Substate state-space

MATLAB function examples

```
close all
clearvars
```

List of functions for the exam:

- sys = ss(F, G, H, D, Ts); or sys = ss(W);
- eigenvalues = eig(sys);
- O = obsv(sys);
- R = ctrb(sys);
- obs_rank = rank(O);
- W = tf(sys); or W = tf(num_coeffs, den_coeffs, Ts);
- W = zpk(W);
- zeros_vec = zero(W);
- pole_vec = pole(W);
- sys_gain = dcgain(W);
- bode(sys)
- impulse(sys)
- step(sys)
- lsim(sys)
- sys = n4sid(data) *
- [U, S, V] = svd(Hqd) *

*explained in the pendulum example and not in this file.

System defined from system matrices (state-space)

Given the following system, define it in MATLAB as a discrete time system with sampling time 1 s:

$$F = \begin{bmatrix} 0.999 & -0.001 \\ 0.002 & 0 \end{bmatrix} \qquad H = [0.5 \quad 2.5] \qquad G = \begin{bmatrix} 0.002 \\ 0 \end{bmatrix} \qquad D = 0$$

```
% Define the system matrices
F = [0.999 -0.001; 0.002 1];
G = [0.002; 0];
H = [0.5 2.5];
D = 0;

% Define system sampling time
Ts = 1; % [s]
```

```
% Define system object
sys = ss(F, G, H, D, Ts);
% If Ts is not provided, sys will be in continuous time. If Ts = -1, the
% sampling period will remain unspecified but the system will be in discrete time.

% We can display the defined system:
sys
```

```
sys =

  A =
            x1      x2
   x1    0.999  -0.001
   x2    0.002       1

  B =
           u1
   x1   0.002
   x2       0

  C =
         x1   x2
   y1   0.5  2.5

  D =
       u1
   y1   0

Sample time: 1 seconds
Discrete-time state-space model.
```

We can now compute the eigenvalues of the system:

```
eigenvalues = eig(sys);

% Display the eigenvalues:
eigenvalues
```

```
eigenvalues = 2×1 complex
   0.9995 + 0.0013i
   0.9995 - 0.0013i
```

```
% Check whether eigenvalues are within unit circle
if all(abs(eigenvalues) < 1)
    disp("Asymptotically stable system");
elseif all(abs(eigenvalues) <= 1)
    disp("Simply stable system");
else
    disp("Unstable system");
end
```

```
Asymptotically stable system
```

We can check controllability and observability of the system:

```
O = obsv(F, H)
```

```
O = 2×2
    0.5000    2.5000
    0.5045    2.4995
```

```
R = ctrb(F, G)
```

```
R = 2×2
    0.0020    0.0020
         0    0.0000
```

```
% If you do not have matrices F, G and H but you have the system object,
% you can use:
O = obsv(sys.A, sys.C);
R = ctrb(sys.A, sys.B);

% One further way to call these functions:
O = obsv(sys);
R = ctrb(sys);

% To check if the system is observable:
nx = size(sys.A, 1); % or nx = size(F, 1)
if rank(O) == nx
    disp("System is observable");
else
    disp("System is NOT observable");
end
```

```
System is observable
```

```
% To check reachability:
nx = size(sys.A, 1); % or nx = size(F, 1)
if rank(R) == nx
    disp("System is reachable");
else
    disp("System is NOT reachable");
end
```

```
System is reachable
```

We can now turn the state space model into a transfer function:

```
W = tf(sys);
% tf command defines a transfer function

% Display W
W
```

```
W =

    0.001 z - 0.00099
  ---------------------
  z^2 - 1.999 z + 0.999

Sample time: 1 seconds
Discrete-time transfer function.
```

```
% If you want to clearly see zero-pole-gain:
W = zpk(W);
% or W = zpk(sys);

% Display W
W
```

```
W =

      0.001 (z-0.99)
  ----------------------
  (z^2 - 1.999z + 0.999)

Sample time: 1 seconds
Discrete-time zero/pole/gain model.
```

```
% To compute separately zeros, poles and gain:
sys_zeros = zero(sys)
```

```
sys_zeros = 0.9900
```

```
% The same function can be also called using a transfer function and not a
% discrete time system object
sys_zeros = zero(W)
```

```
sys_zeros = 0.9900
```

```
sys_poles = pole(W) % or pole(sys)
```

