ARMA(1,1) process:

$$y(t) = \frac{1}{2}y(t-1) + \eta(t) - \eta(t-1), \quad \eta(\cdot) \sim WN(1,9).$$

$$y(t)\left(1 - \frac{1}{2}z^{-1}\right) = \eta(t)\left(1 - z^{-1}\right) \quad \text{so}$$

$$\boxed{y(t) = \underbrace{\frac{1-z^{-1}}{1-\frac{1}{2}z^{-1}}}_{=W(z)}\eta(t)} \quad \text{so} \quad \mathbb{E}[y(t)] = \frac{1-1^{-1}}{1-\frac{1}{2}1^{-1}}\mathbb{E}[\eta(t)] \quad \text{i.e}$$

$$\boxed{\mathbb{E}[\eta(t)] = 0} \quad \text{So} \quad \tilde{y}(t) = y(t) - m_y = y(t) - 0 = y(t) = W(z)e(t)$$

$$\text{i.e} \quad y(t) = W(z)(\eta(t) - 1) \qquad (e(\cdot) \sim WN(0,9)).$$

$$\gamma_y(\tau) = \gamma_{\tilde{y}}(\tau) \quad \text{so} \quad \gamma_y(\tau) = \mathbb{E}[y(t)y(t-\tau)]$$

$$y(t) = \frac{1 - z^{-1}}{1 - \frac{1}{2} z^{-1}} e(t) \quad , \quad e(t) \sim WN(0, 9).$$

Cov fct ?

$$y(t) = \frac{1}{2} y(t-1) + e(t) - e(t-1) \quad \text{so} \quad m_y = 0 \ \& :$$

$$\gamma_y(0) = \mathbb{E}[y(t)^2] = \mathbb{E}\left[\left(\frac{1}{2} y(t-1) + e(t) - e(t-1)\right)^2\right]$$

$$\gamma_y(0) = \frac{1}{4}\gamma_y(0) + 9 + 9 + \underbrace{\cancel{\mathbb{E}[y(t-1)e(t)]}}_{y(t-1) = fct(e(t-1), e(t-2), \dots)} - \mathbb{E}[y(t-1)e(t-1)] + 0$$

$$\frac{3}{4}\gamma_y(0) = 18 - \mathbb{E}[e(t-1)^2] = 18 - 9 = 9$$

So : $\boxed{\gamma_y(0) = \frac{4 \times 9}{3} = 4 \times 3 = 12.}$

$$\gamma_y(1) = \mathbb{E}[y(t)y(t-1)]$$

$$= \frac{1}{2}\mathbb{E}[y(t-1)^2] - \mathbb{E}[e(t-1)^2] = 6 - 9 = -3$$

$\boxed{\gamma_y(1) = -3}$