Neutron Stars

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Introduction

The collapse of a supergiant star after depleting all of its fuel results in the formation of a neutron star. This newly formed object is one of the densest observed celestial bodies and has properties worth admiring. Among these properties are its mass and radius, and methods to determine them rely heavily on thermal and statistical mechanics. Making assumptions to simplify calculations is an important aspect of improving our understanding. This report will explore such methods as well as discuss the relevance of understanding neutron stars.

Discussion

When calculating the radius of a neutron star, certain reasonable assumptions are made to simplify the process. The first of these is to assume $kT << \mu$ to the point where $T = 0$, resulting in the Heaviside Function; the temperature is low enough such that the energy states below the Fermi level have a probability of 1 to be consistent with the Pauli Exclusion Principle[1][2]. Assuming a Fermi gas based on low temperatures is reasonable since the limit where approximations are no longer valid is on the order of 10^{12}K, much hotter than a neutron star at 10^6K[2].

As well, we assumed the neutron star to be a box rather than a sphere, simplifying the kinetic energy calculation by using the energy levels for particles in a box[2]. However, the main assumption that we made was approximating uniform density in order to ignore relativistic effects. This limits the mass of the neutron star to a few solar masses, preventing the particle velocity from reaching relativistic magnitudes[2]. This is because more massive stars have greater energies, and classically, the energy is related to the velocity of the particle by

$$v = \sqrt{\frac{2\epsilon_F}{m}}$$

where ϵ_F is the Fermi level. Neglecting relativistic effects ensures that the mass does not exceed a certain threshold and thereby result in a black hole. Since these are assumptions, a certain accuracy is lost in the calculations. Assuming a uniform density and ignoring relativistic effects simplifies the calculations drastically and prevents the need for Schrodinger's Equation, but does result in an error in the results, albeit small[2].

Experimentally, neutron star properties are determined differently. Measuring a neutron star's mass is most effectively done when it is in a binary system[3]. The gravitational force that each body imparts on the other in tandem with Kepler's laws of motion allows for the calculation of the star's mass[4]. From Kepler's third law, the mass function of a binary system can be derived, which provides a minimum mass for the neutron star. The function is as such[4]

$$f = \frac{M_2^3 \sin^3 i}{(M_1 + M_2)^2} = \frac{K_1^3 P_b}{2\pi G}$$

where M_1 and M_2 are the masses of the bodies in the system, i is the angle at which their orbit is observed, P_b is the orbital period, and K_1 is the radial velocity of M_1. The minimum mass of the neutron star, M_2, can not exceed the Tolman-Oppenheimer-Volkoff Limit, M_{max}, without becoming a black hole, where M_{max} is between 2 and 3 solar masses[5]. Calculating this limit has proven very difficult since the Equation of State (EOS) for neutron stars is still largely unknown.

The EOS relates the mass of the star with its radius by way of a pressure and energy density relation of neutron star matter[6]. Since neutrons are fermions, the EOS derivation is based on Fermi-Dirac statistics that depend on Degeneracy Pressure of a Fermi gas where no more than one fermion can exist in a given quantum state[2]. This lends itself to the mass limit of a neutron star; the neutrons that make up the star matter cannot take up any less space without collapsing into a black hole. Using the Fermi gas EOS, the Tolman-Oppenheimer-Volkoff (TOV) equation that takes General Relativity into account can be integrated to determine the physical properties of the neutron star. The TOV equation is[7]

$$\frac{dp}{dr} = -\frac{G\epsilon(r)\mathcal{M}(r)}{c^2r^2}\left[1+\frac{p(r)}{\epsilon(r)}]\right]\left[1+\frac{4\pi r^3 p(r)}{\mathcal{M}(r)c^2}\right]\left[1-\frac{2G\mathcal{M}(r)}{c^2 r}\right]^{-1}$$

where p is pressure, ϵ the energy density, and $\mathcal{M}$ is the mass within radius r while the three bracketed terms are dimensionless quantities that account for relativistic effects. The TOV equation defines the structure of the star by relating various properties, allowing for radius to be determined by knowing mass, for instance. Since the pressure depends on the EOS, it suggests that there is a maximum mass of a neutron star, as mentioned above[7].

The *NICER* telescope was made to detect thermal X-rays from neutron stars and convert this into useful information, such as the EOS and the mass-radius relation[8]. Determining the EOS depends on the star's mass and radius and until proper measurements have been made, the EOS remains unknown[9].

Figure 1: Render of *NICER* telescope used to detect neutron stars[10]

To find the radius, *NICER* undergoes thorough calibration to detect the thermal X-ray waveform followed by data reduction where background radiation is removed and other interferences are filtered out[11]. From these experiments on a certain star, the radius was found to be between 11.96km and 14.26km,and the mass between 1.3 and 1.59 solar masses[12]. Along with other NICER experiments, these were found to be the most accurate and constrained results; determining these values from thermal X-ray waveforms are the most constrained and they provide a limited range of high density EOS[12].

Conclusion

Neutron stars are phenomena that spawn from the collapse of supergiant stars. Their large masses collapse into an object only several kilometers across, resulting in high dense matter with profound properties. Through the use of thermal and statistical mechanics, these can be measured using advanced telescopes to further our understanding of the universe.

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