Week 5: Collaborative Based RSs – Part I

CS3448: Recommender Systems /

CSX4207/ITX4207: Decision Support and Recommender Systems

Asst. Prof. Dr. Rachsuda Setthawong

Objectives

- To understand the concept of collaborative based filtering approach
- To be familiar with User-based Nearest Neighbor and Item-based Nearest Neighbor algorithms
- To introduce additional proximity measure
- To understand the problems of collaborative based filtering approach

Outlines

- Collaborative Recommendation
- User-based Nearest Neighbor (NN) Recommendation
- Measures to Determine Proximity between Users
- Neighborhood Selection
- Item-based Nearest Neighbor (NN) Recommendation
- Pros and cons of CF based approach
- Problems with CF based approach

Main idea

- To exploit information about the past behavior/opinions of an existing user community.
- To predict which items the target user will most probably like.

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How to Generate Recommendation Using Collaborative Based Filtering Approach

Target
User's Preferences

Recommendation Algorithm(s)

Community's Preferences (user-item rating matrix)

Prediction

Rank

Recommendation

Collaborative: "Tell me what's popular among my peers"

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Pure Collaborative Approaches

- Input:
  - A matrix of given user-item ratings

	Item 1	Item 2	Item 3	Item 4	...
User 1	3	1	2	3	...
User 2	4	3	4	3	...
User 3	3	3	1	5	...
...	...	...	...	...	...

- Outputs:
  - A (numerical) prediction of what degree the target user will like/dislike a certain (unseen) item
  - A list of *k* recommended items

Memory-based VS Model-based Approaches

Memory-based approach	Model-based approach

Memory-based approach

- Modeless (no model created)
- Directly applying rating matrix to find neighbors and to recommend items.
- Time consuming and not scalable
- Example, user-based NN

Model-based approach

- Offline process the raw data.
- At run time, require only the "learned" model to make prediction.
- Update/retrained the model periodically.
- Example, matrix factorization methods, association rule mining, etc.

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User-based Nearest Neighbor (NN) Recommendation – **Main Idea**

- Given
  - A rating database containing:
    - The set of users: $\boldsymbol{U} = \{u_1, \ldots, u_n\}$
    - The set of products (items): $\boldsymbol{P} = \{p_1, \ldots, p_m\}$
    - $\boldsymbol{R}$ as $n \times m$ matrix of ratings $r_{i,j}$ with $i \in 1 \ldots n$, $j \in 1 \ldots m$
      - The rating values: 1 – 5 (1: *strongly dislike* and 5: *strongly like*)
  - The ID of the current (active) user
- Step 1: Find k-NN users that had similar preferences to those of the active user in the past.
- Step 2: Predict ratings of an unseen product p based on the ratings for p made by k-NNs.

Assumptions in Collaborative Approach

1. If users had similar tastes in the past, they will have similar tastes in the future.
2. User preferences remain stable and consistent over time.

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Example 1

- Goal: to determine whether Alice will *like* or *dislike* “Item 5”

Target User →

	Item 1	Item 2	Item 3	Item 4	Item 5
Alice	5	3	4	4	?
User 1	3	1	2	3	3
User 2	4	3	4	3	5
User 3	3	3	1	5	4
User 4	1	5	5	2	1

User-based Nearest Neighbor (NN) Recommendation

- Step 1: Select the value of k for nearest neighbored users to the target user.
- Step 2: Calculate Similarity between the target user and other users (*using Pearson's Correlation Coefficient (PCC)*)
- Step 3: Predicting Product Rating

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Step 1: Select the value of *k* for nearest neighbored users to the target user.

- E.g.,
  - **k = 1 ← select the most similar user to the target user**
  - k = 10 ← select the top-10 most similar users to the target user
  - k = 20 ← select the top-20 most similar users to the target user

Step 2: Calculate Similarity between the target user and other users (*using Pearson's Correlation Coefficient (PCC)*)

- To determine similarity between user preferences.

$$sim(a,b) = \frac{\sum_{p \in P}(r_{a,p} - \bar{r_a})(r_{b,p} - \bar{r_b})}{\sqrt{\sum_{p \in P}(r_{a,p} - \bar{r_a})^2}\sqrt{\sum_{p \in P}(r_{b,p} - \bar{r_b})^2}}$$

- where,
  - $r_{a,p}$, $r_{b,p}$ is a rating of user *a* (*or b*) on product *p*.
  - ** $\bar{r}_a$, $\bar{r}_b$ is the average rating of *the <u>common rated items</u>* of user *a* (*or b*).
  - *P* represents the set of *<u>common rated items</u>* by user *a* and *b*.
- Interpretation (in the range of -1 to 1):
  - -1 = strongly negative correlation
  - 1 = strongly positive correlation

Note: The common rated items =
The items that are both rated by users a and b.

