Random Walks in 2 and 3 Dimensions

Noeloikeau F. Charlot[1]

[1] *Department of Physics & Astronomy, University of Hawaii at Manoa, 2505 Correa Rd, Honolulu, HI, 96822, USA*

In this study random walk algorithms were implemented in two and three dimensions using C++. The net distance travelled after N steps was fit to a power law using Gnuplot and found to scale as $\sqrt{N}$ in each case, confirming theoretical predictions.

INTRODUCTION

Random walks are sequences for which every element s_i is chosen arbitrarily from a set of values m_{ij} permissible to that index. In their simplest form, there exists no functional dependency among elements - in other words, the walk possesses no "memory".

Here we consider such a walk in 2 and 3 dimensions. The coordinates r_i of each step are chosen randomly from within a circle or sphere of unit radius centered on the current position.

PROCEDURE

In two dimensions, the procedure for determining each coordinate was implemented in C++ using the following algorithm:

```
for (i=1; i<=Nmax; i++){
        x += cos(drand48()*2.*M_PI);
        y += sin(drand48()*2.*M_PI);
        r[i] += sqrt(x*x + y*y);}
```

where $drand48()*2.*M_PI$ calls a random angle within $[0, 2\pi]$. For the 3D case it is insufficient to call angles θ and ϕ from within $[0, \pi]$ and $[0, 2\pi]$ because the area element is a function of the polar angle θ [2].

It follows that a new parametrization must be defined such that the selection of available points is uniformly distributed about the surface of the sphere. Following [2] we define parameters $u = cos(\theta)\epsilon[-1, 1]$ and $v = \phi\ \epsilon[0, 2\pi)$ such that

$$x = \sqrt{1-u^2}\cos(v) \tag{1}$$

$$y = \sqrt{1-u^2}\sin(v) \tag{2}$$

$$z = u \tag{3}$$

Using the above definitions, the algorithm for determining the coordinates in 3 dimensions is:

```
for (i=1; i<=Nmax; i++){
        u=drand48()*2.-1.;
        v=drand48()*2.*M_PI;
        x +=(sqrt(1.-u*u))*cos(v);
        y +=(sqrt(1.-u*u))*sin(v) ;
        z += u;
        r[i] += sqrt(x*x + y*y +z*z);}
```

In each case multiple paths were computed and then averaged to obtain a representative value of distance travelled as a function of number of steps taken N. Since each step is independent and of magnitude 1 and mean 0 the average distance travelled is expected to scale as $\sqrt{N}$ (in any number of dimensions).

RESULTS

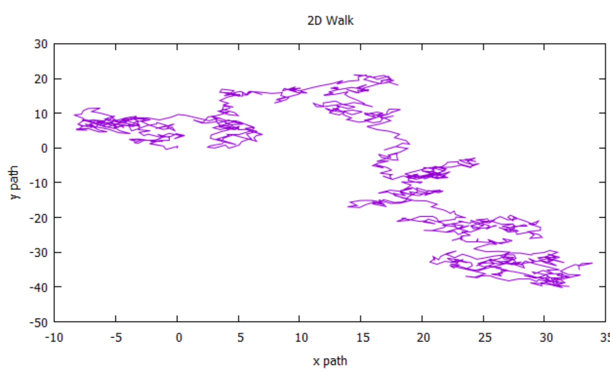

FIG. 1. *2D walk consisting of 1000 steps.*

Fig. 1 displays the results of the 2D walk program after 1000 steps and 10 averages for each step. Note that only a single seed was used in the program for consistency.

Fig. 2 displays the results of a least-squares fit to the net distance travelled after 100 steps using an averaging of 4 trails per data point. Note the convergence to the expected value of 10 units of distance

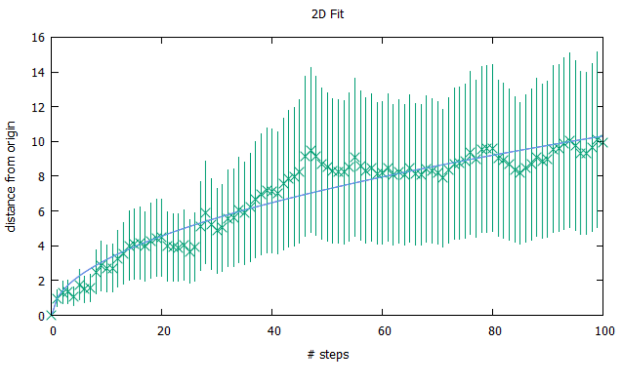

FIG. 2. *Distance from origin as a function of number of steps. Blue line is power law fit, green points are actual distances with error.*

after 100 steps.

