Brackets and Operators:

1. A system is in the state $|\psi\} = A(2|1\} + 3|2\} + |5\})$ where $|n\}$ denotes the normalized eigenstate of an observable O with eigenvalue n. What is the expectation value of O in the state $|\psi\}$?

A) 13/14
B) 1
C) 13/[$14^{1/2}$]
D) 27/14
E) 27

The state $\psi = \frac{1}{\sqrt{6}}\psi_{-1} + \frac{1}{\sqrt{2}}\psi_1 + \frac{1}{\sqrt{3}}\psi_2$ is linear combination of three orthonormal eigenstates of the operator $\hat{O}$ corresponding to eigenvalues -1,1 and 2. What is the expectation value of $\hat{O}$ for this state?

A. 2/3
B. √(7/6)
C. 1
D. 4/3
E. (√3+2√2-1)/√6

(GR0177 #29)

The Hamiltonian operator in the Schrodinger equation can be formed from the classical Hamiltonian by substituting

A. Wavelength and frequency for momentum and energy
B. A differential operator for momentum
C. Transition probability for potential energy
D. Sums over discrete eigenvalues for integrals over continuous variables
E. Gaussian distributions of observables for exact values

(GR8677 #49)

2. Let $|s\}$ and $|t\}$ denote orthonormal states. Let $|\psi_1\} = |s\} + 2i|t\}$ and $|\psi_2\} = 2|s\} + x|t\}$. What must the value of x be such that $|\psi_1\}$ and $|\psi_2\}$ are orthogonal?

A) i
B) -i
C) 1
D) -1
E) i/[$5^{1/2}$]

$$|\psi_1\rangle = 5|1\rangle - 3|2\rangle + 2|3\rangle$$
$$|\psi_2\rangle = |1\rangle - 5|2\rangle + x|3\rangle$$

The states $|1\rangle$, $|2\rangle$ and $|3\rangle$ are orthonormal. For what value of x are the states $|\psi_1\rangle$ and $|\psi_2\rangle$ given above orthogonal?

A. 10
B. 5
C. 0
D. −5
E. −10

(GR0177 #28)

The state of a quantum mechanical system is described by a wave function. Consider two physical observables that have discrete eigenvalues: observable A with eigenvalues $\{\alpha\}$, and observable B with eigenvalues $\{\beta\}$. Under what circumstances can all wave functions be expanded in a set of basis states, each of which is a simultaneous eigenfunction of both A and B?

A. Only if the values $\{\alpha\}$ and $\{\beta\}$ are nondegenerate
B. Only if A and B commute
C. Only if A commutes with the Hamitonian of the system
D. Only if B commutes with the Hamiltonian of the system
E. Under all circumstances

(GR9277 #50)

3. Let ψ_1 and ψ_2 be energy eigenstates of a time-independent Hamiltonian with energies E_1 and E_2. At time t=0, a system is in state $\frac{1}{\sqrt{2}}(\psi_1 - \psi_2)$. At time t, what is the probability that a measurement of the energy of the system will return E_1?

A) 0
B) $\frac{1}{\sqrt{2}}$
C) $\frac{1}{2}$
D) cos[(E_2 - E_1)/ħ]
E) cos[(E_2 + E_1)/ħ]

Let $|n\rangle$ represent the normalized n^{th} energy eigenstate of the one-dimensional harmonic oscillator,

$$H|n\rangle = \hbar\omega\left(n + \frac{1}{2}\right)|n\rangle$$

If $|\psi\rangle$ is a normalized ensemble state that can be expanded as a linear combination

$$|\psi\rangle = \frac{1}{\sqrt{14}}|1\rangle - \frac{2}{\sqrt{14}}|2\rangle + \frac{3}{\sqrt{14}}|3\rangle$$

of the eigenstates, what is the expectation value of the energy operator in this ensemble state?

A. $\frac{101}{14}\hbar\omega$

B. $\frac{43}{14}\hbar\omega$

C. $\frac{23}{14}\hbar\omega$

D. $\frac{17}{\sqrt{14}}\hbar\omega$

E. $\frac{7}{\sqrt{14}}\hbar\omega$

(GR0177 #45)

The energy eigenstates for a particle of mass m in a box of length L have wave functions

$$\phi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$$

and energies

$$E_n = \frac{n^2\pi^2 x^2}{2mL^2}$$

where =1,2,3,... . At time t = 0, the particle is in a state described as follows.

$$\psi(t=0) = \frac{1}{\sqrt{14}}(\phi_1 + 2\phi_2 + 3\phi_3)$$

Which of the following is a possible result of a measurement of energy for the state

A. $2E_1$
B. $5E_1$
C. $7E_1$
D. $9E_1$
E. $14E_1$

(GR0177 #44)

4. Let $|a\}$ and $|b\}$ denote momentum eigenstates with respective eigenvalues a and b where $a \neq b$. What is $\{a|p|b\}$?