**: Ref: http://www.recommenderbook.net/media/corrigenda.pdf

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An Example of Calculating PCC

- To determine similarity between user preferences.

	Item 1	Item 2	Item 3	Item 4	Item 5
Alice	5	3	4	4	?
User 1	3	1	2	3	3

$$sim(Alice, User1) = \frac{\sum_{p \in P}(r_{Alice,p} - \overline{r_{Alice}})(r_{User1,p} - \overline{r_{User1}})}{\sqrt{\sum_{p \in P}(r_{Alice,p} - \overline{r_{Alice}})^2}\sqrt{\sum_{p \in P}(r_{User1,p} - \overline{r_{User1}})^2}}$$

$$\overline{r_{Alice}} = \frac{5+3+4+4}{4} = 4 \qquad \overline{r_{User1}} = \frac{3+1+2+3}{4} = 2.25$$

$$\sqrt{\sum_{p \in P}(r_{Alice,p} - \overline{r_{Alice}})^2} = 1.4142 \qquad \sqrt{\sum_{p \in P}(r_{User1,p} - \overline{r_{User1}})^2} = 1.6583$$

$$sim(Alice, User1) = \frac{(5-4)(3-2.25)+(3-4)(1-2.25)+ \ldots +(4-4)(3-2.25)}{(1.4142)(1.6583)} = 0.85$$

Additional Resource: http://recommender-systems.org/collaborative-filtering/

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An Example of Calculating PCC:

(The similarity between Alice and other users using PCC)

	Pearson Correlation Coefficient
sim(Alice,User1)	0.85
sim(Alice,User2)	0.7
sim(Alice,User3)	0
sim(Alice,User4)	-0.79

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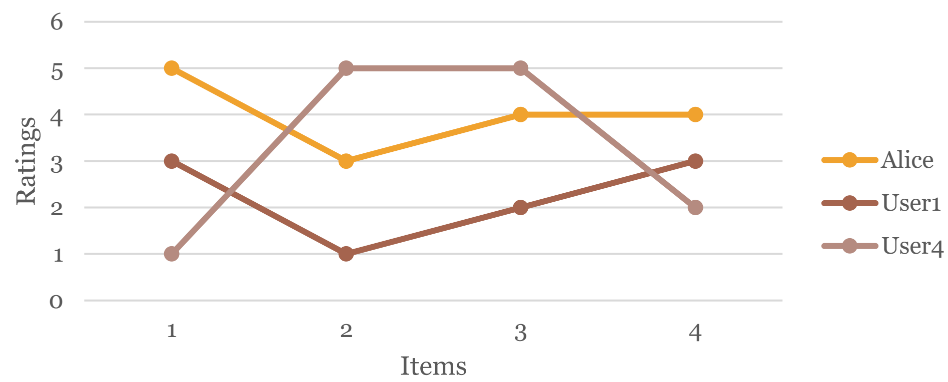
Comparing Alice with Two Other Users Using PCC

The original ratings of common rated items of Alice and other two users:

	Item1	Item2	Item3	Item4
Alice	5	3	4	4
User1	3	1	2	3
User4	1	5	5	2



PCC of Alice and other two users:

	PCC
sim(Alice,User1)	**0.85**
sim(Alice,User4)	**-0.79**

Strongly positive correlation

Strongly negative correlation

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Step 3: Predicting Product Rating

- The prediction is made by adjusting the average rating of the target user with the similarity and the opinion of *k*-NN.
- The formula *pred*(*a*,*p*) for computing a prediction for the rating of the target user *a* for item *p* considers
  - **$\overline{r_a}$: the average rating of all rated items of *a*
  - $r_{b,p}$: *k*-NNs' opinion (deviation from their (individual) means)
  - *sim*(*a*,*b*): relative proximity of the *k* nearest neighbors

$$pred(a,p) = \overline{r_a} + \frac{\sum_{b\in k} sim(a,b)(r_{b,p} - \overline{r_b})}{\sum_{b\in k} sim(a,b)}$$