Fig. 3 displays a scatterplot of endpoints for

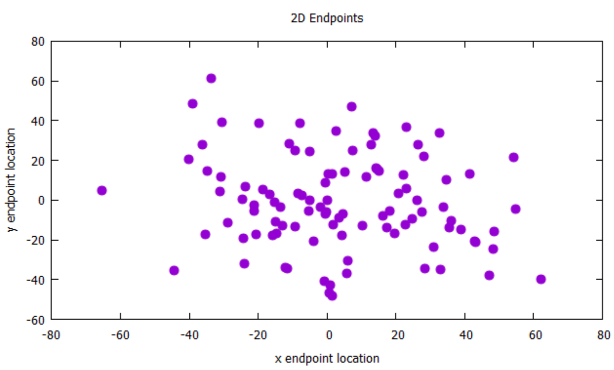

FIG. 3. *Scatterplot of 100 different walk endpoints.*

100 different walks consisting of 1000 steps each. Note the highest density of points occurs within 30 ($\approx \sqrt{1000}$) units from the origin.

Fig. 4 displays the results of a 3D walk consisting of 1000 steps from the origin. Fig. 5 displays the results of a least-squares fit to the net distance travelled after 100 steps using an averaging of 10 trials per data point. Note the convergence to the expected value of 30 ($\approx \sqrt{1000}$) units of distance after 1000 steps.

DISCUSSION

Both the 2D and 3D walks were fit to power laws of the form x^c where c is determined by the fit. In

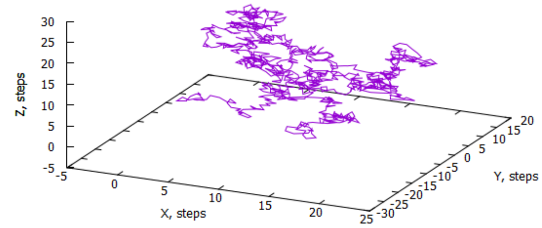

FIG. 4. *3D walk consisting of 1000 steps.*

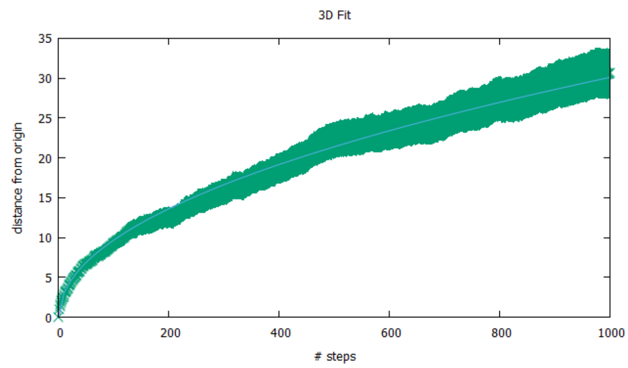

FIG. 5. *Distance from origin as a function of number of steps in 3D. Blue line is power law fit, green points are actual distances with error.*

each case $c \approx 0.50$, to within ± 0.01. This confirms the prediction that the distance scales as $\sqrt{N}$. In general it was observed that the greater the number of averaging events, the more successful the fit. This can be understood as a result of larger sampling sizes reducing the number of outlier steps within the walk.

CONCLUSIONS

In summary, random walks were calculated in both two and three dimensions. The distance travelled during the walk was fit to a power law, and found to scale as $N^{0.50\pm0.01}$, confirming the predicted $\sqrt{N}$ scaling behavior.

REFERENCES

[1] P. Gorham, "Physics 305 Lab 4: Random Walks," *University of Hawai'i at Manoa* (2017). `http://www.phys.hawaii.edu/~gorham/p305/RandomWalks.html`

[2] E. Weisstein, "Sphere Point Picking," *Wolfram Mathworld*(2017). `http://mathworld.wolfram.com/SpherePointPicking.html`