A) a
B) b
C) $|ab|$
D) $|ab|^{1/2}$
E) 0

5. A particle of mass m in a harmonic oscillator potential with angular frequency ω is in the state $\frac{1}{\sqrt{2}}(|1\} + |4\})$. What is the expectation value of p^2 for this particle?

A) $3\hbar\omega/2$
B) $9\hbar\omega/2$
C) $6\sqrt{2}m\hbar\omega/2$
D) $3m\hbar\omega$
E) $6m\hbar\omega$

6. What is the expectation value of the operator $a_+a_- + a_-a_+$ in the ground state of the harmonic oscillator?

A) 0
B) 1
C) $\sqrt{2}$
D) 2
E) $\hbar\omega$

7. A free particle of mass m is in a momentum eigenstate $|p\}$. What is the uncertainty on a measurement of its energy?

A) 0
B) ∞
C) $p^2/2m$
D) p
E) Cannot be determined from the information given

8. A free particle has the wavefunction $sin(kx)$. The particle has:

I. A definite value of position
II. A definite value of momentum
III. A definite value of energy

A) I only
B) II only
C) III only
D) I and II
E) II and III

The operator, $\hat{a} = \sqrt{\frac{m\omega_0}{2\hbar}}(\hat{x} + i\frac{\hat{p}}{m\omega_0})$ when operating on a harmonic energy eigenstate ψ_n with energy E_n, produces another energy eigenstate whose energy is $E_n - \hbar\omega_0$. Which of the following is true?

I. $\hat{a}$ commutes with the Hamiltonian.
II. $\hat{a}$ is a Hermitian operator and therefore an observable.
III. The adjoint operator $\hat{a}^\dagger \neq \hat{a}$

A. I only
B. II only
C. III only
D. I and II only
E. I and III only

(GR9677 #100)

Which of the following is an eigenfunction of the linear momentum operator $-i\hbar\, \partial/\partial x$ with a positive eigenvalue $\hbar k$; i.e., an eigenfunction that describes a particle that is moving in free space in the direction of positive x with a precise value of linear momentum?

A. $\cos kx$
B. $\sin kx$
C. e^{-ikx}
D. e^{ikx}
E. e^{-kx}

(GR8677 #57)

Angular Momentum, Spherical Harmonics

The components of the orbital angular momentum operator $\vec{L} = (L_x, L_y, L_z)$ satisfy the following commutation relations.

$$[L_x, L_y] = i\hbar L_z$$
$$[L_y, L_z] = i\hbar L_x$$
$$[L_z, L_x] = i\hbar L_y$$

What is the value of the commutator $[L_x L_y, L_z]$?

A. $2i\hbar L_x L_y$
B. $i\hbar(L_x^2 + L_y^2)$
C. $-i\hbar(L_x^2 + L_y^2)$
D. $i\hbar(L_x^2 - L_y^2)$
E. $-i\hbar(L_x^2 - L_y^2)$

(GR0177 #43)

9. A spin-zero particle has angular wavefunction $\frac{1}{\sqrt{2}}(Y_3^2(\theta, \varphi) + Y_2^1(\theta, \varphi))$ where $Y_l^m(\theta, \varphi)$ are the normalized spherical harmonics. What is the expectation value of the total spin L^2?

A) $2\hbar^2$
B) $5\hbar^2/2$
C) $3\hbar^2$
D) $9\hbar^2$
E) $18\hbar^2$

A diatomic molecules is initially in the state $\Psi(\Theta, \Phi) = (5Y_1{}^1 + 3Y_5{}^1 + 2Y_5{}^{-1}) / (38)^{1/2}$ where $Y_l{}^m$ is a spherical harmonics. If measurements are made of the total angular momentum quantum number l and of azimuthal angular momentum quantum number m, what is the probability of obtaining the results $l = 5$?

A. 36/1444
B. 9/38
C. 13/38
D. $5/(38)^{1/2}$
E. 34/38

(GR9677 #33)

A system is know to be in the normalized state described by the wave function

$$\psi(\theta, \varphi) = \frac{1}{\sqrt{30}}(5Y_4^3 + Y_6^3 - 2Y_6^0)$$

Where $Y_l{}^m(\theta, \varphi)$ are the spherical harmonics. The probability of finding the system in a state with azimuthal orbital quantum number $m = 3$ is

A. 0
B. 1/15
C. 1/6
D. 1/3
E. 13/15

(GR9277 #28)

Which is the following is the orbital angular momentum eigenfuction $Y_l^m(\theta,\phi)$ in a state for which the operators L^2 and L_z have eigenvalues $6\hbar^2$ and $-\hbar$ respectively?

