Lecture 1: Introduction and Preliminaries
AE4610: Dynamics and Control Laboratory
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1 Objective of this Lecture

- Discuss the syllabus.
- Control of physical world.
- Brief history.
- Important people.
- Summary of basis properties.

2 Control of Dynamical Systems

2.1 Control of Physical World

A control system interacts with the physical world in a feedback loop by measuring the environment via sensors and influencing it via actuators. E.g. a cruise controller is constantly monitoring the speed of the car and adjusts the throttle force so that the speed stays close to the desired cruising speed. The design of the controllers for the physical world requires modeling the dynamics of the physical quantities: to adjust the throttle force, a cruise controller needs a model of how the speed of the car changes with time as a function the throttle force. See Figure 1 and Figure 2.

A control system is an interconnection of components forming a system configuration that will provide a desired output response. The following components are:

- **System:** Aircraft, chemical plant, economic system, internet, biology, etc.

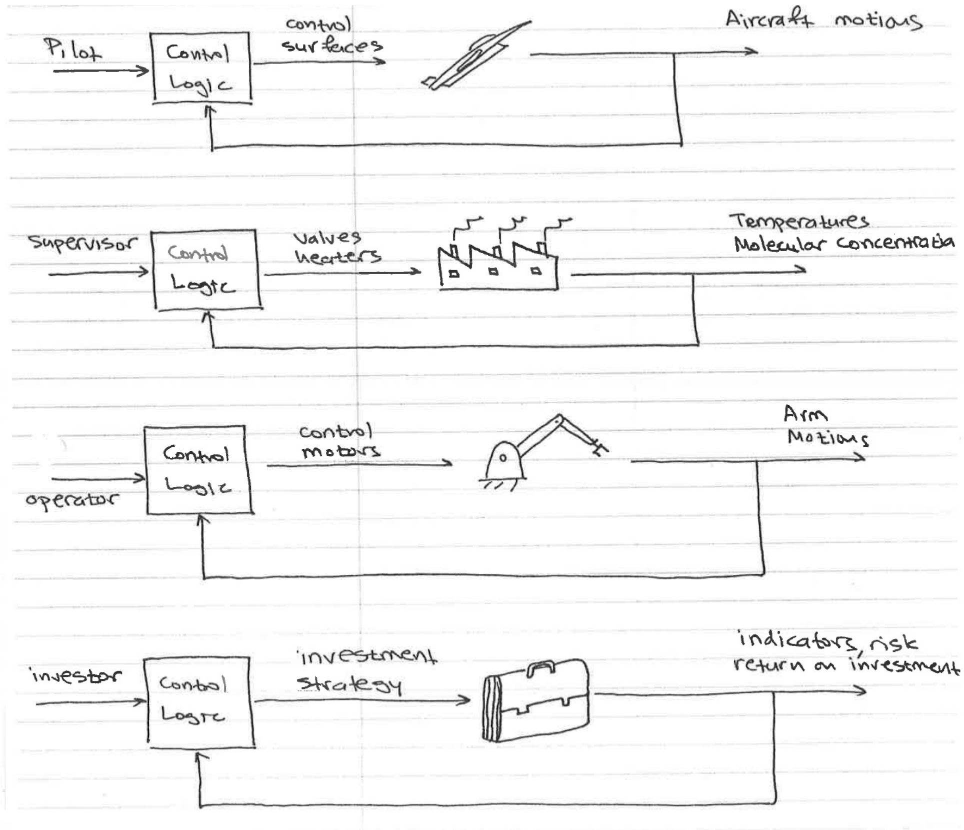

Figure 1: Examples of feedback.

- **Forces:** A set of variables (inputs) which can be manipulated by us.
- **Task:** A set of variables which we wish to exhibit to desirable behavior.
- **Challenges:** Uncertainty, noise, nonlinearities, limited control authority, limited information.
- **System:** We need feedback to stabilize unstable plants and beat uncertainties (signal and modeling).

The theory of **dynamical control systems** is a well-developed discipline with a rich set of mathematical tools for design and analysis, and a basic understanding of these principles is valuable to designers. The traditional control theory focuses on continuous-time systems. Controller consists of discrete software comprising concurrent components operating in multiple possible modes of operation, interacting with the continuously evolving physical environment. Such systems, are called hybrid systems.

- **Analysis:** Given a controller, determine in an intelligent way whether or not a given set of specifications is achieved for the closed-loop system.

