POLITECNICO
MILANO 1863

Local Volatility Model Project

Mathematical Finance II - Longoni Riccardo

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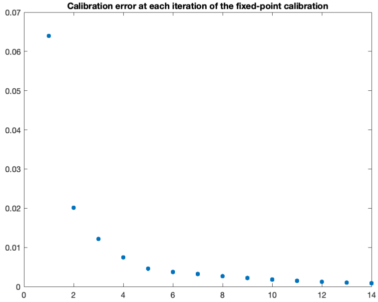
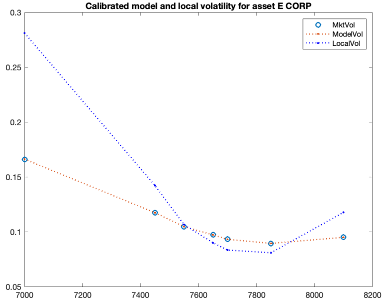
Question 1.1 - Data from E CORP





We can see that the calibration error (left) is decreasing, which confirms that the calibration behaves well. The calibration ends when calibration error is under 0.001. On the right, we can see the market, model and local volatilities, VS market strikes.

Question 1.2 - Data from E CORP

- We extract discount factors and spot price from the market data ;
- We use the "calibrate_r_q" function to find the risk-free rate r and dividend yield q ;
- Using the different values of strikes, we perform MC simulation to price the two call options (valuation formula) as well as the (spot) implied volatility (inversion of B&S formula). We obtain :

```
Monte Carlo LV Price for Kappa = 0.90: 734.5443
Monte Carlo LV Implied Volatility for Kappa = 0.90: 0.1646
Monte Carlo LV Price for Kappa = 1.10: 3.8208
Monte Carlo LV Implied Volatility for Kappa = 1.10: 0.0806
```

The drastic difference in call prices and implied volatilities between the two κ indicates a high sensitivity to the strike parameter. The simulated implied volatilities seem to align quite well with market expectations, suggesting that the model accurately represents market conditions.

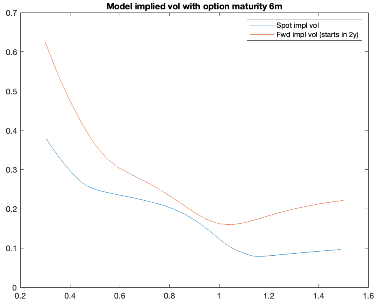
Question 1.3 - Data from E CORP

As before, we use MC simulations to first compute the price and then the implied forward volatility (inversion of B&S formula). We obtain :

```
Forward Starting Option MC price for Kappa = 0.90:  1232.9110
MC Implied Forward Volatility: 0.4786
Forward Starting Option MC price for Kappa = 1.10:  611.2922
MC Implied Forward Volatility: 0.4643
```

- Higher κ results in a lower option price and implied forward volatility, suggesting that the market expects lower volatility for out-of-the-money forward starting options ;
- However, forward starting options seem to be less sensitive to κ than call options.

Question 1.4 - Data from E CORP



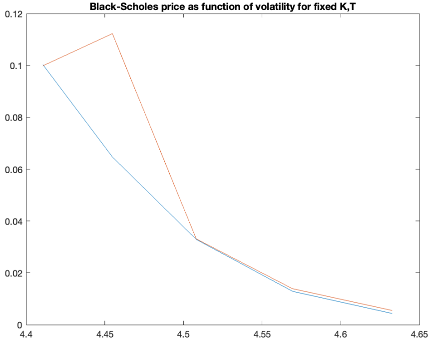
- The implied volatility smiles obtained show how implied volatilities vary with strike prices ;
- We obtain curves showing that volatility increases as the option becomes increasingly in or out -the-money ;

```
Skewness of the spot smile: -0.0339
Skewness of the fwd smile: 1.3298
```

- We can compute the skewness of the spot and forward smiles, i.e the slope of the spot and forward implied volatilities as a function of strike ;
- We can see that the spot smile has a skew close to zero, whereas the forward smile has a positive skew.

