How can vector calculus be used to model and analyze incompressible fluid flow in two-dimensional spaces, and what insights can this provide about real-world fluid systems?

Mathematics AA HL

Word Count: 150

Contents

1 Introduction **2**

1.1 Scope 2

1.2 Background 2

1.3 Fluid dynamics 2

1.4 The Navier-Stokes equations 2

1 Introduction

1.1 Scope

This essay will for simplicity's sake only cover fluid flow in $\mathbb{R}^2$ spaces; an alaysis of fluid flow in $\mathbb{R}^3$ spaces would be orders of magnitude more complex. Furthermore, only incompressible fluids ($\nabla\cdot \mathbf{F} = 0$), such that there are no sinks nore wells, will be analyzed. Test

1.2 Background

Vector calculus is the mathematical study of applying multi-variable calculus to vector valued functions, often for the spaces $\mathbb{R}^2$ and $\mathbb{R}^3$.

$$f : \mathbb{R}^n \to \mathbb{R}^n \ni n > 1, n \in \mathbb{Z}$$
$$\rightsquigarrow \nabla \triangleq \left.\begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \\ \vdots \end{bmatrix}\right\} n \text{ times} \tag{1}$$

1.3 Fluid dynamics

An incompressible fluid is any fluid such that $\nabla\cdot \mathbf{F} = 0$, which is to say that the divergence of the fluid is 0.

1.4 The Navier-Stokes equations

$$\frac{\mathbf{D}f}{\mathbf{Dt}} = \iiint_V \left(\frac{\mathbf{D}\rho}{\mathbf{Dt}} + \rho(\nabla \cdot u)\right)dV \tag{2}$$

Lorem ipsum dolor sit amet