Project for the "Local Volatility Model" course

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1 Instructions

The project has to be done in groups of 2-3-4 persons. Please send as soon as possible the composition of your group by email. The group will then receive the file containing market data (a different file for each group).

Each group must send <u>all</u> Matlab code used to solve the problems and a document (pdf, word, powerpoint, etc..) containing the numerical values obtained and discussing the results.

Deadline: 10 Jan 2024 / 2 Feb 2024

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2 Project

You have a file `MarketData_XY.xls` that contains market data for several assets. In particular, for each asset you will find:

- market volatility surface $\{\sigma_{mkt}(T_i, K_{i,j})\}$ for $i \in \{1, \ldots, N\}$, $j \in \{1, \ldots, M\}$
- market expiry dates $(T_1, \ldots, T_N)$
- market strikes $\{K_{i,j}\}$ (or deltas $\{\Delta_j\}$ for FX assets)
- forwards $(F_0(T_1), \ldots, F_0(T_N))$ and discount factors $(D_0(T_1), \ldots, D_0(T_N))$ at market expiry dates

so that the market price of a call option with expiry T_i and strike $K_{i,j}$ is obtained by Black formula $Bl(F_0(T_i), K_{i,j}, T_i, \sigma_{mkt}(T_i, K_{i,j}), D_0(T_i))$.

Assume that the local volatility function is parameterized by a local volatility matrix $\{v_{i,j}\}$ at the nodes $\{K_{i,j}\}$ with piecewise constant interpolation in time and spline interpolation with flat extrapolation in strike.

Solve the following exercises:

1.1 Consider market data of asset E CORP, contained in file `MarketData_XY.xls`, and calibrate a local volatility model with maximum calibration error of 10bps (i.e., threshold=0.001).

1.2 With the model found above, price two call options with expiry $T = 0.5$ and strikes $K = \kappa \cdot S(0)$ with $\kappa = 0.9, 1.1$, by solving Dupire equation or by Monte Carlo simulation. Compute the relevant implied (spot) volatilities.

1.3 Using a Monte Carlo simulation, price two forward starting option with start date $T_1 = 2$, expiry $T_2 = 2.5$ and strikes $\kappa = 0.9, 1.1$. Compute the implied forward volatilities.

1.4 Using the model implied volatilities above, compute the skew of the spot and forward smile, namely slope of the spot/forward implied volatility as a function of strike κ. What typical feature of the local volatility model do you observe?

2 Consider market data of asset FAIL contained in file `MarketData_XY.xls` and try calibrate a local volatility model. Verify that the calibration procedure fails and explain why (hint: compute market prices $C_0(T_i, K_{i,j})$ of call options via Black's formula and observe the convexity of the map $K_{i,j} \mapsto C_0(T_i, K_{i,j})$ for fixed T_i)

3.1 Consider market data of the FX asset $Y = EUR/USD$, contained in file `MarketData_XY.xls`. Forwards at the expiry dates are given in terms of the spot $Y(0)$, the domestic discount factors $\{D_0^d(T_i)\}$, the foreign discount factors $\{D_0^f(T_i)\}$ according to the formula $F_0(T_i) = Y(0)\frac{D_0^f(T_i)}{D_0^d(T_i)}$. Find market strikes $\{K_{i,j}\}$ from the quoted deltas and calibrate a local volatility model with maximum calibration error of 10bps (i.e., threshold=0.001).

3.2 Consider a plain vanilla option with expiry T_5 (the fifth market expiry provided) and strike $K_{5,2}$ (the strike corresponding to 25-Delta). Moreover consider a digital option with the same expiry T_5 and strike $K_{5,2}$.
Under the Local Volatility model calibrated in [3.1] compute, for both plain vanilla and digital option, Monte Carlo price and confidence interval of level 95% for the option price [1]. Use N=100000, M=100

3.3 Compute the Monte Carlo price and confidence interval of the same options of [3.2] under Black dynamics where the model parameter σ is taken as the market volatility $\sigma_{5,2}^{mkt}$ namely the 25-Delta volatility at expiry T_5. Use N=100000

3.4 Consider the confidence intervals of level 95% for the price of the plain vanilla option under Local Volatility and Black model, as computed in [3.2] and [3.3]. Check that they overlap or at least are very close, and

[1] see slides, and in particular the definition of the so-called Monte Carlo error $\sqrt{\hat{v}_N/N}$ where $\hat{v}_N$ is the sample variance of the Monte Carlo simulation. Also notice that sample variance can be computed using MATLAB *std* function

explain why both models give the same result (up to numerical errors such as calibration error and Monte Carlo Error)
Do the same analysis for the digital option. Do the confidence intervals overlap?