

ARMA(1,1) process:

$$y(t) = \frac{1}{2}y(t-1) + z(t) - \frac{1}{2}z(t-1), \quad z(t) \sim \text{WN}(1,9)$$

$$y(t) \left(1 - \frac{1}{2}z^{-1}\right) = z(t) (1 - z^{-1}) \quad \text{so}$$

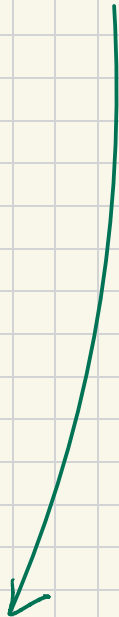
$$\boxed{y(t) = \frac{1-z^{-1}}{1-\frac{1}{2}z^{-1}} z(t)} \quad \text{so} \quad E[y(t)] = \frac{1-z^{-1}}{1-\frac{1}{2}z^{-1}} E[z(t)] \quad \text{i.e.}$$

$\underbrace{\hspace{10em}}_{=W(z)}$

$$\boxed{E[z(t)] = 0} \quad \text{so} \quad \hat{y}(t) = y(t) - m_y = y(t) - 0 = y(t) = W(z)e(t)$$

$$\text{i.e.} \quad y(t) = W(z)(z(t) - 1) \quad (z(t) \sim \text{WN}(0,9))$$

$$\gamma_y(\tau) = \gamma_{\hat{y}}(\tau) \quad \text{so} \quad \gamma_y(\tau) = E[y(t)y(t-\tau)]$$



$$y(t) = \frac{1 - z^{-2}}{1 - \frac{1}{2}z^{-1}} e(t), \quad e(t) \sim \text{WN}(0, 9).$$

Corr $f(t)$?

$$y(t) = \frac{1}{2}y(t-1) + e(t) - e(t-1) \quad \text{so } m_y = 0 \text{ \& } :$$

$$\gamma_y(0) = E[y(t)^2] = E\left[\left(\frac{1}{2}y(t-1) + e(t) - e(t-1)\right)^2\right]$$

$$\gamma_y(0) = \frac{1}{4}\gamma_y(0) + 9 + 9 + \underbrace{E[y(t-1)e(t)] - E[y(t-1)e(t-1)]}_{\substack{y(t-1) = f(t(e(t-1), e(t-2), \dots)) \\ = 0}}$$

$$\frac{3}{4}\gamma_y(0) = 18 - E[e(t-1)^2] = 18 - 9 = 9$$

$$s_0 = \gamma_y(0) = \frac{4 \times 9}{3} = 4 \times 3 = 12.$$

$$\gamma_y(1) = E[y(t)y(t-1)]$$

$$= \frac{1}{2}E[y(t-1)^2] - E[e(t-1)^2] = 6 - 9 = -3$$

$$\gamma_y(1) = -3$$