Modelling Long-Run Relationships in Finance

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Stationarity and Unit Root Testing

- **Why Test for Non-Stationarity?**
  - Non-stationary series can result in spurious regressions.
  - Standard asymptotic analysis assumptions are invalid if variables are not stationary.

Two Types of Non-Stationarity

- **Random Walk with Drift**: $y_t = \mu + y_{t-1} + u_t$
- **Deterministic Trend Process**: $y_t = \alpha + \beta t + u_t$

Stochastic Non-Stationarity

- Model: $y_t = \mu + \phi y_{t-1} + u_t$ (where $\phi > 1$ describes explosive processes, typically ignored).

Impact of Shocks

- AR(1) Model: $y_t = \phi y_{t-1} + u_t$
- Three Cases:
  1. $\phi < 1$: Shocks die away.
  2. $\phi = 1$: Shocks persist indefinitely.
  3. $\phi > 1$: Shocks grow over time.

Detrending Non-Stationary Series

- **Difference Stationarity**: $\Delta y_t = y_t - y_{t-1} = \mu + u_t$
- **Deterministic Trend Stationarity**: Remove deterministic trend.

Testing for Unit Roots

- **Dickey-Fuller Test**: Tests null hypothesis H_0: $\phi = 1$ vs H_1: $\phi < 1$.
- **Augmented Dickey-Fuller (ADF) Test**: Augments DF test to allow for autocorrelation in residuals.
- **Phillips-Perron Test**: Corrects DF test for autocorrelation.

Stationarity Tests

- **KPSS Test**: Tests null hypothesis H_0: y_t is stationary vs H_1: y_t is non-stationary.
- Compare KPSS results with ADF/PP to confirm conclusions.

Cointegration

- **Definition**: Series are cointegrated if a linear combination of them is stationary.
- **Engle-Granger Approach**: Two-step method to test and estimate cointegration relationships.
- **Johansen Test**: Tests for multiple cointegration relationships using a Vector Error Correction Model (VECM).

Error Correction Models (ECM)

- **Specifying an ECM**: Combines first differences and levels of variables:

$$\Delta y_t = \beta_1 \Delta x_t + \beta_2 (y_{t-1} - \gamma x_{t-1}) + u_t \tag{1}$$

- **Error Correction Term**: $(y_{t-1} - \gamma x_{t-1})$ is stationary if y_t and x_t are cointegrated.

Testing for Cointegration in Regression

- Use DF/ADF tests on residuals from cointegrating regression.
- **Engle-Granger (EG) Test**: Tests null hypothesis H_0: residuals contain unit root vs H_1: residuals are stationary.

Johansen Technique for Cointegrated Systems

- Converts VAR to VECM:

$$\Delta y_t = \Pi y_{t-1} + \Gamma_1 \Delta y_{t-1} + ... + \Gamma_{k-1} \Delta y_{t-(k-1)} + u_t \tag{2}$$

- **Johansen Test**: Tests for cointegration by examining the rank of Π matrix via eigenvalues.

Practical Steps

1. Test for non-stationarity using ADF or similar tests.
2. If series are I(1), test for cointegration using Engle-Granger or Johansen method.
3. If cointegration is found, estimate a Vector Error Correction Model.