Modelling Long-Run Relationships in Finance

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Stationarity and Unit Root Testing

⊕ infinite persistence of shocks for non-stationary series.

- **Why Test for Non-Stationarity?**
  - Non-stationary series can result in spurious regressions. high R^2 even if the 2 variables are totally unrelated.
  - Standard asymptotic analysis assumptions are invalid if variables are not stationary. → the "t-ratios" don't follow a t-distribution.

Two Types of Non-Stationarity : "weak form OR covariance" stationarity.

- **Random Walk with Drift**: $y_t = \mu + y_{t-1} + u_t$
- **Deterministic Trend Process**: $y_t = \alpha + \beta t + u_t$

(u_t iid in both cases)

Stochastic Non-Stationarity

- Model: $y_t = \mu + \phi y_{t-1} + u_t$ (where $\phi > 1$ describes explosive processes, typically ignored). We use $\phi = 1$ to characterize non-stationarity.

Impact of Shocks

- AR(1) Model: $y_t = \phi y_{t-1} + u_t$ → so $y_t = \phi(\phi y_{t-2} + u_{t-1}) + u_t$ $= \phi^2 y_{t-2} + \phi u_{t-1} + u_t \ldots$ T substitutions leads to:
- Three Cases:
  1. $\phi < 1$: Shocks die away.
  2. $\phi = 1$: Shocks persist indefinitely.
  3. $\phi > 1$: Shocks grow over time.

→ $\boxed{y_t = \phi^T y_0 + \phi u_{t-1} + \phi^2 u_{t-2} + \ldots + \phi^T u_0 + u_t}$.

Detrending Non-Stationary Series

- For stoch. non-stationarity → **Difference Stationarity**: $\Delta y_t = y_t - y_{t-1} = \mu + u_t$: we induced stationarity by differentiating of log returns.
- For deterministic non-stat. → **Deterministic Trend Stationarity**: Remove deterministic trend. ↓ "detrending"

generalization:
You can differentiate d times.
If $y_t \sim I(d)$ then $\Delta^d y_t \sim I(0)$.

In Fi./Eco. → usually a single unit root. Some are stationary & consumer prices are known to have 2 unit roots.

We can write: $\Delta y_t = u_t + \mu + \delta t + \psi y_{t-1}$ $\psi = \phi - 1$.

tests based on the t-ratio on the y_{t-1} term in the estimated regression of y_t:

$\frac{\hat{\psi}}{\hat{SE}(\hat{\psi})}$ = test statistic. → does not follow the usual t-distribution under the null (non-stat.).

↳ Instead: comparison to critical value: H_0 rejected if < critical value.

Testing for Unit Roots

How do we test for unit root?

- **Dickey-Fuller Test**: Tests null hypothesis H_0: $\phi = 1$ vs H_1: $\phi < 1$. (in $y_t = \phi y_{t-1} + u_t$)
- **Augmented Dickey-Fuller (ADF) Test**: Augments DF test to allow for autocorrelation in residuals.
- **Phillips-Perron Test**: Corrects DF test for autocorrelation. for residuals.

for higher order, continue to test for a further unit root until we rejected H_0.

Stationarity Tests

: to cope w/ the fact that previous tests decide poorly when $\phi \approx 1$.

- **KPSS Test**: Tests null hypothesis H_0: y_t is stationary vs H_1: y_t is non-stationary.
- Compare KPSS results with ADF/PP to confirm conclusions.

Cointegration

- **Definition**: Series are cointegrated of order (d,d) if a linear combination of them is stationary.
- **Engle-Granger Approach**: Two-step method to test and estimate cointegration relationships.
- **Johansen Test**: Tests for multiple cointegration relationships using a Vector Error Correction Model (VECM).

Cointegrating relationship may be seen a long-term relationship. → example: Spot & futures prices.

Error Correction Models (ECM)

- **Specifying an ECM**: Combines first differences and levels of variables:

$$\Delta y_t = \beta_1 \Delta x_t + \beta_2 (y_{t-1} - \gamma x_{t-1}) + u_t \tag{1}$$

- **Error Correction Term**: $(y_{t-1} - \gamma x_{t-1})$ is stationary if y_t and x_t are cointegrated.

(I(0)) ↩ even though the constituents are I(1).

Testing for Cointegration in Regression

- Use DF/ADF tests on residuals from cointegrating regression.
- **Engle-Granger (EG) Test**: Tests null hypothesis H_0: residuals contain unit root vs H_1: residuals are stationary.

Johansen Technique for Cointegrated Systems

- Converts VAR to VECM:

$$\Delta y_t = \Pi y_{t-1} + \Gamma_1 \Delta y_{t-1} + ... + \Gamma_{k-1} \Delta y_{t-(k-1)} + u_t \tag{2}$$

- **Johansen Test**: Tests for cointegration by examining the rank of Π matrix via eigenvalues.

Practical Steps

1. Test for non-stationarity using ADF or similar tests. Consider possible structural break.
2. If series are I(1), test for cointegration using Engle-Granger or Johansen method.
3. If cointegration is found, estimate a Vector Error Correction Model.