

1 Topic A – Space, time and motion

1.1 Subtopic A.1 – Kinematics

A1:P11 Question 1

- a) The total distance is given as the sum of distance traveled during each translation, $2.5 + 3.8 = 6.3$ [km]
b) Displacement is a vector quantity, thus taking the magnitude of the sum of the two vector displacements caused by the movement:

$$\begin{aligned}\left\| \begin{pmatrix} 2.5 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 3.8 \end{pmatrix} \right\| &= \left\| \begin{pmatrix} 2.5 \\ 3.8 \end{pmatrix} \right\| \\ &= \sqrt{2.5^2 + 3.8^2} \approx 4.55 \text{ [km]}\end{aligned}$$

- c) Since this scenario can be set up as a right angle triangle, with the boat as the hypothesis' outer vertex, using trigonometry the angle can be determined as:

$$\tan \theta = \frac{o}{a} \rightsquigarrow \theta = \tan^{-1} \frac{2.5}{3.8} \approx 33.3^\circ$$

A1:P11 Question 2

- a) 15 minutes corresponds to a 90 degree ($\frac{\pi}{2}$ [rad]) rotation on a clock, or one quarter of its perimeter. Therefore the distance travelled by the tip of the pointer must be:

$$s = \frac{2\pi r}{4} = \frac{15\pi}{2} \approx 23.6 \text{ [cm]}$$

The displacement however is the distance between the points $\langle 0, 15 \rangle$ and $\langle 15, 0 \rangle$, thus using Pythagoras' theorem^a:

$$s = \sqrt{15^2 + 15^2} = \sqrt{450} = 21.2 \text{ [cm]}$$

- b) Analogously to part a) but for the 180 degrees (π [rad]) rotation resulting from 30 elapsed minutes:

$$s_{\text{distance}} = \frac{2\pi r}{2} = 15\pi \approx 47.1 \text{ [cm]}$$

$$s_{\text{displacement}} = \sqrt{0^2 + 30^2} = 30 \text{ [cm]}$$

A1:P12 Question 3

Since they are headed in completely opposite directions, $s_{\text{Ada}} + s_{\text{Matt}} = 580$ [m]. Ada's speed of 20 [km h^{-1}]

is approximately $5.56 \text{ [m s}^{-1}\text{]}$, as found by dividing through by 3.6. Since $s = \int v dt$:

$$v_{\text{Ada}}t + v_{\text{Matt}}t = 580$$

$$5.56 \times 60 + v_{\text{Matt}} \times 60 = 580$$

$$\therefore v_{\text{Matt}} \approx 4.11 \text{ [m s}^{-1}\text{]} \equiv \boxed{14.8 \text{ [km h}^{-1}\text{]}}$$

A1:P12 Question 4

First multiply the speed of light by the length of a light-year in seconds to obtain 1 [ly].

$$\begin{aligned} 1 \text{ [ly]} &= 3 \times 10^8 \text{ [m s}^{-1}\text{]} \times (60 \times 60 \times 24 \times 365) \text{ [s]} \\ &\approx 9.46 \times 10^{15} \end{aligned}$$

Then, find the conversion between 1 [au] and one light-year.

$$1 \text{ [au]} \approx \frac{1.5 \times 10^{11}}{9.46 \times 10^{15}} = 1.59 \times 10^{-5} \text{ [ly]}$$

Therefore,

$$5.5 \times 10^5 \text{ [au]} \approx \boxed{8.72 \text{ [ly]}}$$

A1:P14 Question 5

a) Calculating the speed between stations A and B:

$$v = \frac{1000}{80} = 12.5 \text{ [m s}^{-1}\text{]}$$

Multiplying by 3.6 to obtain the speed in $\text{[km h}^{-1}\text{]}$:

$$v = 12.5 \times 3.6 = \boxed{45 \text{ [km h}^{-1}\text{]}}$$

b) $\Delta y = 1800 - 1000 = 800 \text{ [m]}$

c) The train travels 800 [m] in 60 seconds, therefore, analogously to part a):

$$v = \frac{800}{60} \approx 13.3 \text{ [m s}^{-1}\text{]} = \boxed{47.9 \text{ [km h}^{-1}\text{]}}$$

A1:P14 Question 6

a) i)

$$\frac{4 \text{ [m]}}{10 \text{ [m s}^{-1}\text{]}} = \boxed{0.4 \text{ [s]}}$$

ii) The ball traveled for a total of 0.9 [s]. If then, it took 0.4 [s] to the wall, then it took $0.9 - 0.4 = 0.5 \text{ [s]}$.

The speed needed to travel 4 [m] in 0.5 [s] is:

$$\frac{4 \text{ [m]}}{0.5 \text{ [s]}} = \boxed{8 \text{ [m s}^{-1}\text{]}}$$

b)

Note that graph question solutions are not yet done due to lack of time.

A1:P16 Question 7

a) i) Imagining a straight line tangent to the curve (since $v = \frac{ds}{dt}$), it goes roughly 6 [m] in 1 [s], therefore the velocity is $\boxed{6 \text{ [m s}^{-1}\text{]}}$

ii) The tangent line at $t = 5 \text{ [s]}$ goes approximately from $\langle 2.5, 12.5 \rangle$ to $\langle 10, 28 \rangle$, thus

$$\Delta s = 28 - 12.5 = 15.5 \text{ [m]}$$

$$\Delta t = 10 - 2.5 = 7.5 \text{ [s]}$$

Therefore:

$$v = \frac{\Delta s}{\Delta t} = \frac{15.5}{7.5} \approx \boxed{2 \text{ [m s}^{-1}\text{]}}$$

b)

$$v = \frac{\Delta s}{\Delta t} = \frac{24}{12.5} = \boxed{1.92 \text{ [m s}^{-1}\text{]}}$$

A1:P16 Question 8

a) The perimeter of the track is $s = \frac{2\pi r}{2} = 25\pi \text{ [m]}$. Therefore, the average speed is:

$$v = \frac{25\pi}{19} \approx \boxed{4.13 \text{ [m s}^{-1}\text{]}}$$

b) For the velocity, we need the magnitude of the displacement which is 50 [m] since he went from one side of the circle to the other, one length of the diameter ($2r$). Therefore, the average velocity is:

$$v = \frac{50}{19} \approx \boxed{2.63 \text{ [m s}^{-1}\text{]}}$$

A1:P21 Question 9

Since $s = \int v dt$, the sum of the squares under the graph is the distance travelled. Estimating using triangles ($A = \frac{bh}{2}$):

$$\begin{aligned} s &= \frac{4 \times 1}{2} + \frac{2 \times 1}{2} + \frac{2 \times 3}{2} + 11 \\ &= 2 + 1 + 3 + 11 = 17 \text{ [m]} \end{aligned}$$

The triangles in this case underestimate the area, so rounding up we get 20 [m], making the answer \boxed{B} .

A1:P21 Question 10

The tangent line (since $a = \frac{dv}{dt}$) goes from roughly $\langle 0, 4 \rangle$ to $\langle 4, 8 \rangle$, thus:

$$\Delta v = 8 - 4 = 4$$

$$\Delta t = 4 - 0 = 4$$

$$a = \frac{\Delta v}{\Delta t} = \frac{4}{4} = 1 \text{ [m s}^{-1}\text{]}$$

Therefore the answer is A.

A1:P21 Question 11

At $t = 4$ [s], the max speed has been reached. Since the object was accelerated previously, the average speed is below the max line, thus the answer is either A or C. The tangent line (since $a = \frac{dv}{dt}$) at this point, however, is flat, meaning $a_{\text{instant}} = 0$. Thus the answer is A.

A1:P21 Question 12

If the velocity magnitude of the velocity is negative then it is moving in one direction, and if it is positive, then it moves in the oposite direction. Therefore, each time the velocity changes sign, the object is changing direction. The velocity changes sign twice, meaning the answer is B.

A1:P22 Question 13

a) Considering the absolute value of the graph, the velocity is first decreasing between $t = 0$ [s] and 1.5 [s] and then increasing between $t = 1.5$ [s] and 2.5 [s]. What happens however at $t = 1.5$ [s] is that, with the change of sign (vide **A1:21 Question 12**), is that the direction of travel changes.

b) i) The velocity decreases along a straight line, meaning acceleration is constant. Calculating the slope of the line between 2 nice points:

$$a = \frac{\Delta v}{\Delta t} = \frac{-2}{1.5} \approx \boxed{-1.33 \text{ [m s}^{-2}\text{]}}$$

ii) The positive and negatibe parts between $t = 0.5$ [s] and 2.5 [s] will cancel out, thus we only need to compute the area under the curve (since $s = \int v dt$) between $t = 0$ [s] and 0.5 [s]. To compute this we need to know the coordinates of the point $\langle 0.5, ? \rangle$. Since we know $a \approx -1.33 \text{ [m s}^{-2}\text{]}$, we can compute:

$$v_{0.5} \approx v_0 - 1.33 \times 0.5$$

$$= 1.335 \text{ [m s}^{-1}\text{]}$$

Therefore summing the rectangle and triangle below the curve:

$$s = 0.5 \times 1.335 + \frac{(2 - 1.335) \times 0.5}{2} \approx \boxed{0.834 \text{ [m]}}$$

c)

Note that graph question solutions are not yet done due to lack of time.

A1:P25 Question 14

a) Note that if the cart is returning to its origin, then the distance travelled s is 0. Using $s = ut + \frac{1}{2}at^2$:

$$\begin{aligned} 0 &= ut + \frac{1}{2}at^2 \\ &= 3t - 0.5 \times 1.8t^2 \\ \implies 0.9t^2 &= 3t \end{aligned}$$

Dividing by t discards the solution of $t = 0$ (since division by 0 is not mathematically allowed), however this solution is trivial (when the cart begins travelling it is at its origin):

$$\begin{aligned} 0.9t &= 3 \\ \therefore t &= \frac{3}{0.9} \approx [3.33 \text{ s}] \end{aligned}$$

b) When the velocity v is 0 the distance is maximum, since after the velocity changes sign (by passing zero), the object starts moving in the opposite direction. Using $v^2 = u^2 + 2as$:

$$\begin{aligned} 0 &= 3^2 - 2 \times 1.8s \\ \implies s &= \frac{9}{3.6} = [2.5 \text{ m}] \end{aligned}$$

A1:P25 Question 15

a) Converting 100 km h^{-1} to m s^{-1} by dividing by 3.6 means:

$$v = \frac{100}{3.6}$$

Then using $v = u + at$:

$$\begin{aligned} \frac{100}{3.6} &= 0 + 16a \\ \implies a &= \frac{100}{3.6 \times 16} \approx [1.74 \text{ ms}^{-2}] \end{aligned}$$

b) Converting 250 km h^{-1} to m s^{-1} by dividing by 3.6 means:

$$v = \frac{250}{3.6}$$

Using $v^2 = u^2 + 2as$ and the acceleration from a) we derive:

$$\begin{aligned}\frac{250^2}{3.6^2} &= 0 + 2 \times \frac{100}{57.6} s \\ \implies s &= \frac{250^2 \times 57.6}{200 \times 3.6^2} \approx [1400 \text{ [m]}]\end{aligned}$$

