Polarization Phenomena & Crystal Optics

Noeloikeau Charlot

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University of Hawai'i at Manoa

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Abstract

In this study a series of four optical experiments were performed in order to measure the following: the refractive index of BK7 glass, the range of intensities under which a photodiode's response to HeNe laser light is linear, the induced shifts in polarization/phase at dielectric/conductive boundaries, and the orientation of the c-axis of a sapphire crystal. Each individual experiment tested the predictions of particular optical phenomena: Brewster's angle, Malus' Law, reflections from dielectric and conductive surfaces, and birefringence. The values obtained were found to match closely with theoretical expectations and with values in the literature.

1 Background

Physical optics is the study of the interaction of light with itself and with matter in macroscopic settings. At this scale the wavelike properties of light are dominant and its behavior is described by oscillating electric and magnetic fields mutually perpendicular to eachother and to the axis of transmission. The orientation of the electric field vector at any point relative to this axis is the polarization of the wave.

As a light wave propagates through a medium its polarization determines the direction along which a dipole will oscillate. Together with the nature of the medium and the boundary conditions at each surface, the intensity and polarization of light uniquely characterizes a wide variety of phenomena ranging from the color of the sky to the structure of crystals.

1.1 The Brewster Angle

Consider an unpolarized light wave incident upon the boundary of two dielectric mediums having indices of refraction $n_1 < n_2$. By the law of reflection the angle θ relative to the surface normal at which the wave is incident is equal in magnitude to the angle at which it is reflected. Furthermore because light is a transverse wave any radiation emitted by an oscillating dipole must be perpendicular to its axis of oscillation for all θ. This places a constraint on the reflected wave's electric field component within the plane of incidence. Specifically this component must approach zero for angles at which the initial wave would be reflected along the dipole's oscillation axis. For such an angle the transmitted electric field vector must be collinear to the direction of the reflected wave's propagation. It follows that at this angle the reflected wave direction is given by a clockwise 90° rotation from the transmitted wave within the plane of incidence. Consequently $E_{||}^{(R)} = 0$ when $\theta + \theta_T = \frac{\pi}{2}$ where $\theta = \theta_B$ is the Brewster angle. Using Snell's Formula gives $n_1 sin(\theta_B) = n_2 sin(\frac{\pi}{2} - \theta_B)$, or equivalently

$$\theta_B = tan^{-1}(\frac{n_2}{n_1}) \tag{1}$$

1.2 Malus' Law

Anisotropic materials that absorb light along a specific direction serve to polarize the transmitted wave and are known as polarization filters or "polarizers". The expression

$$I = I_0 cos^2(\varphi) \tag{2}$$

is known as Malus' Law and relates the intensity of the light leaving the filter I with the intensity of the light entering the filter I_0 where φ is the angle between the polarization axis of the filter and the initial polarization state of the wave.

1.3 Dielectric & Conductive Boundaries

It can be shown from the boundary conditions on the electric field at a dielectric surface that there is a net change in polarization for reflected waves. These changes are described according to the fresnel coefficients:

$$r_s = \frac{cos(\theta_i) - \sqrt{n_2^2 - sin(\theta_i)}}{cos(\theta_i) + \sqrt{n_2^2 - sin(\theta_i)}} \tag{3}$$

$$r_p = \frac{n_2^2 cos(\theta_i) - \sqrt{n_2^2 - sin(\theta_i)}}{n_2^2 cos(\theta_i) + \sqrt{n_2^2 - sin(\theta_i)}} \tag{4}$$

where $r_s > r_p \; \forall \; (n, \theta)$. The reflected wave's new polarization angle is given by

$$\psi = tan^{-1}(\frac{r_s}{r_p}) \tag{5}$$

It can be shown using Jones' matrices that the relative phase shift between the electric field components of a linearly polarized beam reflecting from a boundary between two mediums $n_1 < n_2$ is given by

$$\phi = cos^{-1}(\frac{I_{+} - I_{-}}{2\sqrt{I_H I_V}}) \tag{6}$$

where $I_{\pm}$ corresponds to $I(\theta = \pm 45^{\underline{o}})$ [i.e, the intensity resulting from a polarizer oriented at $\pm 45^{\underline{o}}$] and $I_{H,V}$ is the intensity resulting from normal incidence upon either a horizontal or vertical polarizer.

