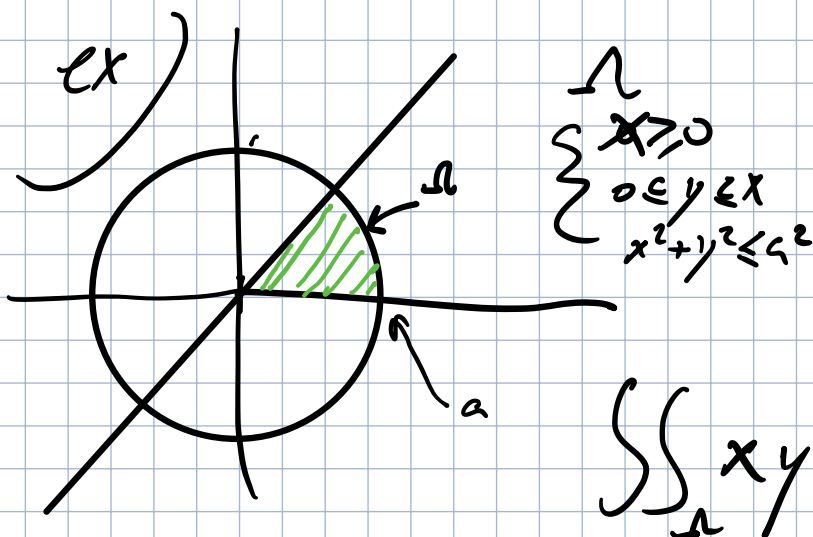


Cirkelområdet



$$\Omega = \begin{cases} x \geq 0 \\ 0 \leq y \leq x \\ x^2 + y^2 \leq a^2 \end{cases}$$

$$\iint_{\Omega} xy \, da$$

① Polar substitution - Byr Integranden

$$x = r \cos(\theta)$$

$$y = r \sin \theta$$

Skalfaktor!
 \downarrow
 $da = dx dy = r \, dr \, d\theta$

$$\iint_{\Omega} r \cos(\theta) r \sin(\theta) r \, dr \, d\theta$$

② Byr

$$= \iint_{\Omega} r^3 \cos(\theta) \sin(\theta) \, dr \, d\theta$$

$$\begin{cases} 0 \leq r \leq a \\ 0 \leq \theta \leq \frac{\pi}{4} \end{cases}$$

③ Byr
Gräns

$$\Rightarrow \int_0^a r^3 \, dr \cdot \int_0^{\pi/4} \cos \sin(\theta) \, d\theta$$

$$\left\{ \begin{array}{l} u = \sin \theta \\ \frac{du}{d\theta} = \cos(\theta) \Rightarrow d\theta = \frac{du}{\cos(\theta)} \end{array} \right\}$$

$$\Rightarrow \frac{a^4}{4} \cdot \int_0^{\pi/4} u \, du = \frac{a^4}{4} \cdot \frac{\sin^2(\frac{\pi}{4})}{2}$$

$$\Rightarrow \frac{a^4}{4} \cdot \frac{1}{2} = \frac{a^4}{8}$$