Univariate Time Series Modelling and Forecasting

Noé Debrois

June 16, 2024

1 Introduction

Univariate time series modelling aims to predict returns using only past values.

2 Notation and Concepts

- **Strictly Stationary Process**: The probability measure for the sequence $\{y_t\}$ is the same as that for $\{y_{t+m}\} \forall m$.
- **Weakly Stationary Process**: A series is weakly stationary if:
  1. $E(y_t) = \mu, \forall t$
  2. $\mathrm{Var}(y_t) = \sigma^2 < \infty$
  3. $\mathrm{Cov}(y_{t_1}, y_{t_2}) = \gamma_{t_2 - t_1}, \forall t_1, t_2$

3 Univariate Time Series Models

3.1 Covariance and Autocorrelation

- For a covariance stationary process, all variances are the same and covariances depend on the difference $t_1 - t_2$.
- **Autocovariances** γ_s: Covariances normalized by variance.
- **Autocorrelation Function (ACF)**: Plotting autocorrelations $\rho_s = \frac{\gamma_s}{\gamma_0}$ against s yields the correlogram.

3.2 White Noise Process

- Defined by $E(y_t) = 0$ and $\mathrm{Var}(y_t) = \sigma^2$ for all t.
- The ACF will be zero except for a peak of 1 at $s = 0$.
- Confidence intervals for significance testing can be constructed, e.g., for 95% CI: $\pm 1.96/\sqrt{T}$.

3.3 Joint Hypothesis Tests

- **Box-Pierce Q-statistic**: $Q = T \sum_{k=1}^{m} \rho_k^2$, asymptotically χ^2 distributed.
- **Ljung-Box statistic**: $\widetilde{Q} = T(T+2) \sum_{k=1}^{m} \frac{\rho_k^2}{T-k}$, used as a general test of linear dependence.

3.4 Moving Average (MA) Processes

- MA(q) model: $y_t = \mu + u_t + \theta_1 u_{t-1} + \theta_2 u_{t-2} + ... + \theta_q u_{t-q}$, where u_t are iid with $E(u_t) = 0$ and $\text{Var}(u_t) = \sigma^2$.
- Example: For $y_t = \theta_1 u_{t-1} + \theta_2 u_{t-2} + u_t$, calculate mean, variance, and ACF.

3.5 Autoregressive (AR) Processes

- AR(p) model: $y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + ... + \phi_p y_{t-p} + u_t$.
- Stationarity condition: Roots of $\phi(L) = 1 - \phi_1 L - \phi_2 L^2 - ... - \phi_p L^p$ lie outside the unit circle.
- Mean and ACF derived using Yule-Walker equations.

3.6 Partial Autocorrelation Function (PACF)

- PACF measures correlation between observations k periods apart, controlling for intermediate lags.
- For AR(p), PACF will be zero after lag p. For MA(q), PACF will decline geometrically.

3.7 ARMA Processes

- ARMA(p,q) model: Combines AR(p) and MA(q) processes: $y_t = \mu + \sum_{i=1}^{p} \phi_i y_{t-i} + \sum_{j=1}^{q} \theta_j u_{t-j} + u_t$.
- Stationarity and invertibility conditions similar to AR and MA processes.

3.8 Summary of ACF and PACF Behaviour

- AR process: Geometrically decaying ACF, number of PACF spikes equals AR order.
- MA process: Number of ACF spikes equals MA order, geometrically decaying PACF.

4 Examples and Exercises

- Exercises on calculating mean, variance, and ACF for given AR and MA processes.
- Example ACF and PACF plots for standard processes.