1.4 Birefringence

A birefringent material is one in which the the structure of the material possesses two indices of refraction. These exist along separate axes oriented relative to one another, and by controlling this orientation it is possible to convert the polarization of incoming light in one state to outgoing light in another. Using geometric optics it can be shown that the angle required to convert an incoming P polarization to an outgoing S polarization is

$$\alpha = tan^{-1}(\frac{sin(\theta_0)}{n_s}) \tag{7}$$

where θ_0 is the angle at which the intensity is zero and n_s is the index of refraction corresponding to S polarized electric field components.

2 Experimental Procedure

2.1 BK7 Refractive Index

The Brewseter angle was used to calculate the index of refraction of BK7 glass. This was accomplished using a HeNe laser set to a pure plane polarization state. The optical axis was centered using a mirror to balance the reflected beam down the incoming beam. BK7 glass was mounted on a rotating platform a distance of approximately 0.75 meters from the start of the beam. The angle was varied until no reflected light was observed. This angle was recorded and used as a reference. A more accurate value was obtained by passing the beam at an angle to a mirror placed colinear to the BK7 surface and displaced forward a approximately 20cm so as to reflect the beam coming off the glass surface toward a photodiode for measurement. An oscilloscope was used to measure the output voltage of a photodiode. Angles at which the current was minimum corresponded to the Brewster angle. Using (1) the index of refraction was then calculated.

2.2 Photodiode Response

Malus' Law was confirmed by testing the regime under which the photodiode's response to the HeNe laser was linear. The same experimental setup was employed with the key difference being that the BK7 glass was replaced with a horizontal optical filter sensitive to the plane of polarization. The laser was tuned to produce a 10V peak response in the photodiode, corresponding to 2V less than the 12V maximum value specified by the manufacturer. The initial angle of the filter was set to a point that produced the minimum observed intensity. The optical filter was then rotated through 5º increments above and below this angle, sweeping through a total of 235º. The intensity of the wave was recorded at each point under minimal ambient light. The resulting data was fit using a relation of the form

$$y = Acos^2(\theta - \theta_0) \tag{8}$$

where θ is the angle at which the data was recorded, θ_0 is the initial angle, and A is a constant fit to the model using Mathematica. Only small excitations corresponding to points below $2V$ were used in the fitting of the model in order to remove the possibility of including nonlinear excitations.

2.3 BK7 & Metal Mirror Reflections

The phase shift of the electric field components of a plane polarized light wave reflecting from a dielectric surface was tested. Using the same experimental setup the intensity of the light was measured as it reflected from BK7 glass at an incident angle of 45º. Using the far right hand side of (3) the phase shift was obtained from the fresnel coefficients.

Next a silvered mirror was mounted in place of the BK7 glass. A vertical polarizer was mounted in the path of the beam permitting the measurement of the vertical component of the intensity. Similarly the horizontal component was measured using a horizontantal polarizer. A single polarizer was tilted at ±45º and the intensity at these points measured.

2.4 Sapphire Crystal Axis

A particular sapphire sample was chosen as a birefringent material. This sample was prebaricated to convert $P \rightarrow S$ polarization states. It was loaded onto an optic in front of the HeNe laser, which was then set to a P polarization and activated. The angle of the crystal was varied and the points at which observed intensity was minimum were recorded. The expected zero-intensity point θ_0 was interpolated by fitting the data to a quadratic of the form

$$y = A(\theta - \theta_0)^2 + B \tag{9}$$

where A and B are constants of the fit.

3 Results

3.1 BK7 Refractive Index

From (1) the index was calculated as n_{BK7} =1.5061±0.005. The index of refraction of air was assumed to be 1 exactly.