A1:P25 Question 16

Using $v^2 = u^2 + 2as$:

$$\begin{aligned}12^2 &= u^2 + 2 \times (-4.3) \times 25 \\ \implies u &= \sqrt{12^2 + 8.6 \times 25} \approx [18.9 \text{ [m s}^{-1}]\boxed{\text{]}}\end{aligned}$$

A1:P25 Question 17

a) To reach maximum velocity and be able to stop in time, the train must accelerate the first half of the distance and then immediately start de-accelerating. Therefore $s = 360$. Then using $v^2 = u^2 + 2as$:

$$\begin{aligned}v^2 &= 0 + 2 \times 1.3 \times 360 \\ \implies v &= \sqrt{720 \times 1.3} \approx [30.6 \text{ [m s}^{-1}]\boxed{\text{]}}\end{aligned}$$

b) The time is minimised if the velocity is maximised. Since the travel is symmetrical, considering the first half ($s = 360$) with maximum acceleration and using $s = ut + \frac{1}{2}at^2$:

$$\begin{aligned}360 &= 0 + \frac{1}{2} \times 1.3 \times t^2 \\ \implies t &= \sqrt{\frac{720}{1.3}}\end{aligned}$$

Since this was only half the distance, the total minimum travel time is $2t$:

$$2t = 2\sqrt{\frac{720}{1.3}} \approx [47.1 \text{ [s]}\boxed{\text{]}}$$

A1:P25 Question 18

a) Since we know the distance, initial velocity (0), and time, we can derive the acceleration through the SUVAT equation $s = ut + \frac{1}{2}at^2$:

$$\begin{aligned}12 &= 0 + 0.5a \times 4^2 \\ \implies a &= \frac{12}{0.5 \times 4^2} = \frac{12}{8} = 1.5 \text{ [m s}^{-2}]\end{aligned}$$

Then using $v = u + at$ we find the velocity at $t = 2$ is:

$$v = 0 + 1.5 \times 2 = 3 \text{ [m s}^{-1}\text{]}$$

The answer matching this is D.

b)

Note that graph question solutions are not yet done due to lack of time.

A1:P34 Question 19

If both projectiles reach the same height, that means that the vertical component of the initial velocity was the same, since:

$$\begin{aligned} u_{1y}t - \frac{1}{2}gt^2 &= \max y = u_{2y}t - \frac{1}{2}gt^2 \\ \implies u_{1y} &= u_{2y} \end{aligned}$$

If the vertical component of the initial velocity is the same, then the time taken to reach the ground is the same. This is because both are experiencing the same gravitational pull. Thus the answer is B.

A1:P34 Question 20

The horizontal distance travelled is proportional to the height $s \propto h$ since the time taken to hit the ground is proportional to the height $t_{\text{final}} \propto h$. The time taken to hit the ground $s_y = -h$, with 0 initial vertical velocity, can be computed as:

$$\begin{aligned} -h &= 0 - \frac{1}{2}gt_{\text{final}}^2 \\ \implies t_{\text{final}} &= \sqrt{\frac{2h}{g}} \end{aligned}$$

Note that the negative solution was discarded as it is physically impossible. Thus we derive that:

$$t_{\text{final}} \propto \sqrt{h}$$

Then, we compute that horizontal distance travelled (with initial horizontal velocity u , with no or negligible air resistance) as:

$$s = ut + 0$$

Therefore, the final distance travelled is s evaluated at t_{final} , meaning:

$$s \propto \sqrt{h}$$

Thus to reach a distance of $2s$, we need a height of $4h$, since $\sqrt{4h} = 2\sqrt{h}$. This means the answer is D.

A1:P34 Question 21

Using $s = ut + \frac{1}{2}at^2$:

$$\begin{aligned}s &= -4t - \frac{1}{2}gt^2 \\ &= -4 \times 1.9 - \frac{g}{2}(1.9)^2 \approx -25.3 \text{ [m]}\end{aligned}$$

This is the displacement of the object over that time, so the tower's height is $h = -s = 25.3$ [m]. The closest answer is therefore **D**.

A1:P34 Question 22

The time to hit the floor is given by, as derived in **A1:P34 Question 20**:

$$t_{\text{final}} = \sqrt{\frac{2h}{g}}$$

The horizontal displacement is then given through $s = ut + \frac{1}{2}at^2$ (noting air resistance is negligible, providing 0 acceleration):

$$\begin{aligned}s &= vt_{\text{final}} \\ &= v \sqrt{\frac{2h}{g}}\end{aligned}$$

The answer is therefore **C**.

A1:P34 Question 23

a) The maximum height is reached when the vertical velocity is 0, since the changing of a sign changes the direction of motion, making the height of the ball strictly decrease post that point. Therefore, using $v = u + at$:

$$\begin{aligned}0 &= u_y - gt_{\text{max}} \\ u_y &= gt_{\text{max}} = 0.9g \approx 8.84 \text{ [m s}^{-1}\text{]}\end{aligned}$$

b) Noting there is no air resistance, the horizontal velocity can be computed using $s = ut + \frac{1}{2}at^2$ as:

$$\begin{aligned}16 &= 0.9u_x + 0 \\ \implies u_x &= \frac{16}{0.9} \approx 17.8 \text{ [m s}^{-1}\text{]}\end{aligned}$$

Using Pythagoras theorem, we get that the total initial velocity is:

$$u = \sqrt{u_x^2 + u_y^2} = \sqrt{17.8^2 + 8.84^2} \approx 19.9 \text{ [m s}^{-1}\text{]}$$

c) Solving the equation $u \sin \theta = u_y$ (or $u \cos \theta = u_x$) gives:

$$\begin{aligned}\sin \theta &\approx \frac{8.84}{19.9} \\ \Rightarrow \theta &\approx \sin^{-1} \frac{8.84}{19.9} \approx 0.4603 \dots [\text{rad}]\end{aligned}$$

Converting to degrees (using $\theta_{\text{deg}} = \frac{180\theta_{\text{rad}}}{\pi}$):

$$\theta \approx [26.4^\circ]$$

d) Using $s = ut + \frac{1}{2}at^2$:

$$\begin{aligned}\max h &= u_y t_{\max} - \frac{1}{2}gt_{\max}^2 \\ &= 8.84 \times 0.9 - \frac{g}{2}0.9^2 \approx [3.98 \text{ [m]}]\end{aligned}$$

A1:P34 Question 24

Since there is 0 initial vertical velocity, the time to reach the net is (using $s = ut + \frac{1}{2}at^2$):

$$\begin{aligned}0.9 - 2.7 &= 0 - \frac{1}{2}gt_{\text{final}}^2 \\ \Rightarrow t_{\text{final}} &= \sqrt{\frac{2 \times 1.8}{g}} \quad (\text{Negative solution not possible}) \\ &\approx 0.605 \text{ [s]}\end{aligned}$$

Then to derive the initial horizontal velocity (with assumed 0 acceleration), we use $s = ut + \frac{1}{2}at^2$ again:

$$\begin{aligned}12 &= u_x t_{\text{final}} + 0 \\ \Rightarrow u_x &\approx \frac{12}{0.605} \approx [19.8 \text{ [m s}^{-1}]\end{aligned}$$

A1:P34 Question 25

a) The initial vertical velocity $u \sin \theta$ is approximately $0.628 \text{ [m s}^{-1}]$. Then using $s = ut + \frac{1}{2}at^2$ and checking for approximate equality at $t = 0.3 \text{ [s]}$:

$$\begin{aligned}-0.25 &\approx 0.628 \times 0.3 - \frac{1}{2}g(0.3)^2 \\ -0.25 &\approx -0.2535 \quad \checkmark\end{aligned}$$

b) i) The initial horizontal velocity $u \cos \theta$ is approximately $8.98 \text{ [m s}^{-1}]$. Using $s = ut + \frac{1}{2}at^2$ with assumed 0 acceleration:

$$s \approx 8.98 \times 0.3 + 0 = [2.69 \text{ [m]}]$$

ii) Only the vertical velocity is changing (due to the assumed 0 horizontal acceleration). Therefore, using

$v = u + at$:

$$v_{\text{final}} \approx 0.628 - 0.3g = -2.32 \text{ [m s}^{-1}\text{]}$$

Then using Pythagoras theorem to calculate the total final velocity:

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{8.98^2 + (-2.32)^2} \approx \boxed{9.27 \text{ [m s}^{-1}\text{]}}$$

c) i)

Note that graph question solutions are not yet done due to lack of time.

ii)

Note that graph question solutions are not yet done due to lack of time.

1.2 Subtopic A.2 – Forces and momentum

A2:P46 Question 1

Disregarding the initial moment, in which force is applied to propell the object, the only force acting on the object is gravity pull. Gravitational pull is always directed downwards, meaning the answer is D.

Note that what is 0 isn't the force, but the velocity (since the sign is changing).

A2:P46 Question 2

Ignore all vertical movement. Using $v^2 = u^2 + 2as$ (with $v = 0$ since the pellet comes to rest):

$$\begin{aligned} 0 &= 200^2 + 2a \times 0.1 \\ \implies a &= -\frac{40000}{0.2} = -200000 \text{ [m s}^{-2}\text{]} \end{aligned}$$

Then using Newton's second law ($F \stackrel{\text{N2}}{=} ma$), and only considering the magnitude (ignoring the – sign):

$$F = 0.002 \times 200000 = 400 \text{ [N]}$$

Thus, the answer is C.

A2:P47 Question 3

The average acceleration is computed using $v = u + at$ as:

$$\begin{aligned} 15 &= 0 + 0.01a \\ \implies a &= \frac{15}{0.01} = 1500 \text{ [m s}^{-2}\text{]} \end{aligned}$$

Then using Newton's second law ($F \stackrel{N2}{=} ma$):

$$F = 0.058 \times 1500 = \boxed{87 \text{ [N]}}$$

A2:P47 Question 4

a) First convert the speed units to $[\text{m s}^{-1}]$ by dividing through by 3.6:

$$\begin{cases} v = 12.5 \text{ [m s}^{-1}] \\ u = 22.2 \text{ [m s}^{-1}] \end{cases}$$

Then using $v^2 = u^2 + 2as$:

$$\begin{aligned} 12.5^2 &= 22.2^2 + 36a \\ \implies a &= \frac{12.5^2 - 22.2^2}{36} \approx -9.35 \text{ [m s}^{-2}] \end{aligned}$$

Therefore, the average force (only considering magnitude) must be:

$$F \stackrel{N2}{=} ma \approx 1200 \times 9.35 \approx \boxed{11220 \text{ [N]}}$$

b) Using $v = u + at$:

$$\begin{aligned} 12.5 &\approx 22.2 - 9.35t \\ \implies t &\approx \frac{12.5 - 22.2}{-9.35} \approx \boxed{1.04 \text{ [s]}} \end{aligned}$$

