# Vektor + Scalar fields!

(Notationen)

$$\left[\vec{F}(x,y,z) = F_1(x,y,z)\vec{i} + F_2(x,y,z)\vec{j} + F_3(x,y,z)\vec{k}\right]$$

- A Scalar field is anothe name for a function med number som out-put.

- $\vec{F}$ är smooth om $F_1, F_2, F_3$ har konter-nuerlig Partiell derivata

- Om $f(x,y,z)$ är Scalar field Så är $\nabla f(x,y,z)$ ett Vector field

  Gradient field!

- Fält linjer!
  - $\vec{r} = \vec{r}(t)$

    $\frac{d\vec{r}}{dt}$ är Parallel till $\vec{F}(\vec{r}(t))$

    ser

    $$\frac{d\vec{r}}{dt} = \lambda(t)\,\vec{F}(\vec{r}(t))$$

Fält linjerna möste Vara

fältlinjerna måste vara

$$\left[\frac{dx}{F_1(x,y,z)} = \frac{dy}{F_2(x,y,z)} = \frac{dz}{F_3(x,y,z)}\right]$$

ex) Find field lines

$$\vec{F}(x,y,z) = -kn\frac{(x-x_0)\vec{i} + (y-y_0)\vec{j} + (z-z_0)\vec{k}}{((x-x_0)^2+(y-y_0)^2+(z-z_0)^2)^{3/2}}$$

Sol: $\vec{F}$ är täljaren

$\vec{F}$ är bara delen med vektorkomponenter (scalärer ska bort)

$$\underbrace{(x-x_0)}_{F_1}\vec{i} + \underbrace{(y-y_0)}_{F_2}\vec{j} + \underbrace{(z-z_0)}_{F_3}\vec{k}$$

$$\frac{dx}{(x-x_0)} = \frac{dy}{(y-y_0)} = \frac{dz}{(z-z_0)}$$

[Integrering]

$$\Rightarrow \ln(x-x_0) + \ln(C_1) = \ln(y-y_0) + \ln(C_2)$$

$$\Rightarrow C_1(x-x_0) = C_2(y-y_0) = C_3(z-z_0)$$

ex$_2$)

ex: Find the field lines of $\vec{F} = xz\vec{i} + 2x^2z\vec{j} + x^2\vec{k}$

soln: Solve the differential equations

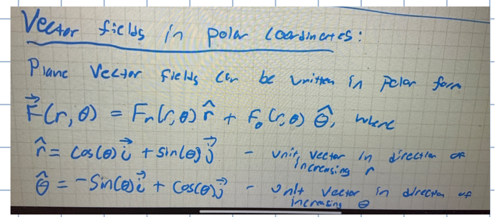
$$\frac{dx}{xz} = \frac{dy}{2x^2z} = \frac{dz}{x^2}$$

① $\frac{dx}{xz} = \frac{dy}{2x^2z}$ ⟷

• Vector fields i polar Coords.

$$\vec{F}(r,\theta) = F_r(r,\theta)\hat{r} + F_\theta(r,\theta)\hat{\theta}$$

$$\hat{r} = \cos(\theta)\vec{i} + \sin(\theta)\vec{j}$$


Vector fields in polar coordinates:

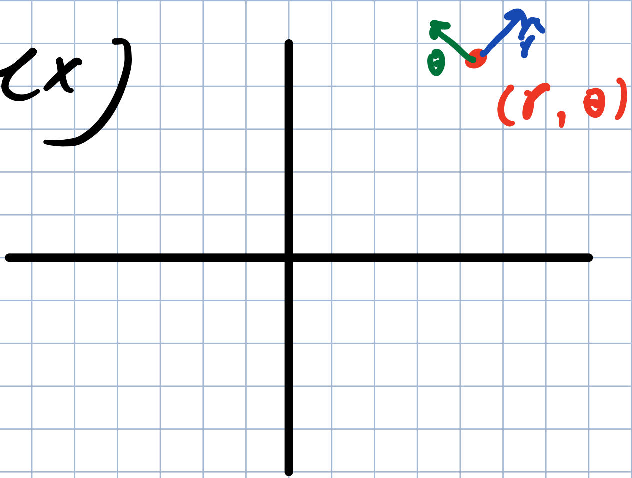
Plane vector fields can be written in polar form

$\vec{F}(r,\theta) = F_r(r,\theta)\hat{r} + F_\theta(r,\theta)\hat{\theta}$, where

$\hat{r} = \cos(\theta)\vec{i} + \sin(\theta)\vec{j}$ - unit vector in direction of increasing $r$

$\hat{\theta} = -\sin(\theta)\vec{i} + \cos(\theta)\vec{j}$ - unit vector in direction of increasing $\theta$

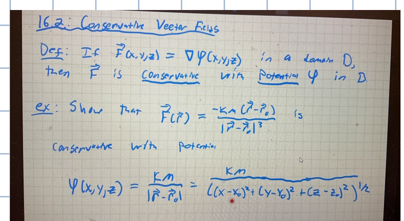
ℓ(x)




$$\hat{\theta} = -\sin(\theta)\hat{i} + \cos(\theta)\hat{j}$$

## Conservative Vector Fields

Def: on $\vec{F}(x,y,z) = \nabla\varphi(x,y,z)$


16.2: Conservative Vector Fields

Def: If $\vec{F}(x,y,z) = \nabla\varphi(x,y,z)$ in a domain D, then $\vec{F}$ is conservative with potential $\varphi$ in D.

ex: Show that $\vec{F}(\vec{r}) = \frac{-km(\vec{r}-\vec{r}_0)}{|\vec{r}-\vec{r}_0|^3}$ is conservative with potential

$$\varphi(x,y,z) = \frac{km}{|\vec{r}-\vec{r}_0|} = \frac{km}{((x-x_0)^2+(y-y_0)^2+(z-z_0)^2)^{1/2}}$$