**: In the above eq., all known ratings of the target user are meant by $\overline{r_a}$ as we are interested in the user's global rating bias.
Ref: http://www.recommenderbook.net/media/corrigenda.pdf

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An Example of Calculating the Predicted rating of Item 5 for Alice based on Ratings of 2-NNs (User1 and User2)

	Item 1	Item 2	Item 3	Item 4	Item 5
Alice	5	3	4	4	?
User 1	**3**	**1**	**2**	**3**	**3**
User 2	**4**	**3**	**4**	**3**	**5**

	Pearson Correlation Coefficient
Sim(Alice,User1)	0.85
Sim(Alice,User2)	0.7

the individual user's **average rating** of ALL rated items

$$\overline{r_{Alice}} = \frac{5+3+4+4}{4} = 4$$

$$\overline{r_{User1}} = \frac{3+1+2+3+3}{5} = 2.4$$

$$\overline{r_{User2}} = \frac{4+3+4+3+5}{5} = 3.8$$

$$pred(Alice, Item5) = \overline{r_{Alice}} + \frac{\sum_{b\in\{User1,User2\}} sim(Alice,b)(r_{b,Item5}-\overline{r_b})}{\sum_{b\in\{User1,User2\}} sim(Alice,b)}$$

$$pred(Alice, Item5) = 4 + \frac{0.85(3-2.4)+0.7(5-3.8)}{0.85+0.7} = 4.88$$

Other Measures to Determine Proximity between Users – 1/2

- Spearman's rank correlation coefficient
  - Case 1: no tied ranks
    - $\rho = 1 - \frac{6\sum d_i^2}{n(n^2-1)}$
    - d = the difference between the two ranks of each item
    - n = number of cases
  - Case 2: having tied ranks

$$\rho = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_i (x_i - \bar{x})^2 \sum_i (y_i - \bar{y})^2}}$$

    - i = paired score

User A score	User B score	Rank(User A's score)	Rank(User B's score)	d	d^2
50	30	1	2	1	1
30	20	3	3	0	0
40	10	2	4	2	4
20	40	4	1	3	9

$$\sum d_i^2 = 1 + 0 + 4 + 9 = 14$$

$$\rho = 1 - \frac{6\sum d_i^2}{n(n^2-1)}$$

$$\rho = 1 - \frac{6 \times 14}{4(4^2-1)} = -0.4$$

Ref: https://en.wikipedia.org/wiki/Spearman%27s_rank_correlation_coefficient
https://statistics.laerd.com/statistical-guides/spearmans-rank-order-correlation-statistical-guide.php

Other Measures to Determine Proximity between Users – 2/2

- Mean squared difference (*msd*) (Shardanand and Maes 1995)
  - Measure the degree of dissimilarity between two user profiles U_a and U_b by the mean squared difference between the two profiles $(U_a - U_b)^2$

$$msd(U_a, Ub) = \frac{\sum_{i=1}^{n}(r_{U_a,i} - r_{U_b,i})^2}{n}$$

- Adjusted cosine similarity
  - To be discussed in Item-based collaborative filtering

Ref: http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.95.586&rep=rep1&type=pdf

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Factors Not Addressed by Pearson Correlation Coefficient

- Different weighting of controversial items and a generally liked item
  - An agreement by two users on a more controversial item has more "value" than an agreement on a generally liked item
- Too few number of co-rated items
- Too few number of agreed items

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Guidelines on Neighborhood Selection

- Select neighbors who have
  - **Positive correlation** with the target user
  - **Rating** on the **predicted items**
- Reduce the size of the neighborhood by
  - Taking only the ***k*** **NNs**.
  - Defining **a minimum threshold** of user similarity.

Further reading: https://pdfs.semanticscholar.org/aeb6/3457163d83f46b9b365381adb4308a161723.pdf

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Effects of Neighborhood Selection Using *k* NNs

- Too high *k*: **noise** is added to the prediction.
- Too small *k*: the **quality** of the prediction may be **negatively affected** ($k < 10$).

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Effects of Neighborhood Selection Using a Min Threshold of User Similarity

- **Too high** threshold: *(get too few neighbors)*
  - May have a **coverage** problem *(**too few unseen items** to suggest)*

- **Too low** threshold: *(get too many neighbors)*
  - The **neighbor sizes** are **NOT significantly reduced**

    → *suggested **items** are **bias to global opinion**.*

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Problems with CF Approaches

- High Computation time for real-time predictions of millions of users and millions of catalog items in large e-commerce sites.