A2:P47 Question 5

a) Noting that Newton's second law states $F \stackrel{N2}{=} ma$, we see that:

$$a = \frac{F}{m} = \frac{400}{11000} = \frac{2}{55} \text{ [m s}^{-2}]$$

Then using $v = u + at$ (and noting that since it was stationary at $t = 0, u = 0$):

$$v = 0 + \frac{2}{55} \times 8 \approx \boxed{0.291 \text{ [m s}^{-1}]}$$

b) Using $s = ut + \frac{1}{2}at^2$:

$$\begin{aligned} s &= 0 + \frac{1}{2} \frac{2}{55} \times 8^2 \\ &= \frac{64}{55} \approx \boxed{1.16 \text{ [m]}} \end{aligned}$$

A2:P47 Question 6

a) The force acting on the electron is only acting along the vertical. Thus, if there is no force acting horizontally, then there is no acceleration horizontally. Without acceleration, the velocity remains constant (Newton's first law).

b) i) The time taken for the electron to travel the 25 [cm] is given by:

$$t = \frac{0.25 \text{ [m]}}{8 \times 10^6 \text{ [m s}^{-1}\text{]}} = 3.125 \times 10^{-8}$$

We then calculate the acceleration acting on the electron using Newton's second law:

$$a \stackrel{\text{N2}}{=} \frac{F}{m} = \frac{6.4 \times 10^{-17}}{9.11 \times 10^{-31}} = \frac{6.4}{9.11} \times 10^{14} \text{ [m s}^{-2}\text{]}$$

Finally using $s = ut + \frac{1}{2}at^2$:

$$\begin{aligned} s &= 0 + \frac{1}{2} \frac{6.4}{9.11} \times 10^{14} \times (3.125 \times 10^{-8})^2 \\ &\approx \boxed{3.43 \text{ [cm]}} \end{aligned}$$

ii) Using $v = u + at$ we get a vertical velocity of:

$$\begin{aligned} v_y &= 0 + \frac{6.4}{9.11} \times 10^{14} \times 3.125 \times 10^{-8} \\ &\approx 2.2 \times 10^6 \text{ [m s}^{-1}\text{]} \end{aligned}$$

Then using $\tan \theta = \frac{v_y}{v_x}$ (v_x is the velocity stated in the question):

$$\theta = \tan^{-1} \frac{2.2 \times 10^6}{8 \times 10^6} \approx \boxed{15.4^\circ}$$

A2:P47 Question 7

a)

Note that graph question solutions are not yet done due to lack of time.

If there is no acceleration, then no force other than gravity is acting on the person. Thus the scale, measuring force, will show just the weight force:

$$F_g \stackrel{\text{N2}}{=} mg = 75 \times 9.82 \approx \boxed{737 \text{ [N]}}$$

b)

Note that graph question solutions are not yet done due to lack of time.

The force driving the elevator upwards will push against the person in the elevator. By Newton's third

law, the person will then experience an equal force, but in the opposite direction. Since the force on the elevator acts upwards, the force on the person acts downwards with an acceleration of $2 \text{ [m s}^{-2}\text{]}$.

The magnitude of the force acting downwards on the person is given as:

$$F_{\downarrow} \stackrel{\text{N2}}{=} ma = 75 \times 2 = 150 \text{ [N]}$$

Therefore the magnitude of the resultant force F_R is given as (F_g was computed in a)):

$$F_R = F_g + F_{\downarrow} \approx 737 + 150 = \boxed{887 \text{ [N]}}$$

A2:P54 Question 8

a) Let the force of tension in each string be denoted F_t . Combined, the vertical component of both string's tension ($F_t \sin \theta$) force, $2F_t \sin \theta$, must balance the weight force of the object, F_g . The weight force of the object is given as:

$$F_g \stackrel{\text{N2}}{=} mg = 2g \approx 19.6 \text{ [N]}$$

Then to derive the angle θ , splitting the 150° in to two gives 75° . Then, in consideration of the right angle triangle on the side towards the weight, we get the angle $\theta = 90 - 75 = 15^\circ$. Finally, we solve the equation:

$$\begin{aligned} 2F_t \sin 15^\circ &= F_g \\ F_t &= \frac{19.6}{2 \sin 15^\circ} \approx \boxed{37.9 \text{ [N]}} \end{aligned}$$

b) The threads can only support a certain amount of tension. As the angle (let this be called ϕ) between the strings increases, the angle θ decreases. This is because (as demonstrated in a)):

$$\theta = 90 - \frac{\phi}{2}$$

The equation for the force of tension is proportional to (as demonstrated in a)):

$$\begin{aligned} F_t &\propto \frac{1}{\sin \theta} = \frac{1}{\sin 90 - \frac{\phi}{2}} \\ &= \frac{1}{\cos \frac{\phi}{2}} \\ \therefore F_t &\propto \frac{1}{\cos \phi} \end{aligned}$$

As ϕ increases towards 180° , $\cos \phi$ increases as well, meaning the force of tension, too, increases. Thus by increasing the angle, the threads are more likely to break.

A2:P54 Question 9

a)

Note that graph question solutions are not yet done due to lack of time.

b) The tension in each string has to support the weight force pulling on it. For the upper string, both A ($m_A = M$) and B ($m_B = 2M$) pull on it. Therefore:

$$\begin{aligned} F_{t\text{up}} &= F_{gA} + F_{gB} \\ &\stackrel{\text{N2}}{=} Mg + 2Mg = \boxed{3Mg} \end{aligned}$$

For the second string, only the weight of B is pulling on it, thus:

$$F_{t\text{down}} = F_{gB} = \boxed{2Mg}$$

A2:P54 Question 10

a) i) To form a 40° angle, the angle formed between the bob, the vertical and the end point of F if placed at the end of the weight force must be 40° . We therefore solve the equation:

$$\begin{aligned} \tan 40^\circ &= \frac{F}{0.5} \\ \implies F &\approx 0.839 \times 0.5 \approx \boxed{0.420 \text{ [N]}} \end{aligned}$$

ii) To stay in equilibrium and not break, the magnitude of the tension force in the string must be equal to the magnitude of the resultant. We thus calculate the magnitude of the resultant, using Pythagoras' theorem, as:

$$F_R \approx \sqrt{0.5^2 + 0.420^2} = \boxed{0.653 \text{ [N]}}$$

b) Since the forces remain unchanged, but there is no tension to hold them back anymore, the bob starts accelerating along the resultant. It thus moves away from the wall at the same direction the equilibrium was in (40° away from the negative vertical).

A2:P59 Question 11

When a spring is cut in half, the spring constant doubles, thus $k' = 2k$. Then, when placed in parallel, the total spring constant is the sum of the individual spring constants, thus:

$$k_{\text{total}} = k' + k' = 4k$$

Using Hooke's law, we know that since the object is in equilibrium:

$$\begin{aligned} F_g &\stackrel{\text{N2}}{=} mg = F_h = -kL \\ \implies mg &= -kL \end{aligned}$$

Applying the new total spring constant:

$$mg = -k'L' = -4kL'$$

Since both equations equal mg :

$$\begin{aligned} -kL &= -4kL' \\ \implies L' &= \frac{L}{4} \end{aligned}$$

The answer is thus A. We also conclude that for some spring constant $k' = nk$, the extension of the new system is $L' = \frac{L}{n}$.

A2:P59 Question 12

The spring constant of a system connected in series is given by:

$$\begin{aligned} \frac{1}{k'} &= \frac{1}{k_1} + \frac{1}{k_2} \\ &= \frac{1}{k} + \frac{1}{2k} = \frac{3}{2k} \\ \therefore k' &= \frac{2k}{3} \end{aligned}$$

We define using Newton's second law and Hooke's law the total extension L by:

$$\begin{aligned} F_g &\stackrel{\text{N2}}{=} mg = F_h = -k'L = -\frac{2k}{3}L \\ \implies mg &= -\frac{2k}{3}L \end{aligned}$$

The extension of the first spring is similarly given as:

$$mg = -kL_1$$

Equating both expressions:

$$\begin{aligned} -kL_1 &= -\frac{2k}{3}L \\ \implies L_1 &= \frac{2L}{3} \end{aligned}$$

Thus the answer fitting the total spring constant and extension is D.

A2:P61 Question 13

Let the density of object 1 and 2 be denoted ρ_1 and ρ_2 , and the density of water ρ_w . Dividing both sides

of the ratio $\frac{\rho_1}{\rho_2}$ by ρ_w gives:

$$\frac{\rho_1}{\rho_2} \stackrel{\text{let}}{=} r = \frac{\frac{\rho_1}{\rho_2}}{\rho_w}$$

The fraction of an object's volume which is below water is given as:

$$V' = \frac{\rho_o}{\rho_w}$$

We know 75% of the volume of object 1 is below the water, and 50% for object 2. Thus:

$$r = \frac{75\%}{50\%} = 1.5$$

This makes the answer A.

A2:P61 Question 14

a) The percentage of the volume of the cuboid under the water is the ratio between the submerged height and the total height. Therefore the percentage is given by:

$$\frac{2 \text{ [cm]}}{10 \text{ [cm]}} = 20\%$$

We then get the density of object using the fact that the ratio between an object's density and the fluid's density is the percentage of its submerged volume:

$$0.2 = \frac{\rho_o}{\rho_w} \implies \rho_o = 0.2\rho_w$$

Finally, we calculate mass using $m = V\rho$:

$$\begin{aligned} m &= 0.25 \times 0.25 \times 0.1 \times 0.2\rho_w \\ &= 0.00125\rho_w = 0.00125 \times 10^3 = \boxed{1.25 \text{ [kg]}} \end{aligned}$$

b) To sink, the whole 10 [cm] has to be submerged. The new mass is given as:

$$m = 1.25 + 4 = 5.25 \text{ [kg]}$$

We then need to calculate the new density of the cuboid, which is done as (using the volume computed in **a)**):

$$\rho_o = \frac{m}{V} = \frac{5.25}{0.25 \times 0.25 \times 0.1} = \frac{5.25}{0.00625} = 840 \text{ [kg m}^{-3}\text{]}$$

Finally, we compute the ratio of the object's density and the water's density:

$$\frac{\rho_o}{\rho_w} = \frac{840}{1000} \leq 1$$

Since the ratio is less than 1, all of the volume will not be submerged, making the object stay afloat.

