

How can vector calculus be used to model and analyze incompressible fluid flow in two-dimensional spaces, and what insights can this provide about real-world fluid systems?

Mathematics AA HL

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1 Introduction

1.1 Scope

This essay will for simplicity's sake only cover fluid flow in \mathbb{R}^2 spaces; an analysis of fluid flow in \mathbb{R}^3 spaces would be orders of magnitude more complex. Furthermore, only incompressible fluids ($\nabla \cdot \mathbf{F} = 0$), such that there are no sinks nor wells, will be analyzed.

1.2 Background

Vector calculus is the mathematical study of applying multi-variable calculus to vector valued functions, often for the spaces \mathbb{R}^2 and \mathbb{R}^3 .

$$f : \mathbb{R}^n \rightarrow \mathbb{R}^n \ni n > 1, n \in \mathbb{Z}$$
$$\rightsquigarrow \nabla \triangleq \left\{ \begin{array}{c} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \\ \vdots \end{array} \right\} n \text{ times} \quad (1)$$

1.3 Fluid dynamics

An incompressible fluid is any fluid such that $\nabla \cdot \mathbf{F} = 0$, which is to say that the divergence of the fluid is 0.

1.4 The Navier-Stokes equations

$$\frac{Df}{Dt} = \iiint_V \left(\frac{D\rho}{Dt} + \rho(\nabla \cdot \mathbf{u}) \right) dV \quad (2)$$

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