Question 2

- Abrupt jumps or discontinuities in the observed market data (like market volatility for example) might make it difficult for the local volatility model to fit the data well ;

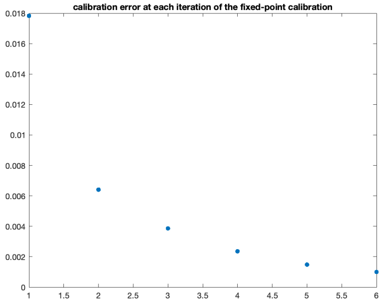


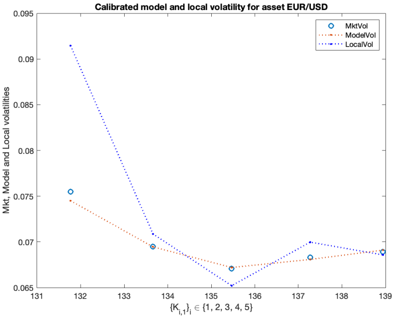
Question 2 bis

- If we fix T, we can check the convexity of $K_{i,j} \to C_0(T, K_{i,j})$: it is indeed not always convex, with some abrupt jumps ;
- The non-convexity of the previous curve might indicate that the model struggles to reproduce the observed option prices consistently across different strikes ;
- When we look at the market volatility, we can see a jump in the cell C14 which can explain the difficulty of the local volatility model to calibrate.

NB : We might need some "Stochastic Volatility Models" to capture these kind of sudden jumps.

Question 3.1





We first compute the (implicitely-defined) strikes by solving numerically $\eta(d_1(K)) = \Delta$. And then we can calibrate the LV model, checking that the calibration error decreases and falls under the threshold.

```
### MONTE CARLO - LOCAL VOLATILITY MODEL ###
Plain Vanilla Option:
Monte Carlo Price: 8.397631
Confidence Interval (95.00%): [8.349679, 8.445584]

Digital Option:
Monte Carlo Price: 0.784405
Confidence Interval (95.00%): [0.781845, 0.786965]
```

For the two options (plain-vanilla and digital), we price them using MC simulation (and the valuation formula). Then, we compute the mean and variance for each of the two options. Using the variance we can compute the MC error and then compute the two confidence intervals (using the confidence level).

```
### MONTE CARLO - BLACK DYNAMICS ###
Plain Vanilla Option:
Monte Carlo Price (under Black dynamics): 8.470223
Confidence Interval (under Black dynamics) (95.00%): [8.415714, 8.524731]

Digital Option:
Monte Carlo Price (under Black dynamics): 0.725324
Confidence Interval (under Black dynamics) (95.00%): [0.722548, 0.728100]
```

We do the same here but this time under Black's dynamics which is simpler in the sense that it doesn't need calibration of a model, the volatility is constant and taken directly from the market. We compute the MC prices and confidence interval in an analogous way.

Question 3.4

- First, we can see that the confidence intervals are very close in both cases (they even overlap for plain-vanilla options) ;
- The Black model assumes constant volatility, while the Local Volatility model allows for more flexibility by considering a volatility surface that can vary. In a sense, Black model is a simplification of LV model. So, in specific scenarios or time periods, the market may exhibit behavior that aligns with the simplified assumptions of the Black model.

Question 3.4 bis

- Why do both models give the same result for plain-vanilla options ?
  - First, plain vanilla options have a continuous payoff depending on the difference between the asset price and the strike price ;
  - Next, if the volatility is relatively constant, the two models should give quite the same results.

Question 3.4 ter

- The confidence intervals don't overlap for digital options...
- Why do models give different results for digital options ?
  - First, digital options have a binary payoff structure so either the option pays a fixed amount or nothing. A small change in modelisation can thus lead to very different conclusions ;
  - Next, the local volatility model captures the impact of changing volatility, which can result in very different price behaviors compared to the constant volatility assumption in the Black-Scholes model.