A. $Y_2^1(\theta,\phi)$

B. $Y_2^{-1}(\theta,\phi)$

C. $\frac{1}{\sqrt{2}}[Y_2^1(\theta,\phi)+Y_2^{-1}(\theta,\phi)]$

D. $Y_2^3(\theta,\phi)$

E. $Y_3^{-1}(\theta,\phi)$

(GR0177 #81)

64. Consider a single electron atom with orbital angular momentum $L = \sqrt{2}\hbar$. Which of the following gives the possible values of a measurement of L_z, the z-component of L ?

(A) 0

(B) 0, $\hbar$

(C) 0, $\hbar$, $2\hbar$

(D) $-\hbar$, 0, $\hbar$

(E) $-2\hbar$, $-\hbar$, 0, $\hbar$, $2\hbar$

95. Let $\hat{\mathbf{J}}$ be a quantum mechanical angular momentum operator. The commutator $\left[\hat{J}_x\hat{J}_y,\ \hat{J}_x\right]$ is equivalent to which of the following?

(A) 0

(B) $i\hbar\hat{J}_z$

(C) $i\hbar\hat{J}_z\hat{J}_x$

(D) $-i\hbar\hat{J}_x\hat{J}_z$

(E) $i\hbar\hat{J}_x\hat{J}_y$

Spin ½, Matrices

86. Which of the following are the eigenvalues of the Hermitian matrix $\begin{pmatrix} 2 & i \\ -i & 2 \end{pmatrix}$?

(A) 1, 0
(B) 1, 3
(C) 2, 2
(D) i, $-i$
(E) $1+i$, $1-i$

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

87. Consider the Pauli spin matrices σ_x, σ_y, and σ_z and the identity matrix I given above. The commutator $[\sigma_x, \sigma_y] \equiv \sigma_x\sigma_y - \sigma_y\sigma_x$ is equal to which of the following?

(A) I
(B) $2i\sigma_x$
(C) $2i\sigma_y$
(D) $2i\sigma_z$
(E) 0

The matrix $A = \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{vmatrix}$ has 3 eigenvalues λ_i defined by $Av_i = \lambda_i v_i$.
Which of the following statements is NOT true?

A. $\lambda_1 + \lambda_2 + \lambda_3 = 0$
B. λ_1, λ_2, and λ_3 are all the real numbers
C. $\lambda_1\lambda_2 = +1$ for some pair roots
D. $\lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_1 = 0$
E. $\lambda_i^3 = +1, i = 1, 2, 3$

(GR9277 #98)

88. A spin-$\frac{1}{2}$ particle is in a state described by the spinor

$$\chi = A\begin{pmatrix} 1+i \\ 2 \end{pmatrix},$$

where A is a normalization constant. The probability of finding the particle with spin projection $S_z = -\frac{1}{2}\hbar$ is

(A) $\frac{1}{6}$

(B) $\frac{1}{3}$

(C) $\frac{1}{2}$

(D) $\frac{2}{3}$

(E) 1

10. What is the expectation value of S_z for a spin ½ particle with the same spinor as given above?
   - A) $-\hbar/6$
   - B) $-\hbar/3$
   - C) $-\hbar/2$
   - D) $\hbar/2$
   - E) $\hbar/3$

11. A spin ½ particle is is initially measured to have $S_z = \hbar/2$. A subsequent measurement of S_x yields $-\hbar/2$. If a third measurement is made, this time of S_z again, what is the probability the measurement yields $\hbar/2$?
   - A) 0
   - B) ¼
   - C) ½
   - D) ¾
   - E) 1

12. A meson is a bound state of a quark and an antiquark, both with spin ½. Which of the following is a possible value of *total* angular momentum $l = 2$?
   - I. $j = 0$
   - II. $j = 1$

III. $j = 2$

IV. $j = 3$

A) I and II
B) I and III
C) II and IV
D) I, II, and III
E) II, III, and IV

13. A deuterium atom, consisting of a proton and neutron in the nucleus with a single orbital electron, is measured to have total angular momentum $j = 3/2$ and $m_j = 1/2$ in the ground state. Let the up and down states of the proton and neutron be given by $|\uparrow\}_P$, $|\downarrow\}_P$ and $|\uparrow\}_N$, $|\downarrow\}_N$ respectively. Assuming the nucleus has no orbital angular momentum, its spin state could be:

I. $\frac{1}{\sqrt{2}}(|\uparrow\}_P |\downarrow\}_N - |\downarrow\}_P |\uparrow\}_N)$

II. $|\uparrow\}_P |\uparrow\}_N$

III. $\frac{1}{\sqrt{2}}(|\uparrow\}_P |\downarrow\}_N + |\downarrow\}_P |\uparrow\}_N)$

IV. $|\downarrow\}_P |\downarrow\}_N$

A) I only
B) I and III
C) II and IV
D) II and III
E) III and IV

The state of a spin ½ particle can be represented using the eigenstate $|\uparrow\rangle$ and $|\downarrow\rangle$ of the S_z operator.

$$S_z\,|\uparrow\rangle = \frac{1}{2}\hbar\,|\uparrow\rangle$$
$$S_z\,|\downarrow\rangle = -\frac{1}{2}\hbar\,|\downarrow\rangle$$

Given the Pauli matrix $\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ which of the following is an eigenstate of S_x with eigenvalue $-\frac{1}{2}\hbar$?