3.2 Photodiode Response

The photodiode response data was fit to (8) and is given below in Figure 1:

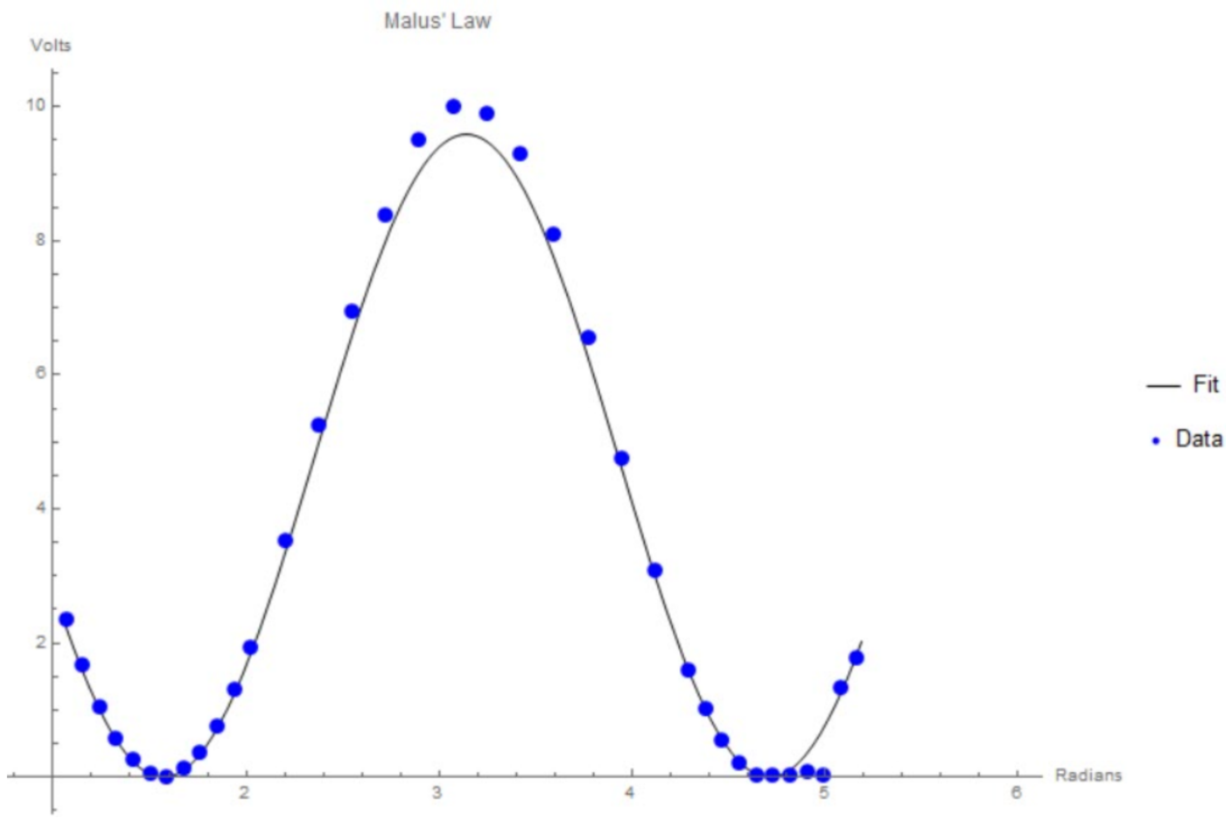

Figure 1: HeNe laser induced photodiode voltage and corresponding fit.

3.3 BK7 & Metal Mirror Reflections

Using (5) the dielectric polarization shift was found to be $\psi = (72.99 \pm 0.02)^{\circ}$. From (6) the conductive phase shift was determined to be $\phi = (25.37 \pm 0.02)^{\circ}$.

3.4 Sapphire Crystal Axis

The minimum intensity angles were fit to (9) and the resulting plot is given below in Figure 2:

