Kinematics of a Bouncing Ball

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Abstract

In this study the nonconservative physics of a bouncing ball is considered using a one-dimensional model on the Earth and on Mars. Energy dissipation in the system is set by the coefficient of restituion, and the resulting height and time of each bounce are calculated using kinematics. These quantities are then compared graphically between the two cases.

1 Background

One of the simplest physical models in which energy is not conserved is that of a bouncing ball. Each time the ball hits the ground, it is compressed, and kinetic energy is lost through friction and heat, reducing the subsequent rebound height. Assuming the only dimension of motion is the vertical, a single parameter C known as the coefficient of restitution determines the percent of energy lost. Letting n denote the number of bounces, the energy of each bounce is

$$E_n = C^n E_0$$

where E_0 is the initial energy. Assuming the rebound event at the floor is instantaneous, the time t_n taken for the ball to rebound to some height h_n is the same as the time taken for the ball to fall from that height back to the floor (due to the time reversal symmetry of Newton's laws). Thus, the time taken for any half trip can be obtained from the kinematic equations and is given by

$$t_n = \sqrt{2E_n/mg^2}$$

Similarly the distance travelled over any half trip can be obtained from the gravitational energy and is simply

$$h_n = E_n/mg$$

2 Procedure

The preceding quantities were coded in C++ using the Cloud9 IDE. The system is modeled by a program which takes in the coefficient of restitution and the number of bounces, and outputs the height of each bounce and the time taken to fall as well as the sum of these quantities over the series of bounces. The height and time taken as a function of energy is given below.

```
float tbounce(float Eb) {
        float t =  sqrt(2*E/(m*g*g));
        return(t); }

float ybounce(float Eb) {
        float y = E/(m*g);
        return(y); }
```

3 Results

The height achieved by and time taken to fall from the first 10 bounces are plotted below with $C = 0.5$, $g = 9.81$, $m = h_0 = 1$.

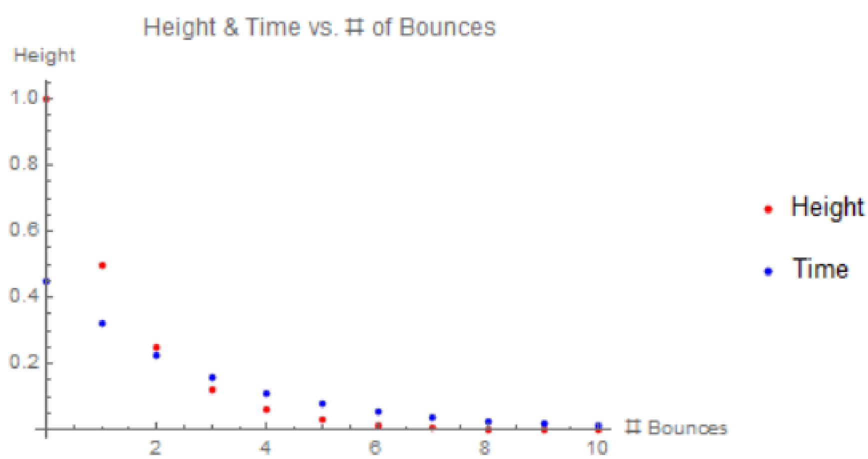

Figure 1: Vertical and temporal range for $n\epsilon[0, 10]$.

These are geometric series whose partial sums approach zero. Note that by the third bounce the time taken to fall exceeds the vertical range. This can be understood as a consequence of the gravitational acceleration determining the characteristic time and length scales. To demonstrate this consider the case for which the same ball is on mars, where $g = 3.74$[1]:

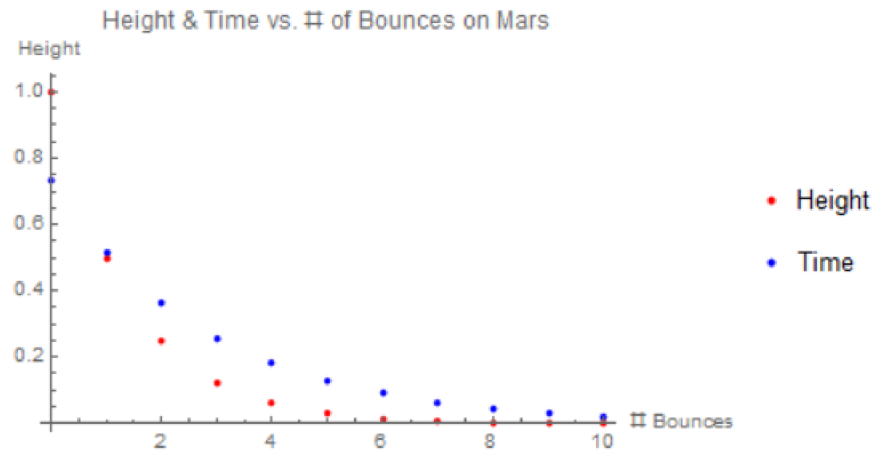

Figure 2: First ten bounces on mars. Note the initial time/height ratio, as compared to Fig. 1.

In this case the ball takes nearly twice as long to fall from the same height. Furthermore the time it takes to do so will always exceed the distance fallen after the initial drop.

4 Discussion

In general the behavior of this system is characterized by its gravitational acceleration. This is in part due to the initial condition coupling the mass and length scales. Further studies could account for this. More interestingly a potential future study could include determining the point at which time exceeds height for arbitrary g.

5 Conclusion

The kinematics of a bouncing ball were explored for a one dimensional system on the Earth and on Mars. Calculations were performed in C++ and graphics generated using Mathematica.

References

[1] "Mars Fact Sheet," NASA Space Science Data Coordinated Archive, December 23 2016. [Accessed 1/22/2017]. http://nssdc.gsfc.nasa.gov/planetary/factsheet/marsfact.html