A. $|\uparrow\rangle$
B. $\frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)$
C. $\frac{1}{\sqrt{2}}(|\uparrow\rangle - |\downarrow\rangle)$
D. $\frac{1}{\sqrt{2}}(|\uparrow\rangle + i\,|\downarrow\rangle)$
E. $\frac{1}{\sqrt{2}}(|\uparrow\rangle - i\,|\downarrow\rangle)$

(GR0177 #83)

14. In the ground state of helium, which of the following gives the total spin quantum numbers of the two electrons?
   A) $s=0$, $m_s=0$
   B) $s=1$, $m_s=1$
   C) $s=1$, $m_s=0$
   D) $s=1$, $m_s= -1$
   E) $s=½$, $m_s=½$

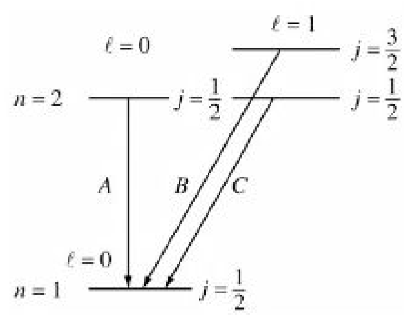
15. Two spin ½ electrons are placed in a one dimensional harmonic oscillator potential of angular frequency ω. If a measurement of S_z of the system returns $\hbar$, which of the following is the smallest possible energy of the system?
   A) $\hbar\omega/2$
   B) $\hbar\omega$
   C) $3\hbar\omega/2$
   D) $2\hbar\omega$
   E) $5\hbar\omega/2$

Selection Rules, Atomic Physics

Which of the following is NOT compatible with the selection rule that controls electric dipole emission of photons by excited states of atoms?

A. Δn may have any negative integral value
B. $\Delta l = \pm 1$
C. $\Delta m_l = 0, \pm 1$
D. $\Delta s = \pm 1$
E. $\Delta j = \pm 1$

(GR8677 #92)



An energy-level diagram of the $n = 1$ and $n = 2$ levels of atomic hydrogen (including the effect of spin-orbit coupling and relativity) is shown in the figure. Three transitions are labeled A, B, and C. Which of the transitions will be possible electric-dipole transition?

A. B only
B. C only
C. A and C only
D. B and C only
E. A, B, and C

(GR0177 #84)

A transition in which one photon is radiated by the electron in a hydrogen atom when the electron's wave function changes from ψ_1 to ψ_2 is forbidden if ψ_1 and ψ_2

A. have opposite parity
B. are orthogonal to each other
C. are zero at the center of the atomic nucleus
D. are both spherically symmetrical
E. are associated with different angular momenta

(GR8677 #48)

16. In the Bohr model of the hydrogen atom, let r_1 and r_2 be the radii of the n=1 and n=2 orbital shells. What is r_2 / r_1?
   A) ½
   B) 1/sqrt2
   C) 1
   D) 2
   E) 4

17. The bohr model is inconsistent with the modern picture of quantum mechanics because it predicts which of the following?
   A) The electron will not lose energy as it orbits the nucleus
   B) The electron is confined to distinct energy shells
   C) Angular momentum of the atom is quantized
   D) The ground state has nonzero angular momentum
   E) The energy levels go as $1/n^2$ where n is the principal quantum number

18. An atom has electron configuration $1s^2 2s^2 2p^3$. A measurement of the total orbital angular momentum of the outermost electron in the ground state could return which of the following?
   A) ħ
   B) ħ/sqrt2
   C) 2ħ
   D) ħ/sqrt6
   E) 3ħ

19. An atom with the electron configuration $1s^2 2s^3$ is forbidden by which of the following?
   A) Conservation of angular momentum
   B) Hund's Rule
   C) The Pauli exclusion principle

D) The uncertainty principle
E) None of the above

20. Which of the following is the most likely decay chain of the 3s state of hydrogen?
A) $3s \rightarrow 1s$
B) $3s \rightarrow 2s \rightarrow 1s$
C) $3s \rightarrow 2p \rightarrow 1s$
D) $3s \rightarrow 2p \rightarrow 2s \rightarrow 1s$
E) The 3s state is stable

The ground state configuration of a neutral sodium atom (Z = 11) is

A. $1s^2\ 2s^2\ 2p^5\ 3s^2$
B. $1s^2\ 2s^3\ 2p^6$
C. $1s^2\ 2s^2\ 2p^6\ 3s$
D. $1s^2\ 2s^2\ 2p^6\ 3p$
E. $1s^2\ 2s^2\ 2p^5$