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Alternative Approach -- Item-based Nearest Neighbor Recommendation

- Offline preprocessing
- Main idea:
  - Compute predictions using the similarity **between items**

	Item 1	Item 2	Item 3	Item 4	Item 5
Alice	5	3	4	4	?
User 1	3	1	2	3	3
User 2	4	3	4	3	5
User 3	3	3	1	5	4
User 4	1	5	5	2	1

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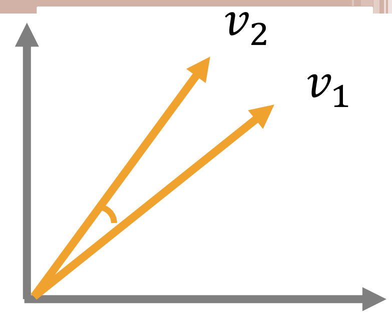

The Cosine Similarity Measure

- Measures the similarity between two *n*-dimensional vectors based on the angle between them.
- $cosine\_sim(v_1, v_2) = \frac{v_1 \cdot v_2}{\|v_1\|\|v_2\|}$
- *Example 3,*

Interpretation:
- Range of possible value is -1 and 1.
- The closer to 1 → the more similar

$$cosine\_sim(Item1, Item5) = \frac{3*3+5*4+4*3+1*1}{\sqrt{3^2+5^2+4^2+1^2}*\sqrt{3^2+4^2+3^2+1^2}} = 0.9941$$

Item 1	Item 5
3	3
4	5
3	4
1	1

- *cosine_sim(**Item1**, Item5) = **0.9941***
- *cosine_sim(Item2, Item5) = 0.7389*
- *cosine_sim(Item3, Item5) = 0.7226*
- *cosine_sim(**Item4**, Item5) = **0.9396***

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Example 2

- Goal: Predict Alice's rating on Item 5 from 2-NN items

  (e.g., 2-NN = Items 1 and 4)

	Item 1	Item 4	Item 5
Alice	5	4	?
User 1	3	3	3
User 2	4	3	5
User 3	3	5	4
User 4	1	2	1

= a weighted average of ratings 4 and 5

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The Problem with Basic Cosine Measure

- Problem: it does not take into account the differences in the average rating behavior of the users.
- Solution: using the *adjusted cosine measure to calculate similarity between items a, b*

$$sim(a,b) = \frac{\sum_{u \in U}(r_{u,a} - \overline{r_u})(r_{u,b} - \overline{r_u})}{\sqrt{\sum_{u \in U}(r_{u,a} - \overline{r_u})^2} \sqrt{\sum_{u \in U}(r_{u,b} - \overline{r_u})^2}}$$

where,

U is the subset of users that rated BOTH items *a* and *b*.

