SF1686 Flervariabelanalys
Exam (08:00-11:00)
Juni 4, 2021

No books/notes/calculators etc. allowed
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This exam consists of three parts: A, B and C; each worth 12 points. The bonus points from the seminars will be automatically added to the total score of part A, which however cannot exceed 12 points.
The thresholds for the respective grades are as follows:

Grade	A	B	C	D	E	Fx
Total sum	27	24	21	18	16	15
of which in part C	6	3	–	–	–	–

For full score you should define your notation; clearly explain the logical structure of your argument in words or symbols; and motivate and explain your argument.

Part A.

Question A1. Consider the vector field $\mathbf{F}(x, y, z) = (2xy, x^2 + 2yz, y^2)$ in $\mathbb{R}^3$.
a) Find the divergence div($\mathbf{F}$). **1 p.**
b) Find the rotation curl($\mathbf{F}$). **1 p.**
c) Find a potential function to $\mathbf{F}$, if it exists. **2 p.**

Question A2. The bull Ferdinand has escaped from the shadow of his beloved tree and is on a hill described by the graph of the function

$$f(x, y) = \frac{x - 2y}{1 + x^2 + y^2}.$$

The exact location of Ferdinand is given by the coordinates $(x, y, z) = (2, 1, 0)$.
a) In which direction in (x, y)-coordinates should Ferdinand go in order to go down the steepest? **2 p.**

b) Determine a trajectory (a curve) γ in (x, y) coordinates so that Ferdinand can walk without changing the height $z = 0$. **2 p.**

Question A3. Consider the function

$$g(x, y, z) = x^2 + 2y^2 + 3z^2.$$

a) Find all points on the level surface $g(x, y, z) = 5$, whose tangent plane is parallel with the plane that is given by the equation $x - 2y + 3z = 0$. **2 p.**

b) Let K be the body which is given by $K = \{(x, y, z) : \ g(x, y, z) < 5\}$. Find the mass of the body K, when the density of K is given by the density function $\rho(x, y, z) = z^2$. **2 p.**

Part B.

Question B1. Let D be the domain in $\mathbb{R}^2$ that is defined by the inequalities $x^2 + 4y^2 \le 4$,

$$f(x,y) = \frac{x+y}{x^2+4y^2+1}.$$

a) Find a parametrisation to the boundary of D: i.e. for the curve $x^2 + 4y^2 = 4$. **2 p.**

b) Compute the maximum value of the function f on the domain D. **4 p.**

Question B2. Compute the curve integral $\int_\gamma (2xy - x^2)dx + (3x+y)dy$ where γ is the positively oriented edge of the bounded area in the first quadrant bounded by the curves $y = x^2$, and $x = y^2$. **6 p.**

Del C.

Question C1. Compute the surface integral $\iint_{\mathcal{S}} f(x,y,z)dS$ where $f(x,y,z) = \frac{z}{\sqrt{x^2+z^2}}$ and the surface $\mathcal{S}$ is given by

$$\mathcal{S} = \{(x,y,z) :\ x^2+y^2+z^2 = 1, \quad z \ge 0, \quad -1/\sqrt{2} < y < 1/\sqrt{2}\,\}.$$

6 p.

Question C2. Let

$$p(a_0, a_1, a_2, x) = x^3 + a_2x^2 + a_1x + a_0$$

be a third degree polynomial with coefficients $a_0, a_1, a_2 \in \mathbb{R}$. We can write the polynomial as

$$(x - x_1)(x - x_2)(x - x_3)$$

where x_1, x_2 and x_3 are the roots of the polynomial. We may consider the roots to be functions of the coefficients a_0, a_1, a_2: $x_1(a_0, a_1, a_2)$, $x_2(a_0, a_1, a_2)$ and $x_3(a_0, a_1, a_2)$. We will assume that the roots are real functions of the coefficients.

a) Use the implicit function theorem to find conditions for the roots $x_1(a_0, a_1, a_2)$, $x_2(a_0, a_1, a_2)$ and $x_3(a_0, a_1, a_2)$ so that they are differentiable w.r.t. a_0, a_1 and a_2. **3 p.**

b) Let $\mathbf{a}(t) = (a_0(t), a_1(t), a_2(t))$ be differentiable curve in $\mathbb{R}^3$ such that $\mathbf{a}(0) = (-6, 11, -6)$. Then

$$p(a_0(0), a_1(0), a_2(0), x) = x^3 - 6x^2 + 11x - 6 = (x-1)(x-2)(x-3).$$

Find $\mathbf{a}'(0)$ if $\frac{dx_1(\mathbf{a}(t))}{dt}\lfloor_{t=0} = 0$, $\frac{dx_2(\mathbf{a}(t))}{dt}\lfloor_{t=0} = 0$ och $\frac{dx_3(\mathbf{a}(t))}{dt}\lfloor_{t=0} = 1$. Here x_1, x_2 and x_3 are the roots such that $x_1(-6, 11, -6) = 1$, $x_2(-6, 11, -6) = 2$ and $x_3(-6, 11, -6) = 3$. **3 p.**

Good Luck!