A2:P62 Question 15

To float in equilibrium, the density of the balloon + the density of the gas have to be equal to the density of air. Therefore:

$$\rho_a = \rho_b + \rho_g$$

The density of the balloon ρ_b (sans gas) is computed as:

$$\rho_b = \frac{m}{V} = \frac{0.014}{\frac{4}{3}\pi(0.15)^3} \approx 0.990 \text{ [kg m}^{-3}]$$

Since the density of air ρ_a was given, we simply solve the original equation:

$$1.2 = 0.99 + \rho_g \implies \rho = 1.2 - 0.99 = \boxed{0.21 \text{ [kg m}^{-3}]}$$

A2:P62 Question 16

a) Using $F_a = \rho g V$ (the density of air ρ_a was given as $1.2 \text{ [kg m}^{-3}]$):

$$\begin{aligned} F_a &= 1.2g \times \frac{4}{3}\pi \times 0.03^3 \\ &\approx \boxed{1.33 \times 10^{-3} \text{ [N]}} \end{aligned}$$

b) Now the force of buoyancy does not support the ball's weight force, meaning the tension will have to make up for it. Previously:

$$\begin{aligned} F_{t\text{before}} &= F_g - F_t \\ &\stackrel{\text{N2}}{=} 0.0018 \times 9.82 - 1.33 \times 10^{-3} \approx 0.0163 \text{ [N]} \end{aligned}$$

Now:

$$F_{t\text{after}} = F_g \stackrel{\text{N2}}{=} 0.0018 \times 9.82 = 0.0177 \text{ [N]}$$

Thus the percentage multiplier is given by the ratio $\frac{F_{t\text{after}}}{F_{t\text{before}}}$:

$$\frac{F_{t\text{after}}}{F_{t\text{before}}} = \frac{0.0177}{0.0163} \approx 1.0859$$

Therefore the percentage change is $1 - \text{the multiplier}$:

$$1 - 1.0859 = \boxed{8.59\%}$$

A2:P67 Question 17

a) First convert the speed $50 [\text{km h}^{-1}]$ to $[\text{m s}^{-1}]$ by dividing by 3.6:

$$u = \frac{50}{3.6} \approx 13.9 [\text{m s}^{-1}]$$

Using $v^2 = u^2 + 2as$:

$$\begin{aligned} 0 &= 13.9^2 + 50a \\ \implies a &\approx -\frac{193}{50} = \boxed{-3.86 [\text{m s}^{-2}]} \end{aligned}$$

b) The maximum force the static friction can handle is given as:

$$\max F_{\mu_s} = \mu_s F_N$$

In this case, $F_N = F_g$. Therefore:

$$\max F_{\mu_s} \stackrel{\text{N2}}{=} 0.45mg \approx 4.419m [\text{N}]$$

The acceleration of the truck will apply force to the box, which is computed using Newton's second law as:

$$F_{\text{truck}} \stackrel{\text{N2}}{=} ma \approx 3.86m [\text{N}]$$

Thus, we see that the force applied by the truck $F_{\text{truck}} < \max F_{\mu_s}$, meaning the static friction will keep the box stationary.

A2:P67 Question 18

The force applied by the dynamic friction opposing the motion is given by:

$$F_{\mu_d} = \mu_d F_N$$

In this case, $F_N = F_g \stackrel{\text{N2}}{=} mg \approx 19.6 [\text{N}]$. Consequently, the acceleration caused by the friction (using Newton's second law) is:

$$a = \frac{F_{\mu_d}}{m} = \frac{0.1 \times 19.6}{2} = 0.982 [\text{m s}^{-2}]$$

Then, using $v^2 = u^2 + 2as$:

$$v = \sqrt{8^2 - 2 \times 0.982 \times 16}$$

$$\approx [5.71 \text{ m s}^{-1}]$$

A2:P67 Question 19

a) Using $s = ut + \frac{1}{2}at^2$:

$$0.5 = 0 + \frac{1}{2}a \times 3.8^2$$

$$a = \frac{1}{3.8^2} \approx [0.0693 \text{ m s}^{-2}]$$

b) The angle the gravitational force (straight down) makes with the ramps incline is 15° . Thus the component of the weight of the box parallel to the ramp is:

$$F_{g\text{ramp}} = F_g \sin 15^\circ$$

$$\stackrel{\text{N2}}{=} 2 \times 9.82 \times \sin 15^\circ$$

$$\approx [5.08 \text{ N}]$$

c) The resultant force experienced by the box is given using Newton's second law as:

$$F_R \stackrel{\text{N2}}{=} ma = 2 \times 0.0693 = 0.139 \text{ N}$$

The two forces working in the direction of motion is the force of friction, the force of gravity parallel to the ramp, and the pull force, meaning:

$$F_R = 8 - F_{g\text{ramp}} - F_\mu$$

$$\implies F_\mu = 8 - F_{g\text{ramp}} - F_R = 8 - 5.08 - 0.139 \approx [2.78 \text{ N}]$$

d) The normal force is the component of gravity perpendicular to the ramp, which is given as:

$$F_N = F_g \cos 15^\circ$$

$$\stackrel{\text{N2}}{=} 2 \times 9.82 \times \cos 15^\circ 19.0 \text{ N}$$

Finally, since frictional force is defined as $F_\mu = \mu_d F_N$:

$$2.78 = \mu_d 19.0$$
$$\therefore \mu_d = \frac{2.78}{19.0} \approx \boxed{0.146}$$

A2:P67 Question 20

a) For the static friction to support the book, $\mu_s F_N \geq F_g$. We choose the minimum ($\mu_s F_N = F_g$). The normal force is in this case the force F being applied to the book against the wall (since the normal force is the wall pushing back on the book). Since the book's weight (F_g) and μ_s were given, we simply solve the equation:

$$0.75F = 12$$
$$\implies F = \frac{12}{0.75} = \boxed{16 \text{ [N]}}$$

b)

Note that graph question solutions are not yet done due to lack of time.

c) The force of dynamic friction opposing the motion is given as:

$$F_\mu = \mu_d F_N$$
$$= 0.6 \times 10 = 6 \text{ [N]}$$

The resultant force F_R is therefore:

$$F_R = F_g - F_\mu$$
$$= 12 - 6 = 6 \text{ [N]}$$

This is half of the original weight force, meaning that since the mass is constant, the acceleration is $\frac{g}{2}$.

This is approximately:

$$\frac{g}{2} = \boxed{4.91 \text{ [m s}^{-2}\text{]}}$$

A2:P72 Question 21

Since the skydiver cannot magically start falling upwards because of the parachute, the direction of the velocity remains constant. At t_1 however, the speed was increasing, meaning there was positive acceleration. At t_2 , the opposite is true, meaning there was negative acceleration. Thus, the answer is B.

A2:P72 Question 22

The terminal velocity is given by the formula:

$$v_t = \frac{(\rho_s - \rho_f)gV}{6\pi\eta r}$$

Since $V = \frac{4}{3}\pi r^3$,

$$v_t \propto \frac{r^3}{r} \implies v_t \propto r^2$$

Therefore, if the radius is 2 times larger, then the terminal velocity is $2^2 = 4$ times larger, making the answer C.

A2:P72 Question 23

There are two forces acting on the ball, buoyancy F_a and drag force F_δ . The buoyancy force is given by Archimede's principle as:

$$F_a = \rho_f g V$$

where ρ_f is the density of the fluid. Since $m = \rho_s V$ and $\rho_s = 1.5\rho_f$:

$$F_a = \rho_f g \frac{m}{1.5\rho_f} = \frac{mg}{1.5} \stackrel{\text{N2}}{=} \frac{F_g}{1.5}$$

The resultant force is zero (since there is no acceleration), and is given by:

$$\begin{aligned} F_R &= 0 = F_g - F_a - F_\delta \\ \implies F_\delta &= F_a - F_g \\ &= \frac{F_g}{1.5} - F_g \\ &= \frac{1.2}{1.5} - 1.2 = -0.4 \text{ [N]} \end{aligned}$$

The magnitude of the drag force is therefore 0.4 [N], making the answer A.

A2:P72 Question 24

a) Drag force F_δ is proportional to speed whilst weight force and buoyancy are constant. Therefore, the drag force keeps increasing as the ball accelerates downwards until it reaches a velocity (the terminal velocity), in which all of the forces balance, and the ball experiences net zero force. This net zero means 0 acceleration, keeping the velocity at this point.

b) i) Since $m = \rho V$ and $V = \frac{4}{3}\pi r^3$,

$$\begin{aligned} m &= 8000 \times \frac{4}{3}\pi \times 0.002^3 \\ &\approx 0.000268 \text{ [kg]} \end{aligned}$$

The weight is then given by Newton's second law as:

$$F_g \stackrel{N2}{=} mg \approx 0.000268 \times 9.82 \approx [2.63 \times 10^{-3} \text{ N}]$$

ii) By Archimede's principle:

$$\begin{aligned} F_a &= \rho g V \\ &= 920 \times 9.82 \times \frac{4}{3}\pi \times 0.002^3 \\ &\approx [3.03 \times 10^{-4} \text{ N}] \end{aligned}$$

c) Using Stoke's law:

$$\begin{aligned} v_t &= \frac{(\rho_s - \rho_f)gV}{6\pi\eta r} \\ &= \frac{(8000 - 920) \times 9.82 \times \frac{4}{3}\pi \times 0.002^3}{6\pi \times 8.4 \times 10^{-2} \times 0.002} \\ &\approx [0.736 \text{ m s}^{-1}] \end{aligned}$$

A2:P75 Question 25

a) Using $v = u + at$ (with v being in the oposite direction of u , thus negative):

$$\begin{aligned} -6 &= 9 + 0.05a \\ \implies a &= \frac{-15}{0.05} = [-300 \text{ m s}^{-2}] \end{aligned}$$

b) By Newton's second law (considering only magnitude):

$$F \stackrel{N2}{=} ma = 0.4 \times 300 = [120 \text{ N}]$$

A2:P75 Question 26

a) i) The pellet looses all momentum, which means the change in momentum is:

$$\Delta p = mv = 0.002 \times 180 = [0.36 \text{ N s}]$$

ii) The acceleration is computed using Newton's second law as:

$$a \stackrel{N2}{=} \frac{F}{m} = \frac{750}{0.002} = 375000 \text{ m s}^{-2}$$

Then using $v = u + at$:

$$0 = 180 - 375000t \implies t = \frac{180}{375000} = \boxed{4.8 \times 10^4 \text{ [s]}}$$

b) Using $v^2 = u^2 + 2as$:

$$\begin{aligned} 0 &= 180^2 - 2 \times 375000s \\ \implies s &= \frac{180^2}{750000} = \boxed{0.0432 \text{ [m]}} \end{aligned}$$

c) Because it assumes constant acceleration (and force) from the average force. This is not realistic, and the acceleration likely varies over time as the pellet penetrates the block.

A2:P77 Question 27

The area under the graph is the change in momentum. Therefore:

$$\begin{aligned} \Delta p &= \frac{(1200 - 0)(3.5 - 2) \times 10^{-3}}{2} + \frac{(1200 - 0)(7 - 3.5) \times 10^{-3}}{2} \\ &= 3 \text{ [N s]} \end{aligned}$$

Since $p = mv$, the change in velocity is thus:

$$\Delta v = \frac{\Delta p}{m} = \frac{3}{0.15} = 20 \text{ [m s}^{-1}\text{]}$$

Thus (noting that since it's a change in direction, we let the original $8 \text{ [m s}^{-1}\text{]}$ be negative):

$$v_{\text{post hit}} = 20 - 8 = 12 \text{ [m s}^{-1}\text{]}$$

Consequently, the answer is B.

A2:P77 Question 28

Let x be the height of one unit square. Each square has a width of 10^{-2} . The change in momentum is (very approximately by counting rough squares) then:

$$\Delta p = 10 \times (x \times 10^{-2}) = \frac{x}{10}$$

Since the change in momentum was 10 [N s] , we let $x \approx 100$. Finally, because F_{max} is 5 units up, $F_{\text{max}} \approx 5x = 500 \text{ [N]}$. The closest answer is therefore D.