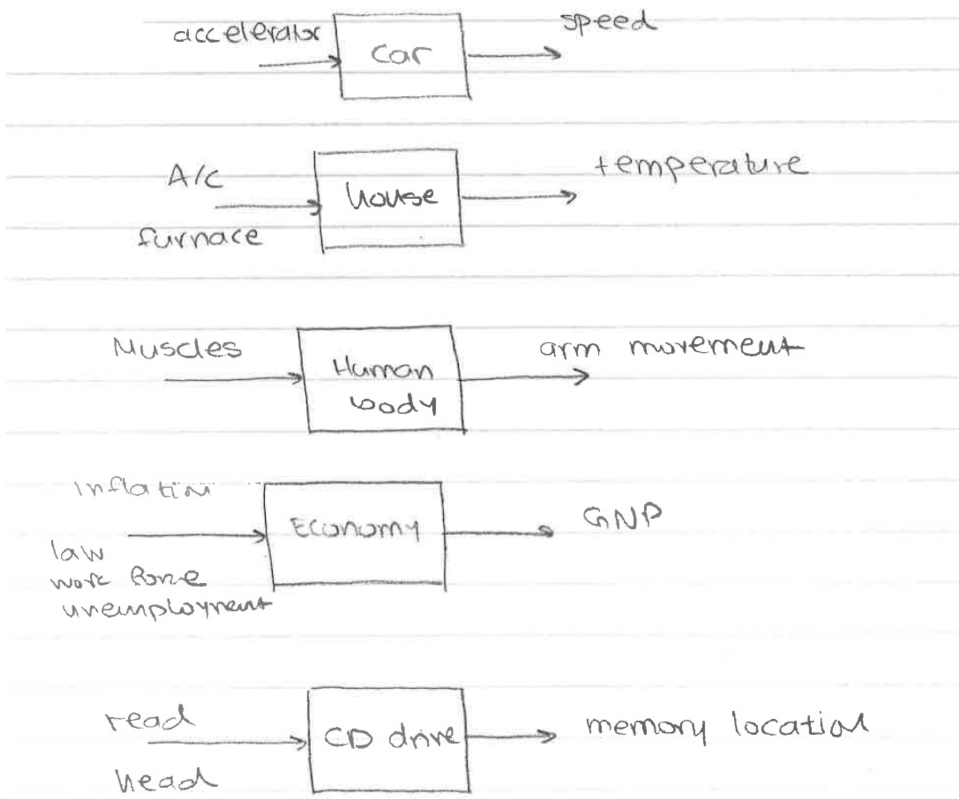

Figure 2: Open-Loop.

- **Synthesis:** Given a set of specifications, find a controller that achieves the desired performance specifications.

3 Dynamical Systems

Controllers such as thermostat for regulating the temperature in a room or a cruise controller for tracking and maintaining the speed of a car interact with the world via sensors and actuators. Most of the controllers up to 19th century were for temperature and speed regulation. The improvement in ships, weapons and electricity gave rise to other and more sophisticated control systems. The modern or state space approach to control was ultimately derived from original work by Poincaré [1] and Lyapunov [2] at the end of the 19th century. The cold war requirements of control engineering centered on aerospace applications. In a conference in Moscow in 1960, Kalman presented his now well known paper on the general theory of control systems which identified the duality between multivariable feedback control and multivariable feedback filtering that was seminal for the development of optimal control. The late 1950s and early 1960s saw the publication of a number of other important works on dynamic programming and optimal control, mostly by Bellman (working in the mathematics department of the RAND Corporation, studied the problem of determining the allocation of missiles to targets so as to inflict the maximum damage. This work led him to formulate the "principle of optimality" and to dynamic programming.), Kalman [3] and Pontryagin [4]. The principal difficulty with dynamic programming is the curse of dimensionality, and even though we now have computing power far beyond anything available to Bellman [5] and Dreyfus [6] we still need to use approximating techniques to handle that. Dynamical problems that involve minimizing or maximizing some performance index have "an obvious and strong analogy with the classical variational formulations of analytical mechanics given by Lagrange and Hamilton." The generalization of Hamilton's approach to geometric optics by Pontryagin in 1956, laid the foundations of optimal control theory. This led to extensive and deep studies of mathematical problems of automatic control. Kalman and Bucy [7] showed the importance of feedback in filtering theory and the duality that existed between the multivariable control problem and multivariable feedback filtering. The Kalman-Bucy filter showed its importance since the controller has a dynamic complexity equivalent to that of the plant being controlled. As a consequence there was a revival of interest in the frequency-response approach. H. Rosenbrock [8] in 1966 developed a systematic attack on the problems of developing frequency response methods for multivariable systems.