(GR9277 #58)

The ground state electron configuration for phosphorus, which has 15 electrons, is

A. $1s^2\ 2s^2\ 2p^6\ 3s^1\ 3p^4$
B. $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^3$
C. $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3d^3$
D. $1s^2\ 2s^2\ 2p^6\ 3s^1\ 3d^4$
E. $1s^2\ 2s^2\ 2p^6\ 3p^2 3d^3$

(GR0177 #17)

The configuration of the potassium atom in its ground state is $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 4s^1$. Which of the following statement about potassium is true:

A. Its n = 3 shell is completely filled
B. Its 4s subshell is completely filled
C. Its least tightly bound electron has l = 4
D. Its atomic number is 17
E. Its electron charge distribution is spherically symmetrical

(GR8677 #30)

The ground state of the helium atom is a spin

A. singlet
B. doublet
C. triplet
D. quartet
E. quintuplet

(GR9277 #59)

The configuration of three electrons is 1s2p3p has which of the following as the value of its maximum possible total angular momentum quantum number?

A. 7/2
B. 3
C. 5/2
D. 2
E. 3/2

(GR9277 #76)

Sodium has eleven electrons and the sequence in which energy levels fill in atom is 1s, 2s, 2p, 3s, 3p, 4s, 3d, etc. What is the ground state of sodium in the usual notation $^{2s+1}L_j$?

A. $^1S_0$

B. $^2S_{1/2}$

C. $^1P_0$

D. $^2P_{1/2}$

E. $^3P_{1/2}$

(GR8677 #84)

A 3p electron is found in the $^3P_{3/2}$ energy level of a hydrogen atom. Which of the following is true about the electron in this state?

A. It is allowed to make an electric dipole transition to the $^2S_{1/2}$ level

B. It is allowed to make an electric dipole transition to the $^2P_{1/2}$ level

C. It has quantum numbers $l = 3, j = 3/2, s = 1/2$

D. It has quantum numbers $n = 3, j = l, s = 3/2$

E. It has exactly the same energy as it would in the $^3D_{3/2}$ level

(GR9677 #41)

In a ^3S state of the helium atom, the possible values of the total electronic angular momentum quantum number are

A. 0 only

B. 1 only

C. 0 and 1 only

D. 0, 1/2, and 1

E. 0, 1, and 2

(GR9277 #31)

Two ions 1 and 2, at fixed separation, with spin angular momentum operators $\mathbf{S}_1$ and $\mathbf{S}_2$, have the interaction Hamiltonian $H = -J\,\mathbf{S}_1 \cdot \mathbf{S}_2$, where $J > 0$. The values of $\mathbf{S}_1{}^2$ and $\mathbf{S}_2{}^2$ are fixed at $S_1(S_1 + 1)$ and $S_2(S_2 + 1)$, respectively. Which of the following is the energy of the ground state of the system?

A. 0

B. $-JS_1S_2$

C. $-J[S_1(S_1 + 1) - S_2(S_2 + 1)]$

D. $-(J/2)[(S_1 + S_2)(S_1 + S_2 + 1) - S_1(S_1+1) - S_2(S_2+1)]$

E. $-(J/2)[(S_1(S_1 + 1) + S_2(S_2 + 1))/(S_1 + S_2)(S_1 + S_2 + 1)]$

(GR9677 #77)

Let $|\alpha\rangle$ represent the state of an electron with spin up and $|\beta\rangle$ the state of an electron with spin down. Valid spin eigenfunctions for a triplet state (^3S) of a two-electron atom include which of the following?

I. $|\alpha\rangle_1|\alpha\rangle_2$

II. $\frac{1}{\sqrt{2}}(|\alpha\rangle_1|\beta\rangle_2 - |\alpha\rangle_2|\beta\rangle_1)$

III. $\frac{1}{\sqrt{2}}(|\alpha\rangle_1|\beta\rangle_2 + |\alpha\rangle_2|\beta\rangle_1)$

A. I only

B. II only

C. III only

D. I and III

E. II and III

GR0177 #82)

A system containing two identical particles is described by a wave function of the form

$$\psi = \frac{1}{\sqrt{2}}\left[\psi_\alpha(x_1)\psi_\beta(x_2) + \psi_\beta(x_1)\psi_\alpha(x_2)\right]$$

Where x_1 and x_2 represent the spatial coordinates of the particles and α and β represent all the quantum numbers, including spin, of the states that they occupy. The particles might be

A. Electrons
B. Positrons
C. Protons
D. Neutrons
E. Deuterons

(GR8677 #89)

The electronic energy levels of atoms of a certain gas are given by $E_n = E_1 n^2$, where $n = 1, 2, 3, \ldots$ Assume that transitions are allowed between all levels. If one wanted to construct a laser from this gas by pumping the $n = 1 \rightarrow n = 3$ transitions, which energy level or levels would have to be metastable?

