Curl

- Curl beskriver rotation

Curl = $+$ och

Formel (2d)

$$\vec{V}(x,y) = \begin{bmatrix} P(x,y) \\ Q(x,y) \end{bmatrix}$$

$$\boxed{\mathrm{Curl}(\vec{V}(x,y)) = \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}}$$

ex)

$$\mathrm{curl}\left(\begin{bmatrix} y^3 - 9y \\ x^3 - 9x \end{bmatrix}\right) = 3x^2 - \cancel{9} - 3y^2 + \cancel{9} = 3x^2 - 3y^2$$

$P_1 = \{3, 0\}$ ger $3 \cdot 3^2 - 0 = 27 \therefore$ moturs rotation på punkt P_1

Curl - 3d

- Vektors längd bestämmer rotations hastighet
- höger hands regeln bestämmer riktningen

$$\bar{V}(x,y) = \begin{bmatrix} y^3 - 9y \\ x^3 - 9x \end{bmatrix} \begin{matrix} P \\ Q \end{matrix}$$

$$\text{Curl}\,\bar{V}_{2d} = 3x^2 - 3y^2 \qquad \text{ser} \qquad \underset{3d}{\text{Curl}}\,\bar{V} = \begin{bmatrix} 0 \\ 0 \\ 3x^2 - 3y^2 \end{bmatrix}$$

Formel! $_{3d}$

$$\bar{V}(x,y,z) = \begin{bmatrix} P(x,y,z) \\ Q(x,y,z) \\ R(x,y,z) \end{bmatrix}$$

$$\boxed{\text{Curl}\,\bar{V} = \begin{bmatrix} \partial/\partial x \\ \partial/\partial y \\ \partial/\partial z \end{bmatrix} \times \begin{bmatrix} P(x,y,z) \\ Q(x,y,z) \\ R(x,y,z) \end{bmatrix}}$$

$$\Rightarrow \begin{bmatrix} \frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z} \\ \frac{\partial P}{\partial z} - \frac{\partial R}{\partial x} \\ \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \end{bmatrix}$$

ex)

$$\begin{bmatrix} \partial/\partial x \\ \cdots \end{bmatrix} \times \begin{bmatrix} xy \\ \cdots \end{bmatrix}$$

$$\begin{bmatrix} \partial/\partial y \\ \partial/\partial z \end{bmatrix} \times \begin{bmatrix} \cos(z) \\ z^2+y \end{bmatrix}$$

$$= \begin{bmatrix} 1+\sin(z) \\ 0 \\ -x \end{bmatrix}$$