A2:P77 Question 29

a) i) The change in momentum between $t = 0$ and 0.5 [s] is 0.75 [N s] . The only force acting to change

this is the weight force. Therefore:

$$F_g = \frac{\Delta p}{t} = \frac{0.75}{0.5} = \boxed{1.5 \text{ [N]}}$$

ii) Using $v = u + at$:

$$v = 0 + 0.5g \approx \boxed{4.91 \text{ [m s}^{-1}\text{]}}$$

b) The change in momentum during the contact (between $t = 0.5$ and 0.6 [s]) is $0.5 - (-0.75) = 1.25$ [N s].

Dividing this by the contact time of 0.1 [s] gives us the average force:

$$\bar{F} = \frac{1.25}{0.1} = \boxed{12.5 \text{ [N]}}$$

A2:P79 Question 30

a) i) The thrust force is given by $F_\tau = \dot{m}v_e$ where $\dot{m} = \frac{dm}{dt}$ is the rate of change of the mass expulsion and v_e the velocity of the explused material. Therefore:

$$F_\tau = 2.8 \times 3.6 \times 10^3 \approx \boxed{10000 \text{ [N]}}$$

ii) Using Newton's second law:

$$a \stackrel{\text{N2}}{=} \frac{F_\tau}{m} = \frac{10000}{4 \times 10^4} = \boxed{0.25 \text{ [m s}^{-2}\text{]}}$$

b) The thrust force remains constant (since \dot{m} and v_e are by design constant), but the mass of the rocket decreases. By Newton's second law:

$$a \stackrel{\text{N2}}{\propto} \frac{1}{m}$$

If m therefore decreases, the acceleration increases. When the rocket then stops thrusting, the acceleration becomes g .

c) The final mass is $m = 40000 - 2.8 \times 25 \times 60 = 35800$ [kg]. Therefore the final acceleration is:

$$a \stackrel{\text{N2}}{=} \frac{F_\tau}{m} = \frac{10000}{35800} \approx 0.279 \text{ [m s}^{-2}\text{]}$$

Using $v = \bar{a}t$:

$$v = \frac{0.25 + 0.279}{2} \times 25 \times 60 \approx \boxed{397 \text{ [m s}^{-1}\text{]}}$$

Note that this answer differs from the one given in the answer sheeet provided by Oxford. However, there ansewr only seems to use the final acceleration, which would assume constant acceleration across, when the question before states clearly this is not the case. I therefore decided to keep my answer which I believe to be more correct.

d) Using $F_\tau = \dot{m}v_e$:

$$65000 = 3.6 \times 10^3 \times \dot{m}$$
$$\implies \dot{m} = \frac{65000}{3600} \approx \boxed{18.1 \text{ [kg s}^{-1}\text{]}}$$

A2:P85 Question 31

a) The only part which changed is that it went from $-p_y$ to p_y where p_y was the magnitude of the vertical momentum. The change is thus:

$$\Delta p = p_y - (-p_y) = 2p_y = 2mv \sin \theta \text{ [kg m s}^{-1}\text{]}$$

This makes the answer C.

b) Re-iterating what was stated in a), the only part which changed was the vertical momentum. It went from pointing down to pointing up. Thus the vector must point up, making the answer D.

A2:P85 Question 32

Recall that the momentum must be conserved. The total momentum (positive direction to the right) before the collision was:

$$p = 2mv - mv = mv \text{ [kg m s}^{-1}\text{]}$$

After the collision, the carts stick together. We thus think of them as one object with combined mass $2m$. Therefore, using the definition $p = mv$:

$$mv = 2m \implies v = \frac{v}{2} = 0.5m \text{ [m s}^{-1}\text{]}$$

This makes the answer A.

A2:P85 Question 33

a) The momentum before the explosion:

$$p = mv = 2 \times 6 = 12 \text{ [kg m s}^{-1}\text{]}$$

Since the small piece stopped, $v = 0$, meaning that the larger piece carries all the momentum. Thus:

$$v \triangleq \frac{p}{m} = \frac{12}{1.5} = \boxed{8 \text{ [m s}^{-1}\text{]}}$$

b) Using $E_k = \frac{1}{2}mv^2$ (note that small piece has 0 kinetic energy since $v = 0$):

$$E_{k\text{before}} = \frac{1}{2} \times 2 \times 6^2 = 36 \text{ [J]}$$

$$E_{k\text{after}} = \frac{1}{2} \times 1.5 \times 8^2 = 48 \text{ [J]}$$

The gain is thus:

$$E_{k\text{after}} - E_{k\text{before}} = 48 - 36 = \boxed{12 \text{ [J]}}$$

A2:P85 Question 34

a) The momentum after the collision is:

$$p_{\text{after}} = 0.002 \times 150 + 0.05 \times 2.4 = 0.42 \text{ [kg m s}^{-1}\text{]}$$

Before the collision, the box had no momentum, meaning the pellet had all of it. Therefore:

$$v = \frac{p}{m} = \frac{0.42}{0.002} = \boxed{210 \text{ [m s}^{-1}\text{]}}$$

b) i) Using $v = u + at$:

$$150 = 210 + a \times 1.5 \times 10^{-4}$$

$$\implies a = \frac{150 - 210}{1.5 \times 10^{-4}} = \frac{-600000}{1.5} = \boxed{-4 \times 10^5 \text{ [m s}^{-2}\text{]}}$$

ii) Using Newton's second law (and only considering magnitude):

$$F \stackrel{\text{N2}}{=} ma = 0.002 \times 4 \times 10^5 = \boxed{800 \text{ [N]}}$$

A2:P85 Question 35

a) The 2000 [kg] truck received all the momentum post-collision since the other truck stopped, meaning:

$$p_{\text{after}} = 2000 \times 6 = 1.2 \times 10^4 \text{ [kg m s}^{-1}\text{]}$$

The momentum before is given by the equation (considering right to be the positive direction):

$$p_{\text{before}} = 6000v - 2000v = 4000v$$

Equating both and solving for v :

$$4000v = 1.2 \times 10^4 \implies v = \frac{1.2 \times 10^4}{4000} = \boxed{3 \text{ [m s}^{-1}\text{]}}$$

b) If the collision is elastic, then all energy is conserved. Before the collision:

$$E_{k\text{before}} = \frac{1}{2} \times 6000 \times 3^2 + \frac{1}{2} \times 2000 \times 3^2 = 36000 \text{ [J]}$$

After the collision:

$$E_{k\text{after}} = \frac{1}{2} \times 2000 \times 6^2 = 36000 \text{ [J]}$$

Since $E_{k\text{before}} = E_{k\text{after}}$, it has been shown that the collision was elastic.

A2:P88 Question 36

a) Since momentum is conserved along all directions, we consider the horizontal. Then the first body contributes 0 momentum, meaning the horizontal component of the body of interest ($p \cos \theta$) must be equal to the original momentum.

$$1.6 \times 1 = 1.6 = 2v \cos \theta$$

For the vertical, the vertical component of the momentum of the body of interest must be equal to, but opposite the momentum of the first body:

$$0.8 \times 1 = 0.8 = 2v \sin \theta$$

Squaring both equations and adding both equations together (and using the identity $\cos^2 \theta + \sin^2 \theta = 1$):

$$1.6^2 + 0.8^2 = 4v^2 \cos^2 \theta + 4v^2 \sin^2 \theta$$

$$\implies 3.2 = 4v^2$$

$$\therefore v = \pm \sqrt{\frac{3.2}{4}}$$

$$\rightsquigarrow v = \boxed{0.894 \text{ [m s}^{-1}\text{]}} \quad (\text{since we only consider the magnitude})$$

Then, we return to either of the original equations. We choose the vertical.

$$0.8 \approx 2 \times 0.894 \sin \theta$$

$$\implies \theta = \sin^{-1} \frac{0.4}{0.894} \approx \boxed{26.6^\circ}$$

b) For the collision to be elastic, the kinetic energy before must equal the kinetic energy post-collision.

For the before energy:

$$E_{k\text{before}} = \frac{1}{2} \times 1 \times 1.6^2 = 1.28 \text{ [J]}$$

For the after energy:

$$E_{k\text{after}} \approx \frac{1}{2} \times 1 \times 0.8^2 + \frac{1}{2} \times 2 \times 0.894^2 \\ \approx 0.32 + 0.799 = 1.12 \text{ [J]}$$

We thus conclude the kinetic energies are not equal before and after the collision (and that it can't be a rounding error), meaning the collision was not elastic.

A2:P88 Question 37

a) The kinetic energy before the collision was:

$$E_{k\text{before}} = \frac{1}{2} \times m \times 200^2 = 20000m$$

The kinetic energy after the collision was:

$$E_{k\text{after}} = \frac{1}{2} \times m \times 160^2 + \frac{1}{2} \times 4m \times v^2$$

Equating the two since energy is conserved:

$$E_{k\text{before}} = E_{k\text{after}} \\ \Rightarrow 20000m = 12800m + 2mv^2 \\ \Rightarrow v = \sqrt{\frac{7200}{2}} = \boxed{60 \text{ [m s}^{-1}\text{]}}$$

b) The vertical component of the momentum of the red body must be equal to, but opposing, the vertical component of the blue body, since the original momentum has no vertical component. Thus:

$$160m \sin 82.8^\circ = 60 \times 4m \times \sin \theta \\ \Rightarrow \theta = \sin^{-1} \left(\frac{160}{60 \times 4} \times \sin 82.8^\circ \right) \\ \approx \boxed{41.4^\circ}$$

A2:P90 Question 38

a) First we convert the volume flow rate of $9 \text{ [L minute}^{-1}\text{]}$ to cubic metres per second as as $1.5 \times 10^{-4} \text{ [m}^3 \text{ s}^{-1}\text{]}$. The mass flow rate is then the volume flow rate times the density of water:

$$\dot{m} = 1.5 \times 10^{-4} \times 1000 = 0.15 \text{ [kg s}^{-1}\text{]}$$

Note that the rate of change of mass is equal to the infinitesimal volume of water moving through the

hose per unit second times the density of the water. We calculate this infinitesimal volume as the cross-sectional area times the infinitesimal distance travelled:

$$\frac{dm}{dt} = \rho_w \times A \times \frac{ds}{dt}$$

Note that since $v \triangleq \frac{ds}{dt}$ (noting that $r = \frac{d}{2}$):

$$\begin{aligned} v &= \frac{\dot{m}}{A\rho_w} \\ &= \frac{0.15}{\pi \times 0.007^2 \times 1000} \\ &\approx [0.974 \text{ m s}^{-1}] \end{aligned}$$

b) Using the formula derived in **a)** for the velocity pre-nozzle:

$$\begin{aligned} v &= \frac{\dot{m}}{A\rho_w} \\ &= \frac{0.15}{\frac{\pi \times 0.007^2}{12} \times 1000} \\ &\approx 11.7 \text{ m s}^{-1} \end{aligned}$$

Using the formula $\Delta p = \dot{m}(v - u)$:

$$\begin{aligned} \Delta p &= 0.15 \times (11.7 - 0.974) \\ &\approx 0.15 \times 10.7 \approx [1.61 \text{ N s}] \end{aligned}$$

c) Since the change in momentum is non-zero, a force is exerted by the water on the hose. The force will want to accelerate the hose, so to keep it stationary one must apply a force to the hose.