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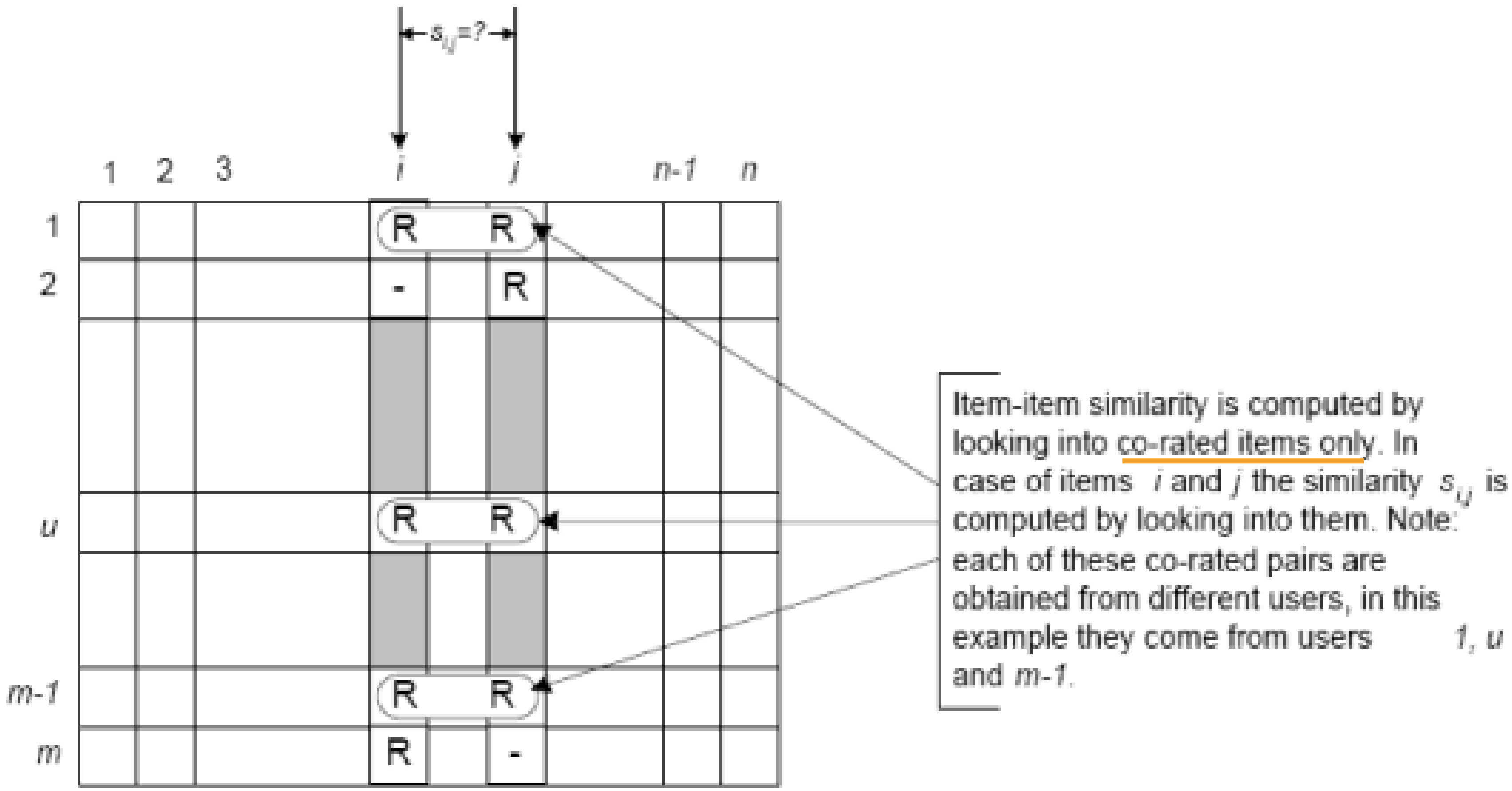
Similarities between Items



Ref. : http://www.cs.carleton.edu/cs_comps/0607/recommend/recommender/itembased.html

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Preprocessing Step: Adjusting Ratings
(Later Used by Adjusted Cosine Measure)

Original ratings database

$(r_{u,a})$

	Item 1	Item 2	Item 3	Item 4	Item 5
Alice	5	3	4	4	?
User 1	3	1	2	3	3
User 2	4	3	4	3	5
User 3	3	3	1	5	4
User 4	1	5	5	2	1

the individual user's **average rating** of ALL rated items

$(\bar{r_u})$

	Avg. Rating
Alice	4
User 1	2.4
User 2	3.8
User 3	3.2
User 4	2.8

Subtract the original ratings by the avg. rating

Mean-adjusted ratings database

$(r_{u,a} - \bar{r_u})$

	Item 1	Item 2	Item 3	Item 4	Item 5
Alice	1.0	-1.0	0	0	?
User 1	0.6	-1.4	-0.4	-0.6	0.6
User 2	0.2	-0.8	0.2	-0.8	1.2
User 3	-0.2	-0.2	-2.2	2.8	0.8
User 4	-1.8	2.2	2.2	-0.8	-1.8

Revised Example 3

Mean-adjusted ratings

	$(r_{u,item1} - \bar{r_u})$	$(r_{u,item5} - \bar{r_u})$:
User1	0.6	0.6
User2	0.2	1.2
User3	-0.2	0.8
User4	-1.8	-1.8

- The adjusted cosine similarity of items *a* and *b*:

$$sim(a,b) = \frac{\sum_{u \in U}(r_{u,a} - \bar{r_u})(r_{u,b} - \bar{r_u})}{\sqrt{\sum_{u \in U}(r_{u,a} - \bar{r_u})^2}\sqrt{\sum_{u \in U}(r_{u,b} - \bar{r_u})^2}}$$

U is the subset of users that rated BOTH items *a* and *b*.