The digital computer (1950-1960) played a huge role in the development of modern control theory. This was mostly due to the replacement of electronic tubes by semiconductors such as diodes, transistors and thyristors, and the replacement of mechanical and electrical components by solid-state and micro-electronic devices. But the great change for controls, with the introduction of digital computers was that ultimately the approximate methods of frequency response or root locus design, developed explicitly to avoid computation, could be replaced by techniques in which accurate computation played a vital role. The digital computer also helped to collect data on-line for optimization and supervisory control, in direct digital control and finally for on-line control in the 1970s.

Control theory made significant strides in the next 100 years. New mathematical techniques made it possible to control, more accurately, significantly more complex dynamical systems than the original flyball governor. These techniques include developments in optimal control in the 1950s and 1960s, followed by progress in stochastic, robust, adaptive and optimal control methods in the 1970s and 1980s. Applications of control methodology have helped make possible space travel and communication satellites, safer and more efficient aircraft, cleaner auto engines, cleaner and more efficient chemical processes, to mention but a few.

Before it emerged as a unique discipline, control engineering was practiced as a part of mechanical engineering and control theory was studied as a part of electrical engineering, since electrical circuits can often be easily described using control theory techniques. In the very first control relationships, a current output was represented with a voltage control input. However, not having proper technology to implement electrical control systems, designers left with the option of less efficient and slow responding mechanical systems. A very effective mechanical controller that is still widely used in some hydro plants is the governor. Later on, previous to modern power electronics, process control systems for industrial applications were devised by mechanical engineers using pneumatic and hydraulic control devices, many of which are still in use today.

The important years of control theory are,

- 1769: James Watt's centrifugal governor for the speed control of a steam engine.
- 1800: Eli Whitney invented the cotton gin. This was one of the key inventions of the Industrial Revolution and shaped the economy of the Antebellum South. Whitney's invention made upland short cotton into a profitable crop, which strengthened the economic foundation of slavery in the United States.
- 1869: James Maxwell developed the theory to control James Watt's governor.
- 1913: Henry Ford developed and manufactured the first automobile that many middle class Americans could afford.
- 1920s: Minorsky worked on automatic controllers for steering ships.
- 1938: H. W. Bode developed asymptotic phase and magnitude plots, now known as Bode plots, which displayed the frequency response of systems clearly
- 1930s: Nyquist developed a method for analyzing the stability of controlled systems. Nyquist did important work on thermal noise, the stability of feedback amplifiers, telegraphy, facsimile, television, and other important communications problems
- 1940s: Frequency response methods made it possible to design linear closed-loop control systems .
- 1950s: The root-locus method due to Evans was fully developed.
- 1960s: State space methods, optimal control, adaptive control and
- 1980s: Learning controls are begun to be investigated and developed.
- Present: and on-going research fields. Recent application of modern control theory includes such non-engineering systems such as biological, biomedical, economic and socio-economic systems,...

3.1 Famous People in Control

The birth of the mathematical control theory has been mostly the result of the work of the following people,

- G. B. Airy (1840): the first one to discuss instability in a feedback control system the first to analyze such a system using differential equations
- J. C. Maxwell (1868): the first systematic study of the stability of feedback control
- E. J. Routh (1877): deriving stability criterion for linear systems
- A. M. Lyapunov (1892): deriving stability criterion that can be applied to both linear and nonlinear differential equations results not introduced in control literature until about 1958

3.2 Basic Elements of Control Theory

3.2.1 Open-Loop Control Systems, Closed-Loop Control Systems and Feedback

Feedback control [9, 10, 11, 12, 13, 14] is the basic mechanism by which systems, whether mechanical, electrical, or biological, maintain their equilibrium or homeostasis. In the higher life forms, the conditions under which life can continue are quite narrow. Feedback control may be defined as the use of difference signals, determined by comparing the actual values of system variables to their desired values (because the system is always making small corrections to get to the right place), as a means of controlling a system. An everyday example of a feedback control system is an automobile speed control, which uses the difference between the actual and the desired speed to vary the fuel flow rate. Since the system output is used to regulate its input, such a device is said to be a *closed-loop control system* (see Figure 4). *Open-loop control system* (see Figure 3) on the other side utilize a controller or control actuator to obtain the desired response but without knowledge of the output.

Feedback is a universal concept that appears in natural systems, interaction of species, and biological systems including the basic cell and muscle control systems in the body. Charles Darwin [15] showed that feedback over long time periods is responsible for natural selection of species. Volterra [16] showed that feedback is responsible for the interactions of predator/prey species in closed ecosystems. Adam Smith [17] showed that feedback occurs in the balance of the international economies of nations. Feedback in human engineered systems has many benefits. Using feedback, one can move the closed-loop poles so that the system performance improves. Performance can be improved in terms of fast transient response, percent overshoot, and oscillation frequency. One can