A2:P92 Question 39

a) The lift force has to oppose the weight force of the weight force, thus the magnitude of the two are equal. Consequently:

$$\begin{aligned} F_l &= F_g = 3 \times 10^3 \times 9.82 \\ &\approx [29500 \text{ N}] \end{aligned}$$

b) The lift force exerted is the rate of change of the momentum of the air:

$$F_l = \dot{p} = v\dot{m} = v \times A\rho \times \frac{ds}{dt}$$

Therefore, since $p = mv$ (and since $v = \frac{ds}{dt}$):

$$\dot{m} = \frac{\dot{p}}{v} = \rho A v$$

c) Using the relationship stated in a):

$$F_l = v\dot{m} = \rho A v^2 = F_g \approx 29500$$

$$v \approx \sqrt{\frac{29500}{95 \times 1.2}} \approx [16.1 \text{ m s}^{-1}]$$

d) The acceleration due to lift is, by Newton's second law:

$$a \stackrel{\text{N2}}{=} \frac{F_l}{m} = \frac{\rho A v^2}{m}$$

We now know that the magnitude of the acceleration due to lift must be 1.2 units greater than the downward acceleration due to gravity. This means:

$$\begin{aligned} \frac{\rho A u^2}{m} &= g + 1.2 \\ \Rightarrow u &\approx \sqrt{\frac{3 \times 10^3 \times (g + 1.2)}{1.2 \times 95}} \approx [17.0 \text{ m s}] \end{aligned}$$

A2:P92 Question 40

a) Using Newton's second law we get that:

$$a \stackrel{\text{N2}}{=} \frac{F}{m} = \frac{6000}{64}$$

Then using $v = u + at$ we can figure out the time needed to produce this worst-case scenario (converting $[\text{hm h}^{-1}]$ to $[\text{m s}^{-1}]$ by dividing by 3.6):

$$t = \frac{\frac{45}{3.6} \times 64}{6000} = [0.1\bar{3} \text{ s}]$$

b) Using $s = ut + \frac{1}{2}at^2$ (noting that the acceleration acts in the opposite direction of the motion):

$$\begin{aligned} s &= \frac{45}{3.6} \times 0.1\bar{3} - \frac{1}{2} \times \frac{6000}{64} \times 0.1\bar{3}^2 \\ &\approx [0.833 \text{ m}] \end{aligned}$$

A2:P96 Question 41

a) One revolution around the sun is $\theta = 2\pi \text{ [rad]}$, and then since we know one year is 365.25 [days], we

can convert this in to seconds:

$$t_{\text{orbit}} = 365.25 \times 24 \times 60 \times 60 \approx 3.16 \times 10^7 \text{ [s]}$$

Therefore the angular velocity is:

$$\omega = \frac{\theta}{t_{\text{orbit}}} = \frac{2\pi}{3.16 \times 10^7} \approx \boxed{1.99 \times 10^{-7} \text{ [rad s}^{-1}\text{]}}$$

b) The circumference of earth's orbit is:

$$P = 2\pi \times 1.5 \times 10^{11} = 3\pi \times 10^{11} \text{ [m]}$$

Thus, using the t_{orbit} determined in **a)**, the speed is:

$$v = \frac{P}{t_{\text{orbit}}} \approx \frac{3\pi \times 10^{11}}{3.16 \times 10^7} \approx \boxed{2.98 \times 10^4 \text{ [m s}^{-1}\text{]}}$$

A2:P96 Question 42

a) If the blades make 670 revolutions in one minute, then:

$$\omega = \frac{670}{60} \approx 11.2 \text{ [Hz]}$$

Knowing that one revolution is 2π [rad]:

$$\omega \approx \boxed{70.4 \text{ [rad s}^{-1}\text{]}}$$

b) Using the formula $v_{\text{tip}} = \omega r$:

$$v_{\text{tip}} \approx 70.4 \times 0.16 \approx \boxed{11.3 \text{ [m s}^{-1}\text{]}}$$

A2:P97 Question 43

Using the formula $a = \omega^2 r$, we deduce:

$$a \propto r$$

Therefore, the ratio is computed as (knowing both have the same angular speed):

$$\frac{a_P}{a_Q} = \frac{R}{\frac{R}{2}} = 2$$

Thus, the answer is C.

A2:P97 Question 44

a) Since the rotation frequency is 5 [Hz]:

$$t = \frac{1}{5} = 0.2 \text{ [s]}$$

The circumference of a neutron star is:

$$P = 2\pi \times 10^4 \text{ [m]}$$

Thus, the speed is:

$$v = \frac{P}{t} = \frac{2\pi \times 10^4}{0.2} = \pi \times 10^5 \approx 3.14 \times 10^5 \text{ [m s}^{-1}\text{]}$$

b) Using the formula $a = \frac{v^2}{r}$:

$$a \approx \frac{(3.14 \times 10^5)^2}{10^4} \approx 9.86 \times 10^6 \text{ [m s}^{-2}\text{]}$$

A2:P98 Question 45

a) We first convert the angular speed to [Hz] by dividing by 60:

$$\omega = \frac{2}{60} = \frac{1}{30} \text{ [Hz]}$$

This means the angular speed in $\text{[rad s}^{-1}\text{]}$ is:

$$\omega = \frac{2\pi}{\frac{1}{30}} \approx 0.209 \text{ [rad s}^{-1}\text{]}$$

b) Since the passengers move along the circumference of the Ferris wheel we use the formula $v = \omega r$, solving for r :

$$r = \frac{v}{\omega} = \frac{3}{0.209} \approx 14.4 \text{ [m]}$$

c) Using $a = \frac{v^2}{r}$:

$$a \approx \frac{3^2}{14.4} = 0.625 \text{ [m s}^{-2}\text{]}$$

A2:P103 Question 46

a) Since there is no inclined plane, the normal force is equal to the weight force. For the box to stay stationary, the frictional force has to be greater than or equal to the centripetal force:

$$F_\mu \geq F_C$$

$$\implies mg\mu \geq m\omega^2 r$$

Thus, solving for the maximum ω (when there is an equality rather than an inequality):

$$\omega \leq \sqrt{\frac{g\mu}{r}} \approx \sqrt{\frac{9.82 \times 0.7}{0.3}} \approx \boxed{4.79 \text{ [rad s}^{-1}\text{]}}$$

b) Using $\theta = \omega t + \frac{1}{2}\alpha t^2$ (there is no acceleration in this problem), and solving for t :

$$t = \frac{\theta}{\omega} \approx \frac{2\pi}{4.79} \approx \boxed{1.31 \text{ [s]}}$$

A2:P103 Question 47

Using $a = \frac{v^2}{r}$ and solving for r , setting $a = 9g$:

$$r = \frac{v^2}{a} \approx \frac{280^2}{9 \times 9.82} \approx \boxed{887 \text{ [m]}}$$

A2:P103 Question 48

a)

Note that graph question solutions are not yet done due to lack of time.

b) The vertical component of the normal force, which is θ angular units from the horizontal, must balance the weight force completely. Thus:

$$\begin{aligned} F_N \sin \theta &= F_g = mg \\ \implies F_N &= \frac{mg}{\sin \theta} \end{aligned}$$

The only remaining unbalanced force is then the horizontal component of the normal force, meaning by Newton's second law, the acceleration of the marble is:

$$\begin{aligned} a &\stackrel{\text{N2}}{=} \frac{F_N \cos \theta}{m} = \frac{\frac{mg}{\sin \theta} \cos \theta}{m} \\ &= \frac{g \cos \theta}{\sin \theta} = \boxed{\frac{g}{\tan \theta}} \end{aligned}$$

c) i) Using $a = \frac{v^2}{r}$ and solving for r gives (and using the expression for a derived in **a**):

$$\begin{aligned} r &= \frac{v^2}{a} = \frac{v^2}{\frac{g}{\tan \theta}} \\ &\approx \frac{1.5^2}{\frac{9.82}{\tan 28^\circ}} \approx \boxed{0.122 \text{ [m]}} \end{aligned}$$

ii) What the question is really asking for is the normal force, since that is the force the bowl pushes against

the marble with. The normal force was defined in a):

$$F_N = \frac{mg}{\sin \theta} = \frac{0.003 \times 9.82}{\sin 28^\circ} \approx [6.28 \times 10^{-2} \text{ m}]$$

d) Re-aranging $a = \omega^2 r$ for ω yields:

$$\omega = \sqrt{\frac{a}{r}}$$

Since $a = \frac{g}{\tan \theta}$ is constant:

$$\omega \propto \frac{1}{\sqrt{r}}$$

Where r is the radius of the path. We therefore conclude that as the ball slips down, and r starts decreasing, ω will increase.

A2:P105 Question 49

- a) At the bottom, since at the bottom the force of tension in the string has to exactly counter-act the weight force of the stone and the centripetal force.
- b) i) Taut means not slackened. We thus want the centripetal force to perfectly balance the tension and weight force. At the top there is no tension due to weight force, so the equation reduces to:

$$\begin{aligned} F_g &= F_C \\ \Rightarrow mg &= m \frac{v^2}{r} \\ \therefore v &= \sqrt{gr} = \sqrt{9.82 \times 0.8} = [2.80 \text{ m s}^{-1}] \end{aligned}$$

ii) The maximum tension (at the bottom) is given by:

$$\max F_t = F_g + F_C = mg + m \frac{v^2}{r}$$

We know the maximum tension force was 10 [N], thus:

$$\begin{aligned} 10 &\approx 0.25 \times 9.82 + 0.25 \times \frac{v^2}{0.8} \\ \Rightarrow v &\approx \sqrt{\frac{10 - 2.455}{0.3125}} \approx [4.91 \text{ m s}^{-1}] \end{aligned}$$

A2:P105 Question 50

- a) This is a tricky question. If a question involves circular motion, then remember that the centripetal force F_C is the resultant of the forces acting on the object. Note that the centripetal force acts downwards in towards the centre of the bridge. The two other forces acting on the car at the top of the bridge is the weight force F_g (downward) and the normal force F_N (upward). Thus, letting downwards towards the

bridge be the direction of positive motion:

$$\begin{aligned} F_C &= F_g - F_N \\ \implies m \frac{v^2}{r} &= mg - F_N \\ \therefore F_N &= mg - m \frac{v^2}{r} = \boxed{m \left(g - \frac{v^2}{r} \right)} \end{aligned}$$

b) i) First converting $50 \text{ [km h}^{-1}]$ to $[\text{m s}^{-1}]$:

$$v = \frac{50}{3.6} \approx 13.9 \text{ [m s}^{-1}]$$

Using the formula derived in **a)**:

$$\begin{aligned} F_N &\approx 1400 \left(9.82 - \frac{13.9^2}{60} \right) \\ &\approx \boxed{9240 \text{ [N]}} \end{aligned}$$

ii) On a horizontal road, $F_N = F_g$, therefore, 10% of the value must be:

$$0.1F_N = 0.1mg \approx 1370 \text{ [N]}$$

Then using the equation derived in **a)**:

$$\begin{aligned} 1370 &= 1400 \left(g - \frac{v^2}{60} \right) \\ \implies v &\approx \sqrt{60 \left(9.82 - \frac{1370}{1400} \right)} \approx \boxed{23.0 \text{ [m s}^{-1}]} \end{aligned}$$

The answer sheet gives the answer in $[\text{km h}^{-1}]$ which is found by taking the answer v found here multiplied by 3.6.

