How can vector calculus be used to model and analyze incompressible fluid flow in two-dimensional spaces, and what insights can this provide about real-world fluid systems?

Mathematics AA HL

Word Count: 86

Contents

1 Introduction

1.1 Scope

This essay will for simplicity's sake only cover fluid flow in $\mathbb{R}^2$ spaces; an alaysis of fluid flow in $\mathbb{R}^3$ spaces would be orders of magnitude more complex. Furthermore, only incompressible fluids ($\nabla \cdot \mathbf{F} = 0$), such that there are no sinks nor wells, will be analyzed. Test

1.2 Background

Vector calculus is the mathematical study of applying multi-variable calculus to vector valued functions, often for the spaces $\mathbb{R}^2$ and $\mathbb{R}^3$.

$$f : \mathbb{R}^n \to \mathbb{R}^n \ni n > 1, n \in \mathbb{Z}$$

$$\rightsquigarrow \nabla \triangleq \left.\begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \\ \vdots \end{bmatrix}\right\} n \text{ times} \tag{1}$$

$$\frac{\mathrm{D}f}{\mathrm{D}t} = \frac{\partial f}{\partial t} + \vec{v} \cdot \nabla f \tag{2}$$

$$\vec{v_1} \otimes \vec{v_2}$$

1.3 Fluid dynamics

An incompressible fluid is any fluid such that $\nabla \cdot \mathbf{F} = 0$, which is to say that the divergence of the fluid is 0.

1.4 The Navier-Stokes equations

$$\frac{\mathbf{D}f}{\mathbf{Dt}} = \iiint\limits_V (\frac{\mathbf{D}\rho}{\mathbf{Dt}} + \rho(\nabla \cdot u))dV \tag{3}$$

Lorem ipsum dolor sit amet [Peyret and Taylor, 2012]

2 References

[Peyret and Taylor, 2012] Peyret, R. and Taylor, T. D. (2012). *Computational methods for fluid flow*. Springer Science & Business Media.