

How can vector calculus be used to model  
and analyze incompressible fluid flow in  
two-dimensional spaces, and what insights  
can this provide about real-world fluid  
systems?

Mathematics AA HL

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# 1 Introduction

## 1.1 Scope

This essay will for simplicity's sake only cover fluid flow in  $\mathbb{R}^2$  spaces; an analysis of fluid flow in  $\mathbb{R}^3$  spaces would be orders of magnitude more complex. Furthermore, only incompressible fluids ( $\nabla \cdot \mathbf{F} = 0$ ), such that there are no sinks nor wells, will be analyzed. Test

## 1.2 Background

Vector calculus is the mathematical study of applying multi-variable calculus to vector valued functions, often for the spaces  $\mathbb{R}^2$  and  $\mathbb{R}^3$ .

$$f : \mathbb{R}^n \rightarrow \mathbb{R}^n \ni n > 1, n \in \mathbb{Z}$$
$$\rightsquigarrow \nabla \triangleq \left. \begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \\ \vdots \end{bmatrix} \right\} n \text{ times} \quad (1)$$

$$\frac{\mathbf{D}T}{\mathbf{D}t} = \frac{\partial}{\partial t} + \vec{v} \cdot \nabla \quad (2)$$

$$\vec{v}_1 \otimes \vec{v}_2$$

## 1.3 Fluid dynamics

An incompressible fluid is any fluid such that  $\nabla \cdot \mathbf{F} = 0$ , which is to say that the divergence of the fluid is 0.

## 1.4 The Navier-Stokes equations

$$\frac{\mathbf{D}f}{\mathbf{D}t} = \iiint_V \left( \frac{\mathbf{D}\rho}{\mathbf{D}t} + \rho(\nabla \cdot \mathbf{u}) \right) dV \quad (3)$$

Lorem ipsum dolor sit amet [Peyret and Taylor, 2012]

## 2 References

[Peyret and Taylor, 2012] Peyret, R. and Taylor, T. D. (2012). *Computational methods for fluid flow*. Springer Science & Business Media.