$$sim(Item1, Item5) = \frac{0.6*0.6 + 0.2*1.2 + (-0.2)*0.8 + (-1.8)*(-1.8)}{\sqrt{0.6^2+0.2^2+(-0.2)^2+(-1.8)^2}*\sqrt{0.6^2+1.5^2+0.8^2+(-1.8)^2}} = 0.8049$$

- *sim(Item1, Item5) =* ***0.8049***
- *sim(Item2, Item5) = -0.9082*
- *sim(Item3, Item5) = -0.7636*
- *sim(Item4, Item5) =* ***0.4331***

Further reading: http://www.cs.carleton.edu/cs_comps/0607/recommend/recommender/itembased.html

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Predicting Product Rating

- **Use** the **previous rated items** of the **target user** by **adjusting** it **with** the similarity between its co-rated items to the *unseen* *(need to be predicted)* item.
- A formula for computing a prediction for the rating of the target user u for item p

$$pred(u,p) = \frac{\sum_{i \in ratedItems(u)} \overbrace{sim(i,p)}^{\text{Pre-calculated}} \times r_{u,i}}{\sum_{i \in ratedItems(u)} sim(i,p)}$$

where i are the items rated by the target user u.

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An Example of calculating the predicted rating of Item for Alice based on 2-NN *(Item1 and Item4)*

	Item 1	Item 4	Item 5
Alice	5	4	?

cosine_sim(Item1, Item5) =	***0.8049***
cosine_sim(Item4, Item5) =	***0.4331***

$$pred(Alice, Item5) = \frac{\sum_{i \in ratedItems(Alice)} \overbrace{sim(i, Item5)}^{\text{Pre-calculated}} \times r_{Alice,i}}{\sum_{i \in ratedItems(Alice)} sim(i, Item5)}$$

$$= \frac{(0.8049 \times 5) + (0.4331 \times 4)}{0.8049 + 0.4331}$$

$$= 4.65$$

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Preprocessing Data for Item-based Filtering

	Item1	Item2	Item3	Item4	Item5
Item1	1	...	...	...	***0.8049***
Item2	...	1	...	...	*-0.9082*
Item3	...	...	1	...	*-0.7636*
Item4	...	...	...	1	***0.4331***
Item5	...	...	...	...	1

- *Offline* precompute of the *item similarity matrix*

$$sim(a,b) = \frac{\sum_{u \in U}(r_{u,a} - \bar{r_u})(r_{u,b} - \bar{r_u})}{\sqrt{\sum_{u \in U}(r_{u,a} - \bar{r_u})^2}\ \sqrt{\sum_{u \in U}(r_{u,b} - \bar{r_u})^2}}$$

- *Online* predict ratings of a product *p* and user *u*,
  1. Determine the set of items *X* that are most similar to *p*.
  2. Build the ***weighted sum*** of *u*'s ratings for these items *X* in the neighborhood.

	Item 1	Item 4	Item 5
Alice	1.0	0	?

$$pred(u,p) = \frac{\sum_{i \in ratedItems(u)} sim(i,p) \times r_{u,i}}{\sum_{i \in ratedItems(u)} sim(i,p)}$$

- Perform subsampling
  - Randomly choose a subset of the data.
  - Ignore customer records with few rating.

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Pros and Cons of CF Based Approach

Pros	Cons
• Not require details of items and users in order to generate recommendations. • Capable of recommending serendipitous items based on similar users' behaviors.	• Requires explicit ratings/opinions from users. • Has a cold start problem (new user/ item). • Has poor accuracy if too few ratings exist. • Is slow (memory-based CF)

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Problems with Rating Scales

- Explicit rating: user might not be willing to provide ratings.
- Implicit rating: user behavior is not easily interpreted as ratings.

Data Sparsity and the Cold-Start Problems

- Data sparsity problem: data the rating matrices tend to be very sparse.
  - Solution: Default voting
- Cold-start problem: hard to generate recommendations for new items.

Suggested Reading Articles

- "Deconstructing Recommender Systems. How Amazon and Netflix predict your preferences and prod you to purchase" By Joseph A. Konstan and John Riedl, IEEE Spectrum, Sep. 2012. http://spectrum.ieee.org/computing/software/deconstructing-recommender-systems
- The Big Promise of Recommender Systems https://www.researchgate.net/publication/220604928_The_Big_Promise_of_Recommender_Systems
- Recommender Systems An Overview https://www.researchgate.net/publication/220604600_Recommender_Systems_An_Overview