kritiska punkter

① Hitta punkterna → $f_x(x,y) = f_y(x,y) = 0$

② klassificera punkterna

Metod #1
P(2) - Taylor ordn 2

Metod #2
Hessematrisen

$$P_2(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b) + \frac{1}{2}\left[f_{xx}(a,b)(x-a)^2 + f_{yy}(a,b)(y-b)^2 + 2f_{xy}(a,b)(x-a)(y-b)\right]$$

$\{f_x$ och $f_y = 0$ om ab är kritisk punkt$\}$

$$Q(x,y) = \frac{1}{2}\left(f_{xx}(a,b)(x-a)^2 + f_{yy}(a,b)(y-b)^2 + 2f_{xy}(a,b)(x-a)(y-b)\right)$$

	ex
om $Q > 0$ ∴ lokalt minimum	$4x^2+y^2$
om $Q < 0$ ∴ lokalt maximum	$-4x^2-y^2$
om $Q \lesseqgtr$ ∴ Sadelpunkt	h^2-hk

Hessematrisen!

$$H(x,y) = \begin{bmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{bmatrix} \Rightarrow H(a,b) = \begin{bmatrix} A & B \\ B & C \end{bmatrix}$$

Om $Det(H(x,y)) = AC - B^2 > 0$ och $A > 0$	Lokal Minpunkt
Om $Det(H(x,y)) = AC - B^2 > 0$ och $A < 0$	Lokalt Maximipunkt
Om $Det(H(x,y)) = AC - B^2 < 0$	Sadelpunkt

ex)

$$f_x = e^{x-y} - 1 + y$$

$$f_y = -e^{x-y} + 1 + x \qquad H = \begin{bmatrix} f_x & f_{yx} \\ f_{xy} & f_y \end{bmatrix}$$

$$f_{xy} = -e^{x-y} + 1$$

$$Det(H) = -e^{(x-y)^2} + e^{x-y} + xe^{x-y} - e^{(x-y)^2} + 2e^{x-y} - 1$$

$$= -2e^{(x-y)^2} +$$

$$Q(h,k) = \frac{1}{2}\left(f_{xx}(a,b)(x)^2 + f_{yy}(a,b)(y)^2 + 2\underbrace{f_{xy}}_{0}(x)(y)\right)$$

$-e^{x-y}+1$

0

$$f_x = e^{x-y} - 1 + y, \quad f_{xx} = e^{x-y} = 1$$

$$f_y = -e^{x-y} + 1 + x \quad f_{yy} = e^{x-y} = 1$$

$$Q = \left(\frac{x^2}{2} + \frac{y^2}{2}\right) > 0 \quad \therefore$$

Lokalt Minimum punkt!