1.3 Subtopic A.3 – Work, energy and power

A3:P111 Question 1

a) Using the definition of work:

$$W = Fs \cos \theta = 0.6 \times 2.5 \times \cos 60^\circ = \boxed{0.75 \text{ [J]}}$$

b) Vide **a)** for explanation

$$W = Fs \cos \theta = 0.6 \times 2.5 \times \cos 90^\circ = \boxed{0 \text{ [J]}}$$

c) Vide a) for explanation

$$W = Fs \cos \theta = 0.6 \times 2.5 \times \cos 160^\circ \approx -1.41 \text{ [J]}$$

A3:P111 Question 2

a) Converting the speed to $[\text{m s}^{-1}]$ by dividing by 3.6:

$$v = \frac{50}{3.6} \approx 13.9 \text{ [m s}^{-1}\text{]}$$

Thus, the distance travelled by the car in one minute is (using $s = ut + \frac{1}{2}at^2$, with $a = 0$ since the speed is constant):

$$s = 13.9 \times 60 \approx 833 \text{ [m]}$$

b) Since the speed is constant, the resultant force of the driving and resistive force must be 0. Thus the magnitude of the two are equal. The magnitude of the resistive force is thus (using the definition of work $W = Fs \cos \theta$, solving for F and noting that $\theta = 90^\circ$):

$$F \approx \frac{190000}{833} \approx 228 \text{ [N]}$$

c) As described in b), the resistive force is equal in magnitude, but opposite in direction to the driving force.

Thus:

$$W_{\text{resistive}} = -W_{\text{driving}} = -190 \text{ [kJ]}$$

A3:P113 Question 3

a) The work is the area under the curve, meaning:

$$W = 60 \times 1.6 \times 10^3 + \frac{40 \times 1.6 \times 10^3}{2} = 1.28 \times 10^5 \text{ [J]}$$

b) i) The driving force at $s = 50 \text{ [m]}$ is $1.6 \times 10^3 \text{ [N]}$, thus taking the resultant (noting the resistive force is opposite in direction) and using Newton's second law, we see:

$$a \stackrel{\text{N2}}{=} \frac{F}{m} = \frac{1.6 \times 10^3 - 400}{1600} = 0.75 \text{ [m s}^{-2}\text{]}$$

ii) The driving force at $s = 100 \text{ [m]}$ is 0. Using the same method as in i):

$$a \stackrel{\text{N2}}{=} \frac{F}{m} = \frac{0 - 400}{1600} = -0.25 \text{ [m s}^{-2}\text{]}$$

A3:P113 Question 4

a) The work done is the area under the curve (under the line being negative). Thus:

$$\begin{aligned} W &= \frac{1}{2} \times 3 \times 6 - \frac{1}{2} \times 2 \times 4 - 5 \times 4 \\ &= 9 - 4 - 20 = \boxed{-15 \text{ [J]}} \end{aligned}$$

b) The graphical way to solve this problem is noting that we need the area above and below the curve to cancel out. The triangle above the curve is almost cancelled out by the one under, including the rectangle from $s = 5$ to 6 [m]. The only area remaining is one unit triangle, which we can get from half a unit square. By traveling an additional 0.25 [m] from $s = 6$ [m], we cover an area of two unit squares divided by 4, equal to half a unit square. Thus by $s = \boxed{6.25 \text{ [m]}}$, we have net zero work done by the force.

A3:P115 Question 5

Using $P = Fv \cos \theta$, solving for F:

$$\begin{aligned} F &= \frac{P}{v \cos \theta} = \frac{180}{1.5 \cos 60^\circ} \\ &= 240 \text{ [N]} \end{aligned}$$

Thus, the answer is D.

A3:P115 Question 6

Using the definition of power:

$$\begin{aligned} P &= \frac{W}{t} = \frac{Fs \cos \theta}{t} \\ \rightsquigarrow P &= Fv \stackrel{\text{N2}}{=} mav \end{aligned}$$

Thus if the power is constant, then as v increases, a has to decrease, starting at a maximum when v is low. Then as the locomotive reaches its "terminal" velocity, the acceleration of the locomotive starts approaching 0. The answer is therefore D. The reason it can't be A is that since the train is driven from rest, it has to gain velocity. Note that the power can still be constant even once the train stops accelerating, since the engine still needs to do work to oppose frictional, air resistance, etc. forces.

A3:P115 Question 7

a) The resultant force is, using Newton's second law:

$$F_R \stackrel{\text{N2}}{=} 2.5 \times 1600 = 4000 \text{ [N]}$$

Thus, since the only forces acting on the force is the resistive force and the driving force:

$$F_R = 4000 = F_{\text{driving}} - F_{\text{resistance}} = F_{\text{driving}} - 500$$

$$\implies F_{\text{driving}} = 4000 + 500 = \boxed{4500 \text{ [N]}}$$

b) i) To derive the distance travelled, we use the SUVAT equation $s = ut + \frac{1}{2}at^2$:

$$s = 0 + \frac{1}{2} \times 2.5 \times 10^2 = 125 \text{ [m]}$$

Thus, by the definition of work:

$$W \triangleq F_{\text{driving}} s \cos \theta = 4500 \times 125 \approx \boxed{5.63 \times 10^5 \text{ [J]}}$$

ii) By the definition of power, using the work found in a):

$$P \triangleq \frac{W}{t} \approx \frac{5.63 \times 10^5}{10} = \boxed{5.63 \times 10^4 \text{ [W]}}$$

c) Since a constant force opposes the motion of the car, to maintain a constant resultant acceleration, the driving force has to be constant too. However, if the car is accelerating, the velocity is increasing. By the definition of power:

$$P \triangleq \frac{W}{t} = \frac{Fs \cos \theta}{t} \stackrel{\text{reduces to}}{=} Fv$$

Thus, if the force is constant:

$$P \propto v$$

This means that the car must increase power to maintain a constant acceleration.

A3:P116 Question 8

a) The work done is the area under the curve. Thus:

$$W = \frac{800 \times 100}{2} + 1200 \times 100 = \boxed{1.6 \times 10^5 \text{ [J]}}$$

b) Using the definition of work, we derive the magnitude of the force acting on the car as:

$$F \triangleq \frac{W}{s} = \frac{1.6 \times 10^5}{100} = 1.6 \times 10^3 \text{ [N]}$$

Thus the acceleration is, using Newton's second law:

$$a \stackrel{\text{N2}}{=} \frac{F}{m} = \frac{1.6 \times 10^3}{2500} = 0.64 \text{ [m s}^{-2}\text{]}$$

Finally, using $v^2 = u^2 + 2as$:

$$v = \sqrt{8^2 + 2 \times 0.64 \times 100} \approx [13.9 \text{ m s}^{-1}]$$

A3:P116 Question 9

a) The work done is the energy needed to halve the kinetic energy. The original kinetic energy is thus:

$$E_k = \frac{1}{2}mv^2 = \frac{1}{2} \times 1.2 \times 2.7^2 \approx 4.37 \text{ J}$$

Thus the work done is:

$$W = -\frac{1}{2}E_k \approx -\frac{1}{2} \times 4.37 = [-2.19 \text{ J}]$$

b) If the kinetic energy is halved, and the mass stays the same, then:

$$\begin{aligned} \frac{1}{2}mv'^2 &= \frac{1}{2}\left(\frac{1}{2}mv^2\right) \\ \implies v'^2 &= \frac{v^2}{2} \\ \therefore v' &= \sqrt{\frac{v^2}{2}} = \sqrt{\frac{2.7^2}{2}} \approx [1.91 \text{ m s}^{-1}] \end{aligned}$$

c) Using $s = \frac{u+v}{2} \times t$:

$$s \approx \frac{1.91 + 2.7}{2} \times 0.9 \approx [2.07 \text{ m}]$$

A3:P120 Question 10

We first need the angle between the gravitational force and the motion of the box. Since the gravitational force is along the negative vertical, the angle is:

$$\theta = 30 + 90 = 120^\circ$$

Then, the gravitational force is computed using Newton's second law as:

$$F_g \stackrel{\text{N2}}{=} mg = 1.5g$$

Finally, using the definition of work:

$$\begin{aligned} W &\stackrel{\Delta}{=} F_s \cos \theta = 1.5g \times 4 \times \cos 120^\circ \\ &= -3g \approx 30 \text{ J} \end{aligned}$$

Thus, the answer is C.

A3:P120 Question 11

The potential is given as:

$$E_p = mgh$$

Note however that the initial height of the pendulum bob above its rest position is exactly one length of the radius, meaning:

$$E_p = mgr$$

Then, at the bottom, all of the potential has been converted into kinetic energy meaning:

$$\begin{aligned} E_k = E_p &\implies \frac{1}{2}mv^2 = mgr \\ \therefore v^2 &= 2gr \end{aligned}$$

Since the bob is traveling in circular motion, it experiences centripetal forces equal to the resultant force F_R . Thus:

$$F_C = \frac{mv^2}{r}$$

Using the form of v^2 derived earlier, we find:

$$F_C = \frac{2mgr}{r} = 2mg$$

Thus, recalling that the centripetal force is the resultant, the acceleration of the bob is given by Newton's second law as:

$$a \stackrel{\text{N2}}{=} \frac{F_R}{m} = \frac{2mg}{m} = 2g$$

Consequently, the answer is C.

