# Lecture Notes

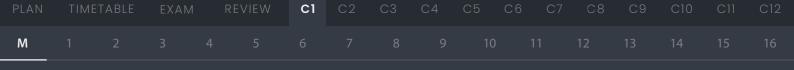
SDE

YEAR 2023/2024

## **Semester Review**

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	Course	Grade	Credit	Comment
1				
2				
3				
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12				



## Course 1

#### | Information

			☐ Major
			Elective
Credit	Time	R	oom
Professor	Phone	E-	mail
Notes			

#### Score

Midterm	Final	Grade

#### Attendance

| Week |
|------|------|------|------|------|------|------|------|
| 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    |
|      |      |      |      |      |      |      |      |
| Week |
| 9    | 10   | 11   | 12   | 13   | 14   | 15   | 16   |
|      |      |      |      |      |      |      |      |

#### Notice

### | Assignment

Deadline	Title	Contents

### | Curriculum

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#### Exam

Date	Title	Contents



Differential Expertise.

Title. END OF THE COURSE: Study of SDEs.

Date. 05/02/2024

Mo Tu We Th Fr Sa Su

Definitions: • solution of a SOF: a) b) g(t) conditions. • Stone solution. g(t) = g(t) + g(t) +

· Uniqueners in law.
· Robbwise uniqueners.

M.B. Note that when we speak about a solution of SDE(6,6), we do not fix a pribri the filtered proba-space and the brownian motion B. When we fix these objects, we will say so explicitly. I several notions of existence and uniqueness of SDEs.

Def [Strong solution]: A SDE has a strong solution, if for every continuous of standard BM  $B = (\Omega, \mathbb{R}, (\mathbb{F}_t)_t, (B_t)_t, \mathbb{R})$  and for every AV of defined on  $(\Omega, \mathbb{F}_t, (\mathbb{R}_t)_t, \mathbb{R})$ ,  $\mathbb{F}_t$ -measurable and following  $\mathcal{S}(\gamma)$ , there exists a process X on  $(\Omega, \mathbb{F}_t, (\mathbb{F}_t)_t, \mathbb{R})$  such that (X,B) is a solution of the SDE (of previous DEF).

The (on DEF) (X,B) is a strong solution of the SDE if X is adapted to the augmented natural filtration of  $B, i \in X$  is  $y_t$ -adapted.

C1

Differential Equation

course. Stochashe Title. END OF THE Course: Study of SDEs.

Date. 05/02/2024

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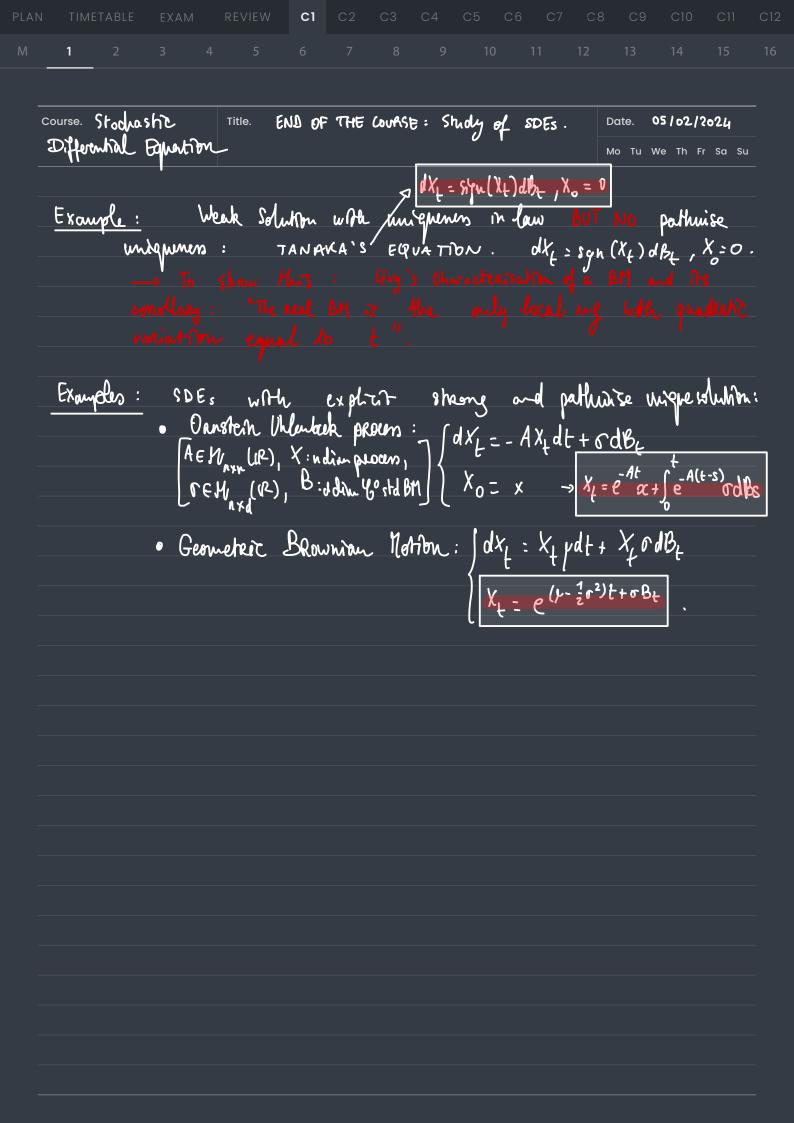
De [weak Solution]: We say that SDE has a weak solution if there exists a BM B: (n, F, (Ft)+, (B+)+, IP), a AV y defined on (I.F. (Fx)x, IP) such that y is Fy-measurable, y ~ L(q) and X defined on (1, F, (Ft), P) such that (X,B) is a solution of the SDE (cf 1th DEF).