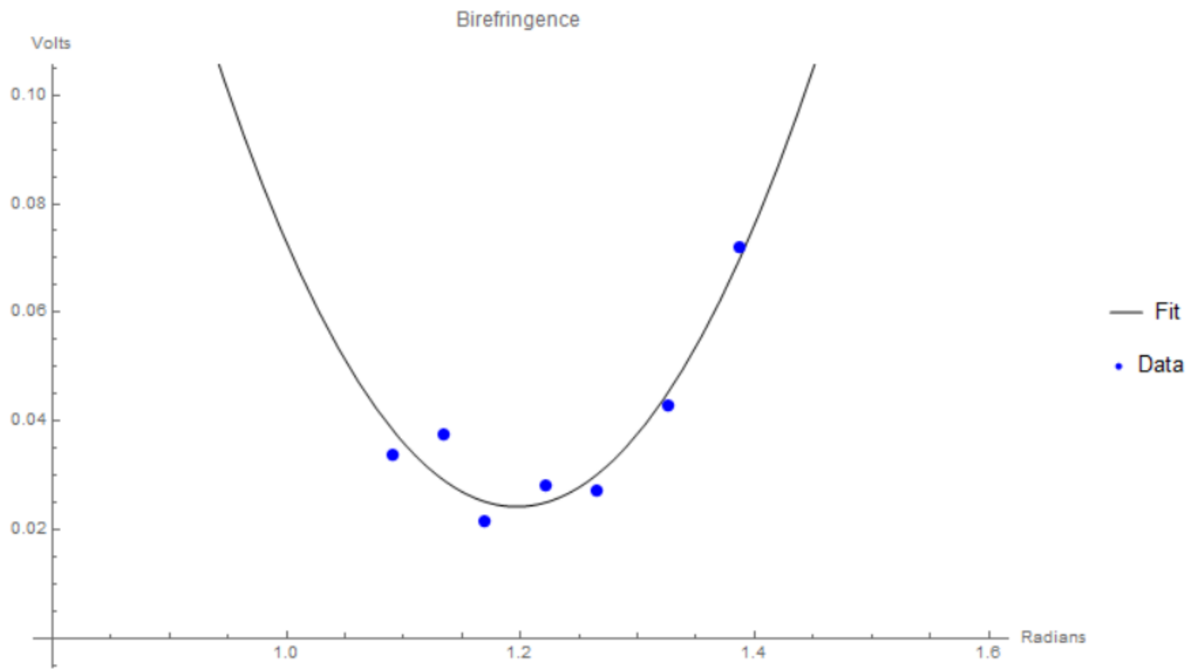

Figure 2: Minimal-intensity points for Sapphire and corresponding fit.

From the fit the zero-intensity angle was estimated at $\theta_0 = 1.19 \pm 0.01$ rad. Using the manufacturer listed index of refraction for Sapphire $n_s = 1.765$ the angle of the c-axis was calculated according to (7) and found to be $\alpha = (27.74 \pm 0.09)°$.

4 Discussion

4.1 BK7 Refractive Index

The calculated value for n_{BK7} aligns closely with the literature standard of 1.5186. Discrepencies are on the order of one part in a hundred and can be understood as a consequence of the varying quality of the sample compared to the tabulated index material as well as the effect of the index of refraction of air.

4.2 Photodiode Response

Figure 1 displays a strong correspondence between the observed data and the fit below 10V. Around 10V the observed data deviates from the fit indicating non-linear behavior in this regime. Malus' Law therefore applies below 10V, demonstrating that the response of the photodiode is linear beneath this threshold. In the context of physical optics this verification process is of practical utility as it can be used to ensure that the operating regime of the photodiode remains linear.

4.3 BK7 & Metal Mirror Reflections

The calculated values of ψ and ϕ are typical for reflections at either surface. In the case of light reflecting from a dielectric (BK7) the magnitude and direction of the wave's electric field components were changed, but not their phase. Conversely in the case of light reflecting from a conductor (silvered mirror) the phase of the wave's electric field components were changed, but not their magnitude or direction. These conditions result from different boundary conditions on the electric field at the two surfaces.

4.4 Sapphire Crystal Axis

The calculated value for the crystal axis angle was found to match the design specifications.

5 Conclusion

In summary several optical experiments involving polarization phenomena were performed using a HeNe laser. Brewster's angle was used to calculate the refractive index of BK7 glass, which was found to be n_{BK7} =1.5061±0.005, matching literature to within one part in a hundred. The regime in which the photodiode exhibited a linear response was measured and fit to Malus' Law, confirming its validity. Predictions regarding the polarization and phase angle resulting from reflections from a dielectric (BK7) and conductor (silvered mirror) were confirmed, having the values $\psi = (72.99 \pm 0.02)^\circ$ and $\phi = (25.37 \pm 0.02)^\circ$, respectively. Finally the angle between the birefringent axes of a custom designed Sapphire plate was calculated as $\alpha = (27.74 \pm 0.09)^\circ$, matching the design specifications.

References

[1] Hecht, Eugene: "Optics". Addison-Wesley, 2002. Print. Chapter 8