Trippel integraler

- över sfäriska kroppar (klot / halvt klot)

14.6.11)

$$\iiint_K (x^2+y^2)\,\underbrace{dV}_{dx\,dy\,dz} \qquad \text{(där } K \text{ är klotet)}$$

$K = x^2+y^2+z^2 \le a^2$ { radie a }

{ Byt till sfäriska koordinater }

$x = r\sin\phi\cos\theta$

$y = r\sin\phi\sin\theta$

$x = r\cos\phi$

byte av koordinater

① Byt integranden

② $dV = dx\,dy\,dz = r^2\sin\phi\,dr\,d\phi\,d\theta$

③ Byt gränserna $\quad 0 \le R \le a$, $0 \le \phi \le \pi$, $0 \le \theta \le 2\pi$

①

$$x^2+y^2 = \underbrace{r^2\sin^2\phi}\cos\theta^2 + \underbrace{r^2\sin^2\phi}\sin^2\theta$$

$$\Rightarrow r^2\sin^2(\phi)$$

$$\iiint_K (x^2+y^2)\,dV = \int_0^{2\pi}\int_0^{\pi}\int_0^{a} r^2\sin^2\phi\; r^2\sin\phi\,dr\,d\phi\,d\theta$$

{ Multiplikationslagen }

$$\Rightarrow \underbrace{\int_0^{2\pi} d\theta}_{\ell} \cdot \int_0^{\pi} \underbrace{\sin^3\phi}_{4/3}\,d\phi \cdot \int_0^{a} \underbrace{r^4}_{5}\,dr = \frac{8\pi}{15}a^5$$

2π

1/3

2/3

klassisk Integral

$$\int \sin^3(x)\,dx$$

$$= \int \sin x \cdot \sin^2(x)\,dx$$

$$= \int \sin x \cdot (1-\cos^2(x))\,dx$$

$u = \cos x \Rightarrow \frac{du}{dx} = -\sin(x)$

$$\Rightarrow \int \sin(x) \cdot (1-u^2)\,\frac{-du}{\sin(x)}$$

$$= \int u^2 - 1\,du$$

$$= \frac{1}{3}\cos^3(x) - \cos(x) + C$$