FOR every BM B = (N. F, (Ft)t, (Bt), (P) we Example: \ \ dX\_t = \ \ X\_t dB\_t X = 1 can find the process such that (X,B)  $X_{t} = e^{\alpha \beta_{t} - \frac{1}{2}\alpha^{2}t}$  (XIB) is a Strong sometion.

Det [Weak uniqueness / uniqueness in law ]: There is uniqueness in law (or weak uniquenes) if, given two solutions  $(\Omega^i, \mathcal{F}^i, (\mathcal{F}_t^i)_t, (X_t^i)_t, (B_t^i)_t, (P^i)$  (if  $f_1, 2_f$ ) possibly defined on two different probability spaces and wat to two different BMs,  $(X^1, B^1)$  and  $(X^2, B^2)$  are equivalent, i.e have the same law, i.e: X2 and X2 have the same law. In other (simplus) words: if all the solutions have the same law.

Det Sathwise Uniqueres: There is pathwise uniqueness if, given two solutions  $(\Omega, \mathcal{F}, (\mathcal{F}_t)_t, (X_t^{\dagger})_t, (B_t), P)$  (if  $\{1,2\}$ ) defined on the same probability space and wat the same BM,  $\chi^2$  and  $\chi^2$  are indistinguishable , i.e.  $\mathbb{P}\left\{\left\{\forall t \in [0 \text{ it}], X_t^2 = X_t^2 \right\}\right\} = 1$ . In allow words: if, whenever the fittered probability space (s. F. (Ft), P) and the (F<sub>t</sub>)<sub>t</sub>-BM are fixed, two solutions X<sup>1</sup> and X<sup>2</sup> such that  $X_0^1 : X_0^2$  Ras are indistryuishable.

11.13. Sterry solution => Weak Solution; Bathwite Unique => Unique ix law.



Course. Stockashi Differential Equation

Title. END OF THE COURSE: Study of SDEs.

08/02/2023 Mo Tu We Th Fr Sa Su

(SDE): 
$$\begin{cases} dX_t = b(t, X_t)dt + O(t, X_t)dB_t \\ X_u = \gamma \end{cases}$$

b: [o,T] × 18h ---> 1Rh  $\sigma: [o_{iT}) \times IR^n \longrightarrow \mathcal{H}_{n\times d}(IR)$ 

1) b and or measurable in (1,x);

2) Sublinear growth: 3M>0 s.t YX(RM, Yte[OIT], [16(+,x)] & M(1+|x1) / [16(+,x)] & M(1+|x1) /

3) Lipschitz antimuty: 3L>0 s.t Vxy Elk", Yte[0,T], {|b(r,x)-b(r,y)| < L|x-y|



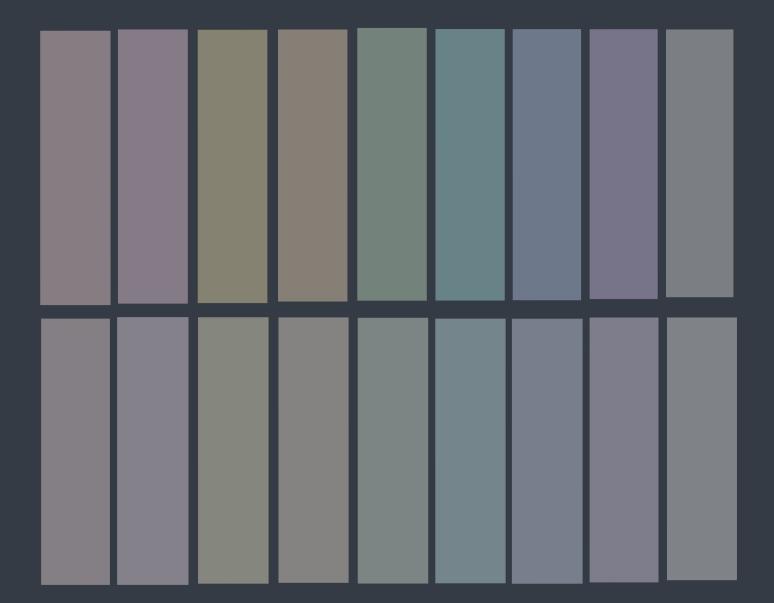
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## **Lecture Notes Sticker**



1 JAN 2 FEB 3 MAR 4 APR 5 MAY 6 JUN

7JUL 8 AUG 9 SEP 10 OCT 11 NOV 12 DEC

## **Lecture Notes Sticker**