A3:P121 Question 12

a) i) By the definition of work:

$$W \triangleq F_s \cos \theta = 3.2 \times 10^4 \times 45 \approx \boxed{1.44 \times 10^6 \text{ [J]}}$$

ii) The helicopter originally had no potential, so the change in energy is equal to the new potential which is given as:

$$E_p \triangleq mgh \approx 2900 \times 45 \times 9.82 \approx \boxed{1.28 \times 10^6 \text{ [J]}}$$

b) The helicopter not only gained potential, but also kinetic energy. The difference between the work done and the potential gained is the new kinetic energy of the helicopter (since it accelerated from rest).

c) Using the reasoning from b):

$$W - E_p \approx 1.44 \times 10^6 - 1.28 \times 10^6 \approx E_k \triangleq \frac{1}{2}mv^2$$

$$\implies v \approx \sqrt{\frac{2 \times (1.44 \times 10^6 - 1.28 \times 10^6)}{2900}} \approx [10.5 \text{ m s}^{-1}]$$

A3:P121 Question 13

Be careful here not to think that the entire kinetic energy of the pellet becomes converted to the kinetic energy of the block when they move together. We instead apply the conservation of momentum. The mass of the two objects intertwined is:

$$m_{\text{combined}} = 0.25 + 0.0018 = 0.2518 \text{ [kg]}$$

Thus:

$$m_{\text{pellet}} \times v_{\text{pellet}} = m_{\text{combined}} \times v$$

$$\implies v = \frac{0.0018 \times 200}{0.2518} \approx 1.43 \text{ [m s}^{-1}\text{]}$$

The kinetic energy of the combined body is then:

$$E_k \triangleq \frac{1}{2}mv^2 = \frac{1}{2} \times 0.2518 \times 1.43^2 = 0.257 \text{ [J]}$$

The maximum height reached is then given by converting all the kinetic energy to potential, meaning:

$$0.257 = E_p \triangleq mgh$$

$$\implies h = \frac{0.257}{mg} \approx \frac{0.257}{0.2518 \times 9.82} \approx [0.104 \text{ [m]}]$$

A3:P121 Question 14

a) The question asks for speed (energy), not time. The ball has the same amount of initial kinetic energy however the ball is oriented, and gains the same amount from the initial potential. Thus when it hits the ground it has the same kinetic energy, which means identical speeds regardless of the initial angle.

b) The initial kinetic energy was:

$$E_{k\text{initial}} = \frac{1}{2}mv^2 = \frac{1}{2} \times m \times 20^2 = 200m$$

The energy gained for the final kinetic energy is the potential possessed in the beginning. The initial potential was:

$$E_p = mgh = 2.5mg$$

Thus the final kinetic energy is

$$\begin{aligned} E_{k\text{final}} &= E_{k\text{initial}} + E_p \\ &= 200m + 2.5mg = m(200 + 2.5g) \end{aligned}$$

Thus we compute the final velocity using the definition of kinetic energy as:

$$\begin{aligned} E_{k\text{final}} &\triangleq \frac{1}{2}mv^2 = m(200 + 2.5g) \\ \implies v &= \sqrt{400 + 5g} \approx \boxed{21.2 \text{ [m s}^{-1}\text{]}} \end{aligned}$$

A3:P123 Question 15

The change in elastic energy is given by:

$$\begin{aligned} \Delta E_H &= E_{H\text{final}} - E_{H\text{initial}} \\ &= \frac{1}{2}k(2\Delta x)^2 - \frac{1}{2}k\Delta x \\ &= \frac{1}{2}k(4\Delta x - \Delta x) = 3 \times \frac{1}{2}k\Delta x = 3E_{H\text{initial}} \end{aligned}$$

Consequently, the answer is C.

A3:P123 Question 16

When the block has returned to its uncompressed length, the elastic energy has been converted in to kinetic energy. Thus:

$$\begin{aligned} E_k &\triangleq \frac{1}{2}mv^2 = E_H = \frac{1}{2}k(\Delta x)^2 \\ \implies v &= \sqrt{\frac{k(\Delta x)^2}{m}} \end{aligned}$$

Then note how the force exerted on the block is due to the spring force, meaning by Hooke's law:

$$F_H \triangleq -k\Delta x = k\Delta x$$

Therefore, we can substitute in F_H to the expression for the velocity:

$$v = \sqrt{\frac{F\Delta x}{m}}$$

This makes the answer B.

A3:P123 Question 17

a) i) If the spring is balanced, then the force used to hold it still is equivalent to the spring force, by

Newton's third law. Thus:

$$60 = F_H \stackrel{\Delta}{=} -k\Delta x$$

$$\implies k = \frac{60}{0.03} = \boxed{2000 \text{ [N m}^{-1}\text{]}}$$

ii) Using the definition of elastic potential as $E_H \stackrel{\Delta}{=} \frac{1}{2}k(\Delta x)^2$:

$$E_H = \frac{1}{2} \times 2000 \times 0.03^2 = \boxed{0.9 \text{ [J]}}$$

b) The work done is converted to elastic potential, meaning the new elastic potential is given by:

$$E'_H = E_H + 0.7 = 0.9 + 0.7 = 1.6 \text{ [J]}$$

Therefore, by the definition of elastic potential as $E_H \stackrel{\Delta}{=} \frac{1}{2}k(\Delta x)^2$:

$$1.6 = E'_H \stackrel{\Delta}{=} \frac{1}{2}k(\Delta x)^2$$

$$\implies \Delta x = \sqrt{\frac{3.2}{2000}} = 0.04 \text{ [m]} = \boxed{4 \text{ [cm]}}$$

A3:P123 Question 18

a) i) Using the definition of kinetic energy as $E_k \stackrel{\Delta}{=} \frac{1}{2}mv^2$:

$$E_k = \frac{1}{2} \times 0.6 \times 2^2 = \boxed{1.2 \text{ [J]}}$$

ii) The gravitational force of the block is given by Newton's second law as:

$$F_g \stackrel{\Delta}{=} mg \approx 0.6 \times 9.82$$

Then, by the definition of work as $W \stackrel{\Delta}{=} Fs \cos \theta$:

$$W = F_g \times 0.051 \approx 0.6 \times 9.82 \times 0.051 \approx \boxed{0.300 \text{ [J]}}$$

iii) The initial kinetic energy E_k of the block has been converted to elastic potential. However, the block has also gained energy from the block losing potential, this is equivalent to the work done by the force of gravity. Thus:

$$E_k \rightarrow E_H + E_p$$

$$\implies E_H = E_k - E_p \approx 1.2 - 0.3 = \boxed{0.9 \text{ [J]}}$$

b) Using $E_H = \frac{1}{2}k(\Delta x)^2$, using the result from **a)**:

$$0.9 \approx \frac{1}{2}k(0.051)^2$$

$$\implies k = \frac{2 \times 0.9}{0.051^2} \approx \boxed{692 \text{ [N m}^{-1}\text{]}}$$

c) The forces acting on the block are the force of gravity and the spring force (by Hooke's law) $F_H = -k\Delta x$. Thus, the resultant is:

$$F_R = F_H - F_g \stackrel{\text{N2}}{=} 692 \times 0.051 - 0.6 \times 9.82 = 29.4 \text{ [N]}$$

Therefore, by Newton's second law:

$$a \stackrel{\text{N2}}{=} \frac{F_R}{m} \approx \frac{29.4}{0.6} = \boxed{49 \text{ [m s}^{-2}\text{]}}$$

A3:P125 Question 19

Since $P = \frac{W}{t}$:

$$W = Pt = 270 \times 200 = 54000 \text{ [J]}$$

The potential energy gained by the cyclist is given as:

$$\Delta E_p = mg\Delta h = 85 \times g \times 50 = 41700 \text{ [J]}$$

Thus, the efficiency is:

$$\eta = \frac{\Delta E_p}{W} \approx \frac{41700}{54000} \approx \boxed{77.3\%}$$

A3:P125 Question 20

Converting the speed of $50 \text{ [km h}^{-1}\text{]}$ to $\text{[m s}^{-1}\text{]}$:

$$v = \frac{50}{3.6} = 13.\bar{8} \text{ [m s}^{-1}\text{]}$$

The kinetic energy gained (since there was none at the start, when the car was at rest) is:

$$E_k = \frac{1}{2}mv^2 = \frac{1}{2} \times 1600 \times 13.\bar{8}^2 \approx 154000 \text{ [J]}$$

Thus, if this is only 65% of the energy transferred, then the total energy transferred is:

$$E = \frac{E_k}{65\%} \approx \frac{154000}{0.65} \approx \boxed{2.37 \times 10^5 \text{ [J]}}$$

A3:P128 Question 21

a) Since the energy losses listed are all the energy losses, $100\% - \text{losses} = \eta$, where η is the efficiency.

Thus:

$$\begin{aligned}\eta &= 100\% - 27\% - 15\% - 5\% \\ &= 1 - 0.27 - 0.15 - 0.05 = 0.53 = \boxed{53\%}\end{aligned}$$

b)

Note that graph question solutions are not yet done due to lack of time.

1.4 Subtopic A.4 – Rigid body mechanics

A4:P133 Question 1

a) Using $\omega_f = \omega_i + \alpha t$:

$$\alpha = \frac{\omega_f - \omega_i}{t} = \frac{15 - 5}{20} = \boxed{0.5 \text{ [rad s}^{-2}\text{]}}$$

b) Using $\omega_f^2 = \omega_i^2 + 2\alpha\theta$, solving for θ :

$$\begin{aligned}\theta &= \frac{\omega_f^2 - \omega_i^2}{2\alpha} = \frac{15^2 - 5^2}{2 \times 0.5} \\ &= 200 \text{ [rad]}\end{aligned}$$

Therefore, the amount of revolutions is:

$$\frac{200}{2\pi} \approx \boxed{31.8 \text{ [revolutions]}}$$

A4:P133 Question 2

a) Using $\omega_f = \omega_i + \alpha t$:

$$\omega_f = 0 + 4.7 \times 20 = 94 \text{ [rad s}^{-1}\text{]}$$

Multiplying by 60 (for the amount per minute) and dividing by 2π (for the amount in revolutions), gives:

$$\frac{94 \times 60}{2\pi} = \boxed{898 \text{ [revolutions minute}^{-1}\text{]}}$$

b) Using $\omega_f^2 = \omega_i^2 + 2\alpha\theta$, solving for θ :

$$\theta = \frac{\omega_f^2 - \omega_i^2}{2\alpha} = \frac{94^2 - 0}{2 \times 4.7} = 940 \text{ [rad]}$$

Thus, in revolutions:

$$\frac{940}{2\pi} \approx \boxed{150 \text{ [revolutions]}}$$

A4:P133 Question 3

a) Converting the speed of 1800 [revolutions minute⁻¹] to [rad s⁻¹]:

$$\omega = \frac{1800 \times 2\pi}{60} = 188 \text{ [rad s}^{-1}\text{]}$$

Using $\omega_f = \omega_i + \alpha t$, solving for α :

$$\alpha = \frac{\omega_f - \omega_i}{t} = \frac{188 - 0}{0.1} = 1880 \text{ [rad s}^{-2}\text{]}$$

b) Using $\theta = \left(\frac{\omega_f + \omega_i}{2}\right)t$, solving for t :

$$t = \frac{2\theta}{\omega_f + \omega_i} \approx \frac{2 \times 5 \times 2\pi}{0 + 188} \approx 0.334 \text{ [s]}$$

A4:P133 Question 4

a) Using $\omega_f^2 = \omega_i^2 + 2\alpha\theta$, solving for α :

$$\alpha = \frac{\omega_f^2 - \omega_i^2}{2\theta} = \frac{50^2 - 10^2}{2 \times 20 \times 2\pi} \approx 9.55 \text{ [rad s}^{-1}\text{]}$$

b) i) Using $\theta = \left(\frac{\omega_f + \omega_i}{2}\right)t$, solving for t :

$$t = \frac{2\theta}{\omega_f + \omega_i} \approx \frac{2 \times 20 \times 2\pi}{50 + 10} \approx 4.19 \text{ [s]}$$

ii) Using $\omega_f^2 = \omega_i^2 + 2\alpha\theta$, solving for ω_f :

$$\omega_f \approx \sqrt{10 + 2 \times 9.55 \times 10 \times 2\pi} \approx 34.8 \text{ [rad s}^{-1}\text{]}$$

Thus, by $\omega_f = \omega_i + \alpha t$, solving for t :

$$t = \frac{\omega_f - \omega_i}{\alpha} \approx \frac{34.8 - 10}{9.55} \approx 2.60 \text{ [s]}$$

$$a^2 + b^2 = c^2$$

2 Topic B – The particulate nature of matter

2.1 Subtopic B.1 – Thermal energy transfers