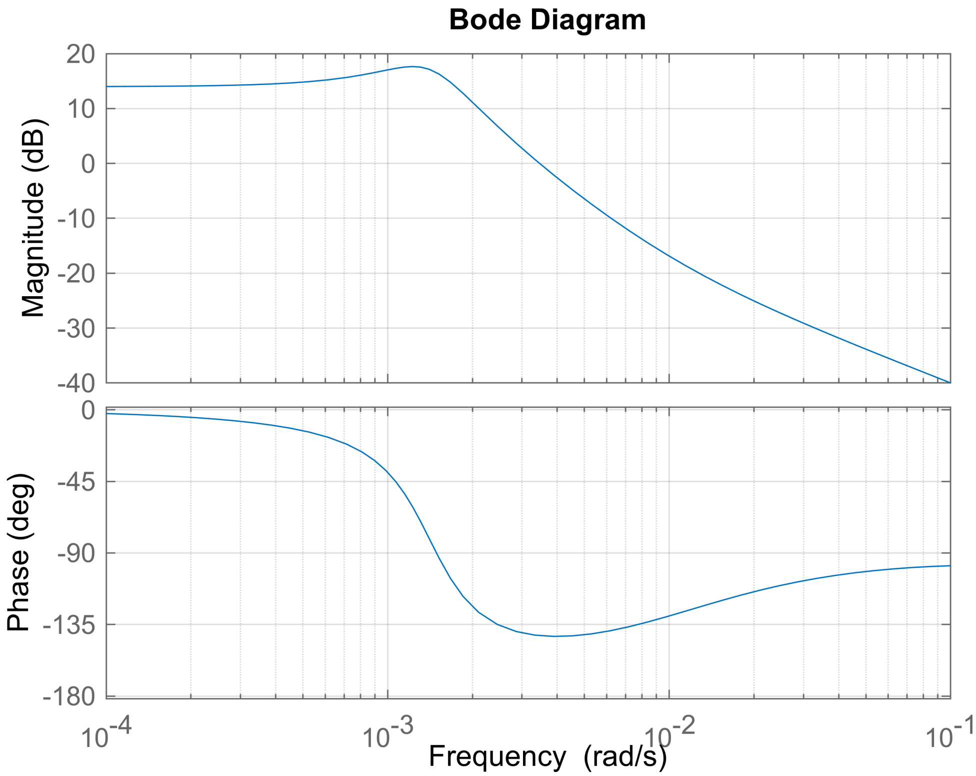
```
sys_poles = 2×1 complex
   0.9995 + 0.0013i
   0.9995 - 0.0013i
```

```
sys_gain = dcgain(W) % or dcgain(sys)
```

```
sys_gain = 5.0000
```

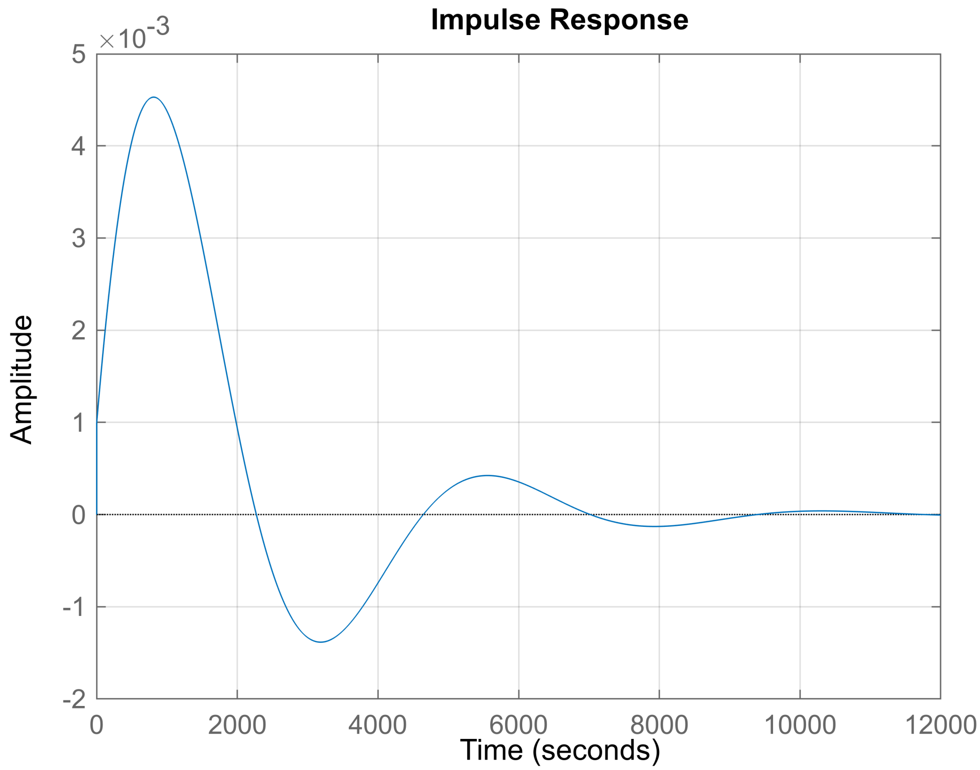
We can check the bode diagram of the system:

```
bode(sys) % or bode(W)
hold on; grid minor; clf;
```



