PHYSICS 122 WORKSHEET 2 1/22/2018

NAME: Key SECTION:

1. We denote the dimensions of a quantity by the bracket symbol around that quantity. For example $[\Delta x] = L$; that is to say, the dimension of displacement is length, denoted L. Other relevant dimensions include time T, mass M, and charge Q. A *system of units* associates with every dimension a unit value against which all other quantities in that dimension are measured (e.g, in SI units, the unit of length is the meter m and all other lengths are measured in meters, the unit of charge is the Coulomb C and all other charges are measured in Coulombs, etc.).

i) What are the dimensions of velocity v? What are its units in SI?

ii) Momentum is defined as $p = mv$ where m is mass and v is the velocity. What are the dimensions of p? What are its units in SI? $[V] = LT^{-1}$; m/s

iii) Force is defined as $F = ma$ where a is the acceleration and m is the mass. What are the dimensions of a and F? What are the units of a and F in SI? $[a] = LT^{-2}$; m/s^2

$[F] = MLT^{-2}$; $\frac{Kg\,m}{s^2}$

iv) Work is defined as $W = F\Delta x$ where F is the force and Δx is the displacement. What are the dimensions of W? What are its units in SI? $[W] = ML^2T^{-2}$; $\frac{Kg\,m^2}{s^2}$

2. SI units include prefixes to denote what power of 10 a quantity is measured in. For example, centimeters are one hundredths of a meter, denoted cm. Hence a c prefix before a unit is equivalent to a coefficient of 10^{-2} on that unit, as $cm = (10^{-2})m$. Similarly a k prefix is equivalent to 10^3, etc.

i) Convert meters m to kilometers km. Given: $kila = k = 10^3$. $1\,Km = 1000\,m$

ii) Convert milligrams mg to kilograms kg. Given: $milli = m = 10^{-3}$. $1\,mg = 10^{-6}\,Kg$

iii) Convert microseconds (s) to megaseconds (Ms). Given: $micro = \mu = 10^{-6}$, $mega = M = 10^6$. $1\,\mu s = 10^{-12}\,Ms$

3. To convert between systems of units you multiply by the ratio of the new unit to the old. For example, to convert minutes (old) to seconds (new); $(1min)\frac{60s}{1min} = 60s$. In general $old\frac{new}{old} = new$.

ii) Convert one meter to miles. Given: (1mile = 1609.344m). $1\,m = \frac{1}{1609.344}\,mile$

ii) Convert one mile per hour ($mile/hr$) to meters per second (m/s). $\frac{1\,mile}{hr}\;\frac{1\,hr}{60s}\;\frac{1609.344\,m}{mile}$

iii) Convert ($kg \cdot m/s$) to ($lb \cdot mile/hr$). Given: $1lb = 0.45359237kg$.

$$\frac{1\,Kg\,m}{s} \cdot \frac{1\,lb}{0.4536\,Kg} \cdot \frac{60s}{hr} \cdot \frac{1\,mile}{1609.34\,m}$$

Example: determining the units and dimensions of G, the newtonian gravitational constant. The expression for the force between two gravitating masses is

$$F_g = G\frac{M_1 M_2}{r^2} \tag{1}$$

To determine G, we rearrange the expression to isolate it on one side:

$$\frac{F_g r^2}{M_1 M_2} = G \tag{2}$$

and now cancel units:

$$\frac{kg \cdot m}{s^2}\frac{m^2}{kg^2} = \frac{m^3}{kg \cdot s^2} \tag{3}$$

Thus the units of G are $m^3 kg^{-1} s^{-2}$ and $[G] = L^3 M^{-1} T^{-2}$.

4: Determine the units and dimensions of ϵ_0 given the expression for the electric force between two charges A and B separated by a distance r. *Hint:* 4π is just a number and so has no dimension.

$$F_e = \frac{1}{4\pi\epsilon_0}\frac{Q_A Q_B}{r^2} \tag{4}$$

$$[\varepsilon_0] = \left[\frac{Q_A Q_B}{F_e r^2}\right] = Q^2 L^{-2} T^2 L^{-1} M^{-1} = \frac{Q^2 T^2}{L^3 M} \; ; \; \frac{C^2 s^2}{m^3 Kg}$$

5: Do the same for

$$k = \frac{1}{4\pi\epsilon_0} \rightarrow [K] = [\varepsilon_0]^{-1} = \frac{L^3 M}{Q^2 T^2} \; ; \; \frac{m^3 Kg}{C^2 s^2} \tag{5}$$

and

$$E = k\frac{Q}{r^2} \quad [E] = [K] Q L^{-2} = \frac{LM}{Q T^2} \; ; \; \frac{m \cdot Kg}{C s^2} \tag{6}$$

Also express the electric field E in terms of Newtons $N = kg \cdot m \cdot s^{-2}$ (the unit of force) and Coulombs C (the unit of charge).

