number of jumps $N(T)$ of a compound Poisson process on the interval $[0,T]$ is a Poisson random variable with parameter λT.

Very easy to generate .

- Conditionally on $N(T)$, the exact moments of jumps on this interval have the same distribution as $N(T)$ independent random numbers, uniformly distributed on this interval, rearranged in increasing order (see Proposition 2.9).

ALGORITHM 6.2 Improved algorithm for compound Poisson process

ON MATLAB:

N= poissrnd(T* lambda)

- *Simulate a random variable N from Poisson distribution with parameter* λT. *N gives the total number of jumps on the interval* $[0,T]$.

jumpT = sort(rand(1,N) *T)

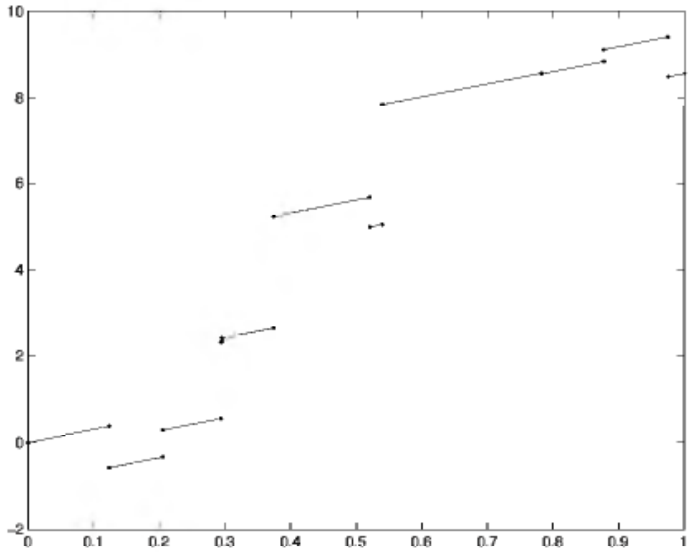
- *Simulate* N *independent r.v.,* U_i*, uniformly distributed on the interval* $[0,T]$. *These variables correspond to the jump times.*
- *Simulate jump sizes:* N *independent r.v.* Y_i *with law* $\frac{\nu(dx)}{\lambda}$.

The trajectory is given by :

$$X(t) = bt + \sum_{i=1}^{N} 1_{U_i<t} Y_i.$$

Depending on the model :
- Kou : $Exp(\lambda_+$ or $\lambda_-)$ ...
- Merton : $N(\mu_j, \sigma_j^2)$ ...

Figure 6.1 depicts a typical trajectory of compound Poisson process, simulated using Algorithm 6.2.



This is only drift + jumps (no BM here).

FIGURE 6.1: Typical trajectory of a compound Poisson process. Here jump size distribution is standard normal, the jump intensity is equal to 10 and the drift parameter is equal to 3. → Like Merton model.

When the Lévy process has a Gaussian component and a jump component of compound Poisson type (in this book, such a process is called a *jump-diffusion*), one can simulate the two independent components separately. The following algorithm gives a discretized trajectory for a process of this type with characteristic triplet (σ^2, ν, b). Like BM + CP Proc.

ALGORITHM 6.3 Simulating jump-diffusions on a fixed time grid

Simulation of $(X_1, \ldots, X_n)$ *for* n *fixed times* $t_1, \ldots, t_n$.

- *Simulate* n *independent centered Gaussian random variables* G_i *with variances* $\mathrm{Var}(G_i) = (t_i - t_{i-1})\sigma^2$ *where* $t_0 = 0$. *A simple algorithm for simulating Gaussian random variables is described in Example 6.2.*
- *Simulate the compound Poisson part as described in the Algorithm 6.2.*

The discretized trajectory is given by

$$X(t_i) = \underbrace{bt_i}_{\text{"drift"}} + \underbrace{\sum_{k=1}^{i} G_k}_{\text{"BM"}} + \underbrace{\sum_{j=1}^{N} 1_{U_j < t_i} Y_j}_{\text{CP Proc.}}.$$

A typical trajectory of process simulated by Algorithm 6.3 is shown in Figure 6.2.