A. $n = 1$ only
B. $n = 2$ only
C. $n = 1$ and $n = 3$ only
D. $n = 1$, $n = 2$, and $n = 3$
E. None

(GR9677 #99)

Perturbations:

21. A harmonic oscillator Hamiltonian of angular frequency ω is perturbed by a potential of the form $V = K(a_- - a_+)^2$ where K is constant and $a_-|n\} = \sqrt{n}\,|n-1\}$, $a_+|n\} = \sqrt{n+1}|n+1\}$. What is the first order energy shift of the state with unperturbed energy $3\hbar\omega/2$?

A) $-3K$
B) $-K$
C) 0
D) K
E) $2K$

In perturbation theory, what is the first order correction to the energy of a hydrogen atom (Bohr radius a_0) in its ground state due to the presence of a static electric field E?

A. 0
B. eEa_0
C. $3eEa_0$
D. $\dfrac{8e^2Ea_0^3}{3}$
E. $\dfrac{8e^2E^2a_0^3}{3}$

(GR9277 #99)

22. A particle in the ground state of an infinite square well between x=0 and x=a is subject to the perturbation $\Delta H = Vx$ where V is constant. What is the first order shift in the energy?

A) $\frac{V\sqrt{2a^3}}{\pi}$
B) $-Va/2$
C) 0
D) $Va/2$
E) $2Va/\pi$

23. A particle of mass m is subject to a square well potential of size a, and and is found to have energy E. The well is now expanded slowly to size 2a. What is E^1/E where E^1 is the expectation value of the energy of the particle after the expansion has finished?

A) 0
B) 1
C) 1/sqrt2
D) 1/2
E) 1/4

A particle of mass M is in an infinitely deep square well potential V where

$$V = 0 \quad \text{for} \quad -a \leq x \leq a, \qquad \text{and}$$
$$V = \infty \quad \text{for} \quad x < -a, a < x.$$

A very small perturbing potential V' is superimposed on V such that

$$V' = \epsilon\left(\frac{a}{2} - |x|\right) \quad \text{for} \quad -\frac{a}{2} \leq x \leq \frac{a}{2} \quad \text{, and}$$

$$V' = 0 \quad \text{for} \quad x < -\frac{a}{2}, \quad \frac{a}{2} < x.$$

If $\psi_0, \psi_1, \psi_2, \psi_3, \cdots$ are the energy eigenfunctions for a particle in the infinitely deep square well potential, with ψ_0 being the ground state, which of the following statements is correct about the eigenfunction ψ'_0 of a particle in the perturbed potential $V + V'$?

A. $\psi'_0 = a_{00}\psi_0, \quad a_{00} \neq 0$

B. $\psi'_0 = \sum_{n=0}^{\infty} a_{0n}\psi_n$ with $a_{0n} = 0$ for all odd values of n.

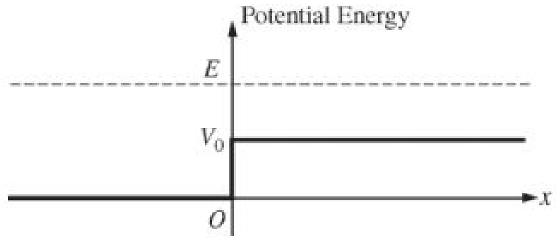
C. $\psi'_0 = \sum_{n=0}^{\infty} a_{0n}\psi_n$ with $a_{0n} = 0$ for all even values of n.

D. $\psi'_0 = \sum_{n=0}^{\infty} a_{0n}\psi_n$ with $a_{0n} = 0$ for all values of n.

E. None of the above

(GR8677 #96)

Wavefunctions and Free Particles:



89. An electron with total energy E in the region $x < 0$ is moving in the $+x$-direction. It encounters a step potential at $x = 0$. The wave function for $x \le 0$ is given by

$$\psi = Ae^{ik_1x} + Be^{-ik_1x}, \text{ where } k_1 = \sqrt{\frac{2mE}{\hbar^2}};$$

and the wave function for $x > 0$ is given by

$$\psi = Ce^{ik_2x}, \text{ where } k_2 = \sqrt{\frac{2m(E - V_0)}{\hbar^2}}.$$

Which of the following gives the reflection coefficient for the system?

(A) $R = 0$

(B) $R = 1$

(C) $R = \frac{k_2}{k_1}$

(D) $R = \left(\frac{k_1 - k_2}{k_1 + k_2}\right)^2$

(E) $R = \frac{4k_1k_2}{(k_1 + k_2)^2}$

A free particle with initial kinetic energy E and de Broglie wavelength λ enters a region in which it has potential energy V. What is the particle's new de Broglie wavelength?

