Line Integrals

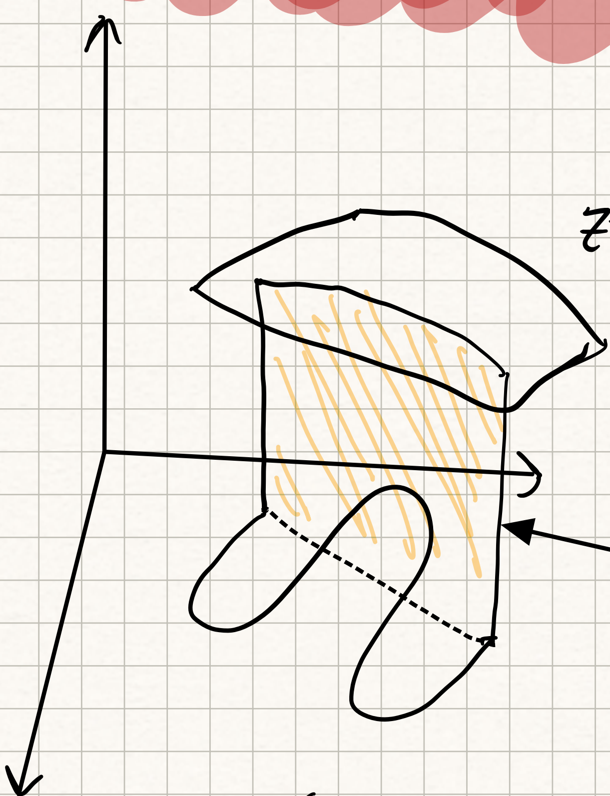
Potentiall funktion

$$\vec{F}(x,y,z) = 2xy,\ x^2 + 2yz,\ y^2 + 2z$$

- Gradienten av ϕ

$$F(x,y,z) = \phi\left(\frac{d\phi}{dx}, \frac{d\phi}{dy}, \frac{d\phi}{dz}\right)$$

Kurv Integralen är Arbetet på fältet

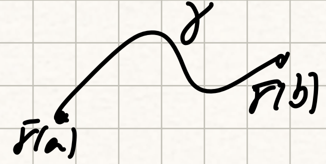


$z = f(x,y)$

$$\int_\gamma f(x,y)\, ds$$

Def: $\displaystyle\int_\gamma f(x,y)\, ds = \int_a^b f(\bar{r}(t)) \left\|\frac{d\bar{r}}{dt}\right\| dt$

- dt $\quad \bar{r}(t)$ parameter γ



Parametrisering av γ

Steg 1:

$$\begin{cases} x(t) = \cos(t) \\ y(t) = \sin(t) \end{cases}$$ där t har gränser

Steg 2:

$$\frac{d\bar{r}}{dt} = [x'(t), y'(t)] = [-\sin(t), \cos(t)]$$

$$\downarrow$$

$$\left\| \frac{d\bar{r}}{dt} \right\| = \sqrt{\cos^2 + (-\sin^2)} = 1$$

Kurvintegral Vektorfält

· Arbete $\quad \int_\gamma \bar{F} \cdot d\bar{r} = \int_\gamma (P(x,y), Q(x,y)) \cdot \overbrace{[dx, dy]}^{dr}$

vektorfält

$$\int_\gamma \bar{F} \cdot d\bar{r} = \int_a^b \vec{F}(\bar{r}(t)) \cdot \frac{d\bar{r}}{dt} \, dt$$

ex) kurvintegral

$\int \bar{F} \cdot d\bar{r} \qquad \bar{F} = (-y, x)$

$$\int_\gamma H \cdot d\gamma \qquad H(x,y) = (-y, x)$$

$$\gamma \begin{cases} y = 1 - x^2 \\ x \geq 0 \\ y \geq 0 \end{cases}$$

Steg 1:

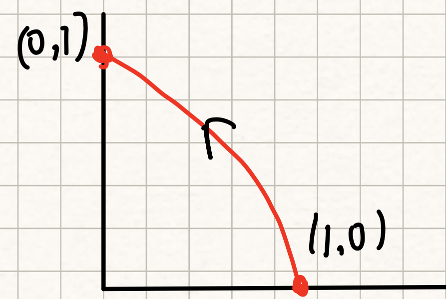
Param

$x = t$

$y(t) = 1 - t^2$

$\bar{r}(t) = (t, 1 - t^2)$

då t



$t : 1 \to 0$

pga x går $1 \to 0$

Steg 2

$$\bar{H}(\bar{r}(t)) = (-y, x) = (-1 + t^2, t)$$

$$\frac{d\bar{r}}{dt} = (1, 2t)$$

$$\bar{H}(\bar{r}(t)) \cdot \frac{d\bar{r}}{dt} = -1 + t^2 - 2t^2 = -t^2 - 1$$

Steg 3

$$\int_\gamma \bar{H} \cdot d\bar{r} = \int_1^0 -t^2 - 1 \, dt = \frac{4}{3}$$

1. Parametrisera
   - och hitta gränser i rätt riktning
2. Gör skalär produkt $H(\bar{r}(t)) \cdot \frac{d\bar{r}}{dt}$
3. utför integralen av skalär produkten