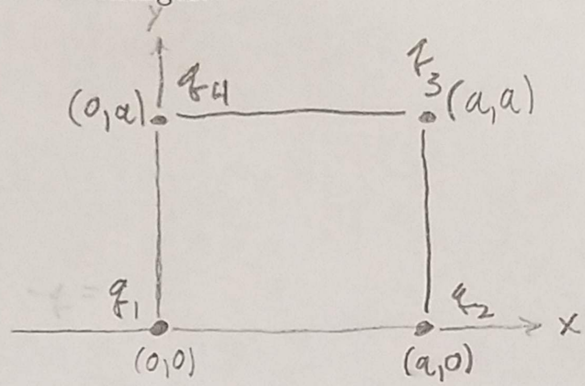
$$[K] = \frac{ML}{T^2}\frac{L^2}{Q^2} \; ; \; N\frac{m^2}{C^2}$$

$$[E] = \frac{ML}{T^2}\frac{1}{Q} \; ; \; N\frac{1}{C}$$

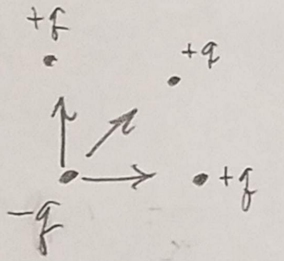
"force per charge"

Now let's calculate the electric field due to a sequence of charges.

6. Consider four charges arranged as follows: q_1 at $(0,0)$, q_2 at $(a,0)$, q_3 at (a,a), and q_4 at $(0,a)$. Draw the charges.



i) If $q_2 = q_3 = q_4 = q$, and $q_1 = -q$, solve for the net force on charge 1.

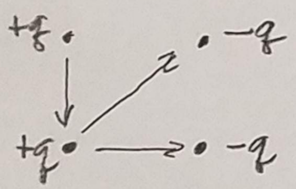


$$\Sigma \vec{F} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14}$$

$$= kq^2\left(\frac{1}{a^2}\hat{x} + \frac{1}{(\sqrt{2}a)^2}(\hat{x}+\hat{y}) + \frac{1}{a^2}\hat{y}\right)$$

$$= \frac{kq^2}{a^2}\left(\frac{3}{2}\hat{x} + \frac{3}{2}\hat{y}\right) = \frac{3kq^2}{2a^2}(\hat{x}+\hat{y})$$

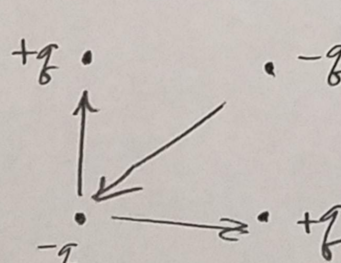
ii) If $q_2 = q_3 = -q$, and $q_1 = q_4 = +q$, solve for the net force on charge 1.



$$\Sigma \vec{F} = kq^2\left(+\frac{1}{a^2}\hat{x} - \frac{1}{a^2}\hat{y} + \frac{1}{2a^2}(\hat{x}+\hat{y})\right)$$

$$= \frac{kq^2}{a^2}\left[\frac{3}{2}\hat{x} - \frac{1}{2}\hat{y}\right]$$

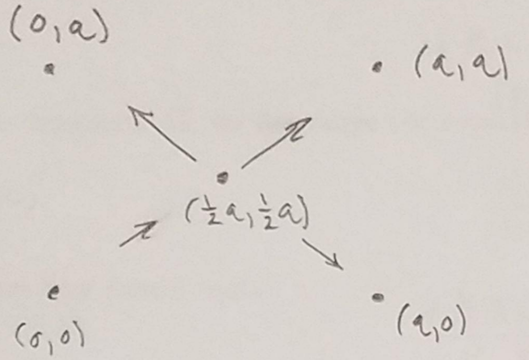
iii) Do the same for $q_1 = q_3 = -q$, and $q_2 = q_4 = +q$.



$$\Sigma \vec{F} = \frac{kq^2}{a^2}\left[\hat{x} + \hat{y} - \frac{1}{2}(\hat{x}+\hat{y})\right]$$

$$= \frac{kq^2}{2a^2}(\hat{x}+\hat{y})$$

Consider the same system of charges as in **6i)**, but place a charge $q_0 = -q$ at the point $(\frac{1}{2}, \frac{1}{2})$. What is the net force on q_0? What is the electric field at that point?



$\Sigma \vec{F} =$ on center charge

$$\Sigma \vec{F} = \frac{kq^2}{a^2}\left[\frac{1}{\left(\frac{\sqrt{2}}{2}\right)^2}(\hat{x}+\hat{y}) + \frac{1}{\left(\frac{\sqrt{2}}{2}\right)^2}(\hat{x}-\hat{y}) + \frac{1}{\left(\frac{\sqrt{2}}{2}\right)^2}(-\hat{x}+\hat{y}) + \frac{1}{\left(\frac{\sqrt{2}}{2}\right)^2}(\hat{x}+\hat{y}) \right]$$

$$= \frac{kq^2}{a^2}(2)\left[\cancel{\hat{x}} \cancel{+\hat{y}} \cancel{+\hat{x}} \cancel{-\hat{y}} \cancel{-\hat{x}} + \hat{y} + \hat{x} + \hat{y} \right] = \frac{4kq^2}{a^2}(\hat{x}+\hat{y})$$

$$\vec{E}\left(\tfrac{1}{2}a, \tfrac{1}{2}a\right) = \frac{\Sigma \vec{F}}{-q} = -\frac{4kq}{a^2}(\hat{x}+\hat{y})$$