A. $\lambda(1 + E/V)$
B. $\lambda(1 - V/E)$
C. $\lambda(1 - E/V)^{-1}$
D. $\lambda(1 + V/E)^{1/2}$
E. $\lambda(1 - V/E)^{-1/2}$

(GR0177 #46)

Which of the following functions could represent the radial wave function for an electron in an atom? (r is the distance of the electron from the nucleus; A and b are constants.)

I. Ae^{-br}

II. $A \sin(br)$

III. A/r

A. I only
B. II only
C. I and II only
D. I and III only
E. I, II, and III

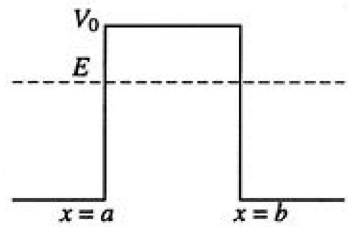
(GR0177 #30)

Quantum Mechanics - Schrodinger Equation

The solution to the Schrödinger equation for a particle bound in a one-dimensional, infinitely deep potential well, indexed by quantum number n, indicates that in the middle of the well the probability density vanishes for

A. The ground state ($n = 1$) only
B. States of even n ($n = 2, 4, \ldots$)
C. States of odd n ($n = 1, 3, \ldots$)
D. All states ($n = 1, 2, 3, \ldots$)
E. All states except the ground state

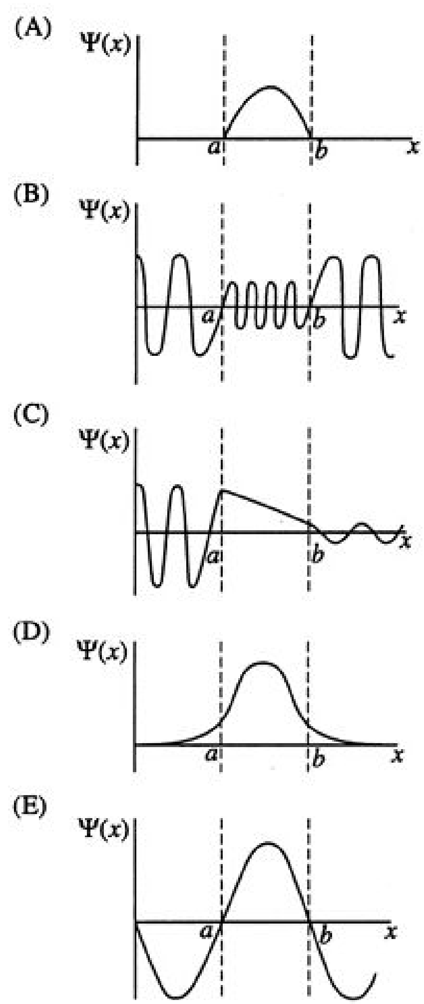
(GR9677 #51)



Consider a potential of the form

$V(x) = 0, x \leq a$
$V(x) = V_0, a < x < b$
$V(x) = 0, x \geq b$

As shown in the figure above. Which of the following wave functions is possible for a particle incident from the left with energy $E < V_0$.



(GR9677 #18)

Quantum Mechanics - Gaussian Wave Packet

A Gaussian wave packet travels through free space. Which of the following statement about the wave packet are correct for all such wave packets?

I. The average momentum of the wave packet is zero
II. The width of the wave packet increases with time, as $t \to \infty$.
III. The amplitude of the wave packet remains constant with time.
IV. The narrower the wave packet is in momentum space, the wider it is in coordinate space.

A. I and III only
B. II and IV only
C. I, II, and IV only
D. II,III, and IV only
E. I, II,III, and IV only

(GR9677 #76)

The wave function of a particle is $e^{i(kx-\omega t)}$ where x is distance, t is time, and k and are ω positive real numbers. The x–component of the momentum of the particle is

A. 0
B. $\hbar\omega$
C. $\hbar k$
D. $\hbar\omega/c$
E. $\hbar k/\omega$

(GR9277 #01)

If a freely moving electron is localized in space to within Δx_0 of x_0, its wave function can be described by a wave packet

$$\psi(x,t) = \int_{-\infty}^{\infty} e^{i(kx-\omega t)} f(k)dk$$

where $f(k)$ is peaked around a central value k_0. Which of the following is most nearly the width of the peak in k?

