EM Waves in Medical Imaging

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PHY293

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Introduction

The use of electromagnetic waves in medical imaging has been a prominent method in locating and diagnosing various diseases that cannot be topologically identified. A developed understanding of electromagnetic waves has allowed for their effective application in the medical field, demonstrating the link between physics and medicine. This report will discuss the relevant physics, various wave behaviour, and properties of magnetic resonance imaging to highlight the pertinent phenomena.

Discussion

The transfer of energy in light-speed transverse waves allows for use in the medicinal imaging, primarily as a result of the way the waves interact with various tissues in the body.

For effective application in medical imaging, a thorough understanding of electromagnetic waves is required. J.C. Maxwell was responsible for deriving the four *Maxwell's Equations* that govern the behaviour of EM waves. The equations describe the relationships between electric charge, current, and fields and how they generate magnetic fields and are as follows[1]:

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}$$

While their derivation is overlooked here, the appreciation for them is not. The simplest solution to the second order partial differential equations for the electric field and magnetic field is surprisingly straightforward. The PDE resembles the general wave equation in 1D

$$\frac{\partial^2 E}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2}$$

$$\frac{\partial^2 B}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 B}{\partial t^2}$$

with solutions[2]:

$$E(x,t) = E_{max} cos(kx - \omega t)$$

$$B(x,t) = B_{max} cos(kx - \omega t)$$

with k as the wavenumber, related to wavelength by $k = \frac{2\pi}{\lambda}$. Since the two waves propagate perpendicularly, the resultant EM wave travels in the direction of their cross product, the x direction. Additionally, the solutions can be decomposed to Fourier transforms[3], one for the electric field and one for the magnetic field.

With the equation groundwork complete, the properties of EM waves can be discussed to gain an understanding of their importance in medical imaging.

Magnetic Resonance Imaging uses a magnetic field to align the spin of the hydrogen atoms' axes in the human body. A radio wave is then added to cause the axes of the hydrogen nuclei to oscillate at their resonant frequency. The frequency of this radio wave depends on the magnetic field strength and the frequency of the hydrogen spin. The MRI then measures the deviation of the axes of the atoms and so, the fundamental wave property employed in MRI is resonance, hence the name[4]. This axial deviation of the proton is known as *precession* and the *Larmor Frequency*, ω, is related to the magnetic field, B_0, by the relation, $\omega = \gamma B_0$, where γ is the gyromagnetic ratio in MHz/T[5]. Therefore, the resonance process here is engineered in order to achieve the medical image.

A fundamental property of electromagnetic waves is their ability to propagate in a vacuum. A medium is not required for energy to travel and so, EM waves are unaffected by wave dispersion in vacuums[6]. The speed of the wave is a function of the wavelength while the angular frequency is a function of the wavenumber, known as the *Dispersion Relation*, $\omega = v(k) \cdot k$,[6]. As such, the *Dispersion Relation* in a vacuum is linear and the angular frequency is proportional to the wavenumber since the wave speed, v, is equal to the speed of light, c. However, the electromagnetic fields that arise in MRI do not propagate through a vacuum, they propagate through human tissue. The tissue causes beta dispersion where high frequency waves can pass through while low frequency waves cannot[7]. The permittivity of tissue, ϵ_t, is different than it is for a vacuum, resulting in a decrease in phase speed. Additionally, both reflection and refraction can occur as the wave interacts with the tissue medium[8].

Standard practice is to use standing waves for MRI. However, at high magnetic fields, MRI equipment does not have a large field of view, as is the nature of the equipment[9]. To improve this, a proposition of using travelling waves to has been brought forth[10]. The conventional standing wave method is more efficient, but does require a lot of equipment positioned close to the subject. Each method has its drawbacks, but the upside potential is more important.

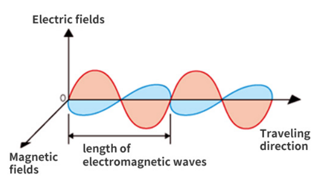

Figure 1: Propagation of electromagnetic waves[11]

Conclusion

Electromagnetic waves play a large role in our world, particularly in the field of medical imaging. Various wave properties and behaviour are exploited to improve our technology. The understanding of EM waves that Maxwell put forth has been instrumental in developing a harmless and effective method of medical imaging. The use of resonance, wave dispersion, and travelling waves (among other wave phenomena) has helped save countless lives and will continue to do so.

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