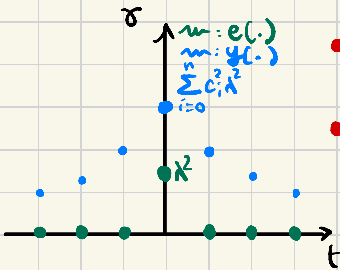
MA(n):

Definition: $y(t) = c_0 e(t) + c_1 e(t-1) + \dots + c_n e(t-n) = \sum_{i=0}^{n} c_i e(t-i)$, $e(\cdot) \sim WN(0, \lambda^2)$.

Properties:

- Stationary;
- $m_y(t) = m_y = 0 \quad (\forall t)$;
- $\gamma_y(\tau) = \mathbb{E}[(y(t)-m_y)(y(t-\tau)-m_y)] = \mathbb{E}[y(t)y(t-\tau)] = \begin{cases} \left(\sum_{i=0}^{n-|\tau|} c_i c_{i+|\tau|}\right)\lambda^2 & \text{if } |\tau| \leq n; \\ 0 & \text{otherwise.} \end{cases}$



Particular case: $Var(y(t)) = \gamma_y(0) = \sum_{i=0}^{n} c_i^2 \lambda^2$.

Generalisation: MA(∞) $\quad y(t) = \sum_{i=0}^{+\infty} c_i e(t-i)$, $e \sim WN(0, \lambda^2)$, with $\sum_{i=0}^{+\infty} c_i^2 < +\infty$.

↳ $m_y = 0$;

↳ $\gamma_y(0) = \sum_{i=0}^{+\infty} c_i^2 \cdot \lambda^2 < +\infty$;

↳ $\gamma_y(\tau) = \sum_{i=0}^{+\infty} c_i c_{i+\tau} \lambda^2$.

MA(∞) is stationary. It can model almost all SP of this course.

Property: MA(n) process has: - n non-trivial zeros;
- n poles, all lying @ the origin.

⟺ All MA(n) processes are asympt. stable.

AR(m):

Why? It allows us to have $\gamma(\tau) \neq 0$ for all τ using a finite set of coefficients (more handy than MA(∞)).

Definition: $y(t) = a_1 y(t-1) + \dots + a_m y(t-m) + e(t)$, $e(\cdot) \sim WN(0, \lambda^2)$.

NB: $AR(1) \equiv MA(\infty)$ with $c_0 = 1$, $c_1 = a$, $c_2 = a^2$, ... : $y(t) = ay(t-1) + e(t) = a^2 y(t-2) + e(t-1) + e(t)$ ...

"operatorial representation": $Z(x(t))$ ...

WE NEED ADDITIONAL TOOLS TO STUDY STATIONARITY OF MODELS HAVING AR PARTS.

Property: AR(m) process has:
- m zeros lying @ the origin;
- m non-trivial poles.

$\Leftrightarrow$ All AR(m) are minimum phase.

ARMA(m,n):

Definition: $y(t) = a_1 y(t-1) + \dots + a_m y(t-m) + c_0 e(t) + \dots + c_n e(t-n)$, $e(\cdot) \sim WN(0, \lambda^2)$.

NB: ARMAX(m,n,p,k) also exist: $y(t) = a_1 y(t-1) + \dots + a_m y(t-m) +$
$c_0 e(t) + c_1 e(t-1) + \dots + c_n e(t-n) +$
$b_0 u(t-k) + b_1 u(t-k-1) + \dots + b_p u(t-k-p)$.

$\begin{cases} p: \text{order of the exogenous part} \\ k: \text{pure input/output (i/o) delay} \end{cases}$

TF: $\boxed{\text{ARMA}}$: $y(t) = \dfrac{c_0 + c_1 z^{-1} + \dots + c_n z^{-n}}{1 - a_1 z^{-1} - \dots - a_m z^{-m}} e(t) = \dfrac{C(z)}{A(z)} e(t) = W(z) e(t)$ ← "Discr. Time TF".

$\boxed{\text{ARMAX}}$: $y(t) = \dfrac{c_0 + c_1 z^{-1} + \dots + c_n z^{-n}}{1 - a_1 z^{-1} - \dots - a_n z^{-n}} e(t) + \dfrac{b_0 + b_1 z^{-1} + \dots +}{1 - a_1 z^{-1} - \dots - a_n z^{-n}} z^{-k} u(t) = \dfrac{C(z)}{A(z)} e(t) + \dfrac{B(z)}{A(z)} z^{-k} u(t)$