Exam: Version C

The objective of this report is to build portfolios using different allocation strategies and discuss their performances, using as investment universe 11 sector indices and 5 factor indices of the S&P500.
The S&P 500 sector indices can be broadly classified into **cyclical**, **defensive**, and **sensitive** categories based on their typical behavior in response to economic cycles.

- **Cyclical Sectors.** They tend to perform well during economic expansions but may underperform in recessions, as they are more sensitive to changes in consumer demand and investment spending. They are: Consumer Discretionary, Financials, Materials, Real Estate, Industrials.

- **Defensive Sectors.** They tend to be less affected by economic cycles as they provide essential goods and services that consumers continue to need regardless of economic conditions. They are: Consumer Staples, Utilities, Health Care.

- **Sensible Sectors.** They are somewhat affected by the economic cycle but less dramatically than cyclical sectors. They can be sensitive to changes in interest rates, inflation, and global trends but often exhibit stability due to innovation. They are: Energy, Information Technology, Communication Services.

Exam guidelines: You must send me via email all the matlab codes and a report of 10-15 pages (maximum) in which you discuss your results with the help of tables and plots.

Description of files:

- "prices.xlsx" contains the prices of 11 sector indices and 5 factor indices of the S&P500 from January 2021 to October 2024

- "capitalizations.xlsx" contains the market capitalization for each index at 25/20/2024

Part A

Use prices from 01/01/2023 to 31/12/2023.

1. Compute the efficient frontier under the standard constraints, i.e. $\sum_i^N w_i = 1$ and $0 \leq w_i \leq 1 \; \forall i \in [1, .., N]$. Compute the Minimum Variance Portfolio, named Portfolio A, and the Maximum Sharpe Ratio Portfolio, named Portfolio B, of the frontier.

2. Compute the efficient frontier under the following constraints (to be considered all at once):

   - Standard constraints,
   - The total exposure to sensible sectors has to be lower than 50% and the total exposure on defensive sectors has to be greater than 30%,
   - Limit the exposure of the most volatile sectors (Communication Services and Energy ($0.05 \leq w_i \leq 0.1$)
   - The total exposure of cyclicals has to be equal to the defensive sectors.

   Compute the Minimum Variance Portfolio, named Portfolio C, and the Maximum Sharpe Ratio Portfolio, named Portfolio D, of the frontier.

3. Compute the frontiers in step 1 and 2 using the resampling method in order to obtain 2 robust frontier. For each frontier save the Minimum Variance Portfolios, named Portfolios E and F, and the Maximum Sharpe Ratio Portfolios, named Portfolios G and H, of the frontiers.

4. Compute the portfolio frontier, under standard constraints, using the Black-Litterman model with the following views (to be considered all at once):

   - **View on Performance of Energy Sector:** In a scenario of rising commodity prices, one could assume that Energy has better prospects compared to other sectors more exposed to commodity costs (Energy annual performance of 3%).
   - **View on the Momentum vs. Quality factor:** In times of economic instability, you might expect the Quality factor to outperform Momentum, as investors favor companies with strong fundamentals, while Momentum may be hurt by rapid price corrections (Annual overperformance of 1%).

   Compute the Minimum Variance Portfolio, named Portfolio I, and the Maximum Sharpe Ratio Portfolio, named Portfolio L, of the frontier.

5. Compute the Maximum Diversified Portfolio (Portfolio M) and the Maximum Entropy (in risk contributions) Portfolio (Portfolio N), under the following constraints (to be considered all at once):

   - Standard constraints,
   - The total exposure on defensive sectors has to be less than 20%,

- Assuming that you have a benchmark portfolio (capitalization weighted portfolio), the sum of the difference (in absolute value) of the weights in the benchmark portfolio and the optimal weights has to be equal to 30%.

6. Compute the portfolio (Portfolio P), using the Principal Component Analysis, that maximizes its expected return under the following constraints (to be considered all at once):

   - Standard constraints,
   - The volatility of the portfolio has to be equal or less than a target volatility of $\sigma_{tgt} = 0.75$

   You have to use the minimum number of factors that explains more than the 95% of the cumulative variance.

7. Compute the Portfolio that maximizes, under standard constraints, the Expected Shortfall-modified Sharpe Ratio (i.e. the risk in the formula of Sharpe Ratio is the Expected Shortfall), named Portfolio Q, using the historical method.

8. Discuss the characteristics of all the portfolios you have computed in steps 1-7 in terms of performance, risk and diversification using performance and diversification metrics, with the help of plots and tables. Use as benchmark the equally weighted Portfolio. Discuss the results.

Part B

Use the portfolio allocations computed from steps 1 to 7 (from portfolio A to Q) to evaluate the out-of-sample performance of the portfolios in the period 01/01/2024 to the end of the available data. Discuss how the results are changing from the ones calculated in point 8.