A. $\Delta k = \dfrac{1}{x_0}$

B. $\Delta k = \dfrac{1}{\Delta x_0}$

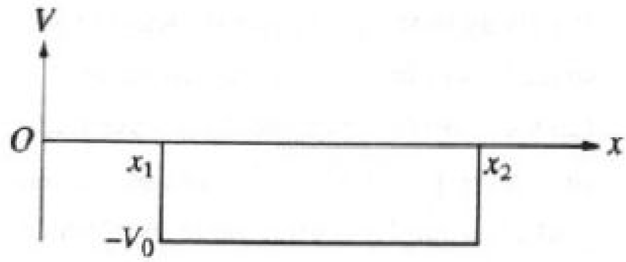
C. $\Delta k = \dfrac{\Delta x_0}{x_0^2}$

D. $\Delta k = \left(\dfrac{\Delta x_0}{x_0}\right) k_0$

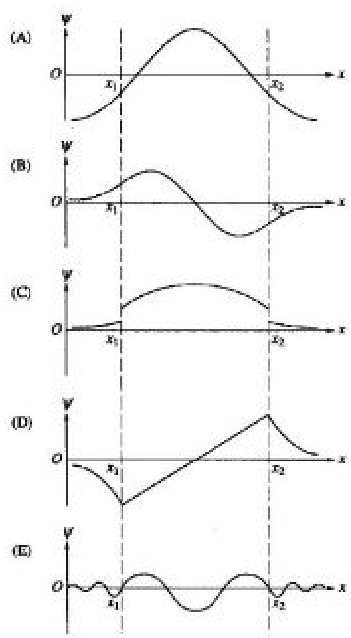
E. $\Delta k = \sqrt{k_0^2 + \left(\dfrac{1}{x_0}\right)^2}$

(GR9277 #27)

Quantum Mechanics, Particle in a box



An attractive, one-dimensional square well has depth V_0 as shown above. Which of the following best shows a possible wave function for a bound state?



(GR9277 #29)

A particle of mass m is confined to an infinitely deep square-well potential:

$V(x) = \infty, x \leq 0, x \geq a$
$V(x) = 0, 0 < x < a$

The normalized eigenfunction, labeled by the quantum number n, are

$$\psi_n = \sqrt{\frac{2}{a}} \sin \frac{n\pi x}{a}$$

For any state n, the expectation value of the momentum of the particle is

A. 0

B. $\frac{\hbar n\pi}{a}$

C. $\frac{2\hbar n\pi}{a}$

D. $\frac{\hbar n\pi}{a}(\cos n\pi - 1)$

E. $\frac{-i\hbar n\pi}{a}(\cos n\pi - 1)$

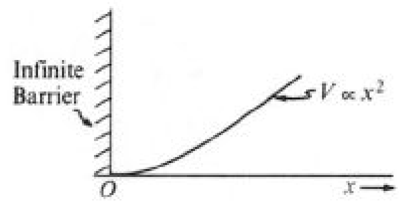
(GR9277 #51)

A measurement of energy E will always satisfy which of the following relationships?

A. $E \leq \pi^2\hbar^2/8ma^2$
B. $E \geq \pi^2\hbar^2/2ma^2$
C. $E = \pi^2\hbar^2/8ma^2$
D. $E = n^2\pi^2\hbar^2/8ma^2$
E. $E = \pi^2\hbar^2/2ma^2$

(GR9277 #53)

Quantum Mechanics - Harmonic Oscillator



The energy levels for the one-dimensional harmonic oscillator are $h\nu(n + ½)$, $n = 0,1,2,\cdots$ How will the energy levels for the potential shown in the graph above differ from those for the harmonic oscillator?

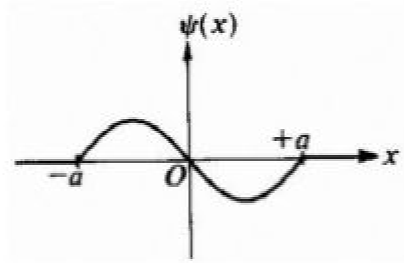
A. The term ½ will be changed to ³⁄₂
B. The energy of each level will be doubled.
C. The energy of each level will be halved.
D. Only those for even values of n will be present.
E. Only those for odd values of n will be present.

(GR9277 #89)

The wave function $\psi(x) = A \exp(-b^2x^2/2)$, where A and b are real constants, is a normalized eigenfunction of the Schrodinger equation for a particle of mass M and energy E in a one dimensional potential $V(x)$ such that $V(x) = 0$ at $x = 0$. Which of the following is correct?

A. $V = \hbar^2b^4/2M$
B. $V = \hbar^2b^4x^2/2M$
C. $V = \hbar^2b^6x^4/2M$
D. $E = \hbar^2b^2(1 - b^2x^2)$
E. $E = \hbar^2b^4/2M$

(GR8677 #18)



The figure above shows one of the possible energy eigenfunctions $\psi(x)$ for a particle bouncing freely back and forth along the x-axis between impenetrable walls located at $x = -a$ and $x = +a$. The potential energy equals zero for $|x| > a$. If the energy of the particle is 2 electron volts when it is in the quantum state associated with this eigenfunction, what is its energy when it is in the quantum state of lowest possible energy?

A. 0 eV
B. $1/\sqrt{2}$ eV
C. 1/2 eV
D. 1 eV
E. 2 eV

(GR8677 #90)