

Divergens Sats

• kompakt kropp \mathbb{R}^3

med sluten Rand

$$\oint_S \vec{F} \cdot d\vec{S} = \iiint_K \operatorname{div} \vec{F} dV$$

\nearrow Sluten \uparrow Rikning ut?

$dV = dx dy dz$

Underlåtande förklaring

• Centroiden av en kropp K

\nwarrow Centrum

om K har centroid (x_c, y_c, z_c)

$$\iiint_K x dV = x_c \cdot \operatorname{Vol}(K)$$

ex)

$$(x-2)^2 + y^2 + (z-3)^2 \leq 9$$

$$\vec{F}(x, y, z) = (x^2, y^2, z^2)$$

Gauss Sats

$$\oint_S \vec{F} \cdot \vec{n} = \iiint_K \operatorname{div} \vec{F} = \iiint_K (2x + 2y + 2z)$$

$$\oint F \cdot dS = \iiint_K \operatorname{div} F \, dV = \iiint_K (x+y+z) \, dV$$

↑ γ closed pointing out

$$(x-1)^2 + (y+1)^2 + (z-2)^2 \leq 4$$

$$\iiint_K (x+y+z) \, dx \, dy \, dz$$

$$\operatorname{Vol}(K) = \frac{4\pi \cdot 2^3}{3} = \frac{24\pi}{3} = 8\pi$$

$$\begin{cases} x = x_c \cdot \operatorname{Vol}(K) = 8\pi \cdot 1 \\ y = y_c \cdot \operatorname{Vol}(K) = -8\pi \\ z = z_c \cdot \operatorname{Vol}(K) = 8\pi \cdot 2 = 16\pi \end{cases}$$

$$2 \left(\iiint_K x \, dx \, dy \, dz + \iiint_K y \, dx \, dy \, dz + \iiint_K z \, dx \, dy \, dz \right)$$

$$\Rightarrow 2 \left(\operatorname{Vol}(K) \cdot x_c + y_c \cdot \operatorname{Vol}(K) + z_c \cdot \operatorname{Vol}(K) \right)$$

$$= 2 \left(8\pi - 8\pi + 16\pi \right) = \underline{\underline{32\pi}}$$