We can now simulate the system over time and plot the results:

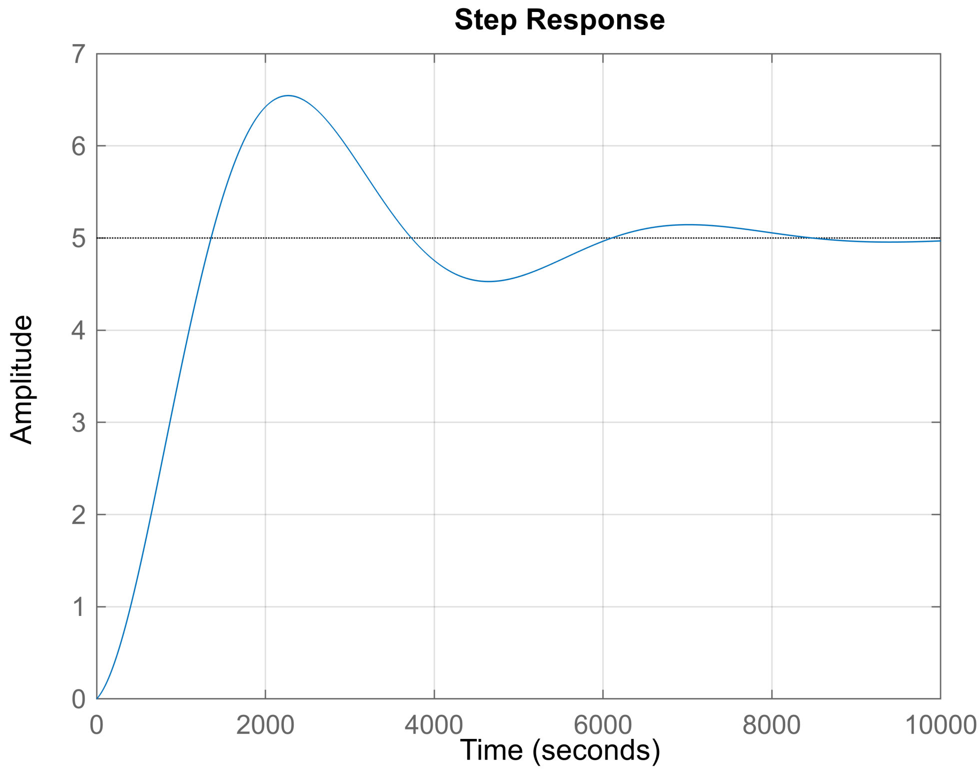
```
% To plot the impulse response of the system:
impulse(sys)
hold on; grid minor; clf;
```




```
% We can also get the impulse response vectors in time:
[ir_data, ir_time] = impulse(sys);
% Display ir_data
ir_data
```

```
ir_data = 10673×1
         0
    0.0010
    0.0010
    0.0010
    0.0010
    0.0010
    0.0010
    0.0011
    0.0011
    0.0011
      ⋮
```

```
% The same can be done for the step response:
step(sys)
hold on; grid minor; clf;
```

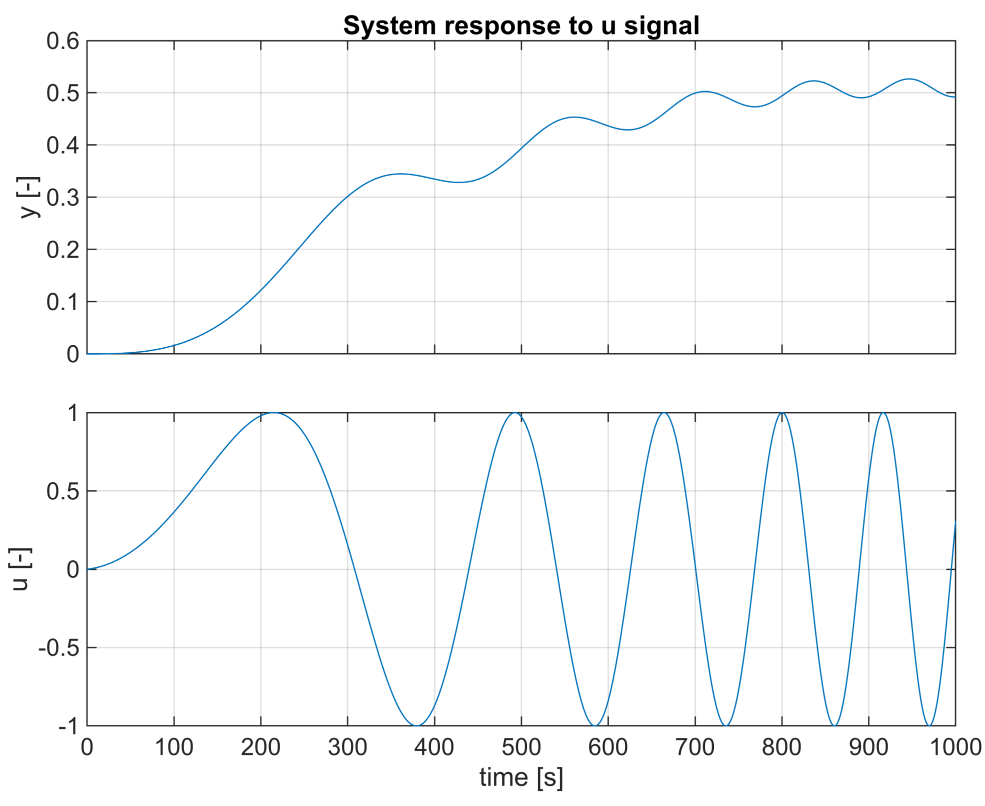



```
% We can also get the step response vectors in time:
[step_data, step_time] = step(sys);
% Display ir_data
step_data
```

```
step_data = 10673×1
         0
    0.0010
    0.0010
    0.0010
    0.0010
    0.0010
    0.0010
    0.0011
    0.0011
    0.0011
      ⋮
```

```
% Finally, we can simulate the system with any input signal:

% Define time vector time_vec
end_time = 1000;
time_vec = (0:Ts:end_time)';

% Define u_vec (vector of the input)
f_start = 0.0001; f_end = 0.01; amplitude = 1;
```

```
u_vec = amplitude*sin(2*pi*(f_start*time_vec + (f_end-f_start)/end_time/2*time_vec.^2));

% Simulate system response
y_vec = lsim(sys, u_vec, time_vec);

%% Experiment plot
figure;
tiledlayout(2, 1, 'TileSpacing', 'compact');

ax(1) = nexttile; hold on; grid on; box on;
title('System response to u signal');
plot(time_vec, y_vec);
ylabel('y [-]');
xticklabels('');

ax(2) = nexttile; hold on; grid on; box on;
xlabel('time [s]');
ylabel('u [-]');
plot(time_vec, u_vec);

linkaxes(ax, 'x'); clear ax;  clf;
```



System defined from transfer function

Define a discrete time system of the following transfer function (sampling time 1 s):

$$W(z) = \frac{1e-3 \cdot z - 0.99e-3}{z^2 - 1.999z + 0.999}$$ (Positive power notation)

```
% Define the coefficients of the numerator and denominator.
numerator_coeffs = [1.0000   -0.9900]*1e-3;
denominator_coeffs = [1.0000   -1.999 0.999];

% Define sampling time
Ts = 1; % [s]

W = tf(numerator_coeffs, denominator_coeffs, Ts)
```

```
W =

    0.001 z - 0.00099
  ---------------------
  z^2 - 1.999 z + 0.999

Sample time: 1 seconds
Discrete-time transfer function.
```

```

% Now we can compute one possibile realization in state-space of the system
% by using the function ss:
sys = ss(W)
```

```
sys =

  A =
           x1      x2
   x1   1.999  -0.999
   x2       1       0

  B =
           u1
   x1  0.0625
   x2       0

  C =
             x1        x2
   y1     0.016  -0.01584

  D =
       u1
   y1   0

Sample time: 1 seconds
Discrete-time state-space model.
```

Question examples:

- Given the following discrete-time system: $F = \begin{bmatrix} 0 & 2 \\ \frac{1}{2} & 3 \end{bmatrix}$ $H = [1 \quad 0]$ $G = \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix}$ $D = 0$, define it as discrete state-space system in MATLAB and check the observability of the system.
- Given a MATLAB discrete-time state-space system named *sys,* compute the eigenvalues and show the transfer function in zero-pole-gain form.
- Given the following transfer function: $W(z) = \frac{0.5 \cdot z - 0.5}{z^2 - 3z - 1}$, define it in MATLAB and find a state-space realization. Then compute its poles.
- Given the discrete time MATLAB system *sys* and a known input vector *u* and its corresponding time vector *t*, write the code to compute the response of *sys* to *u*.
- Write the MATLAB code to compute the first 10 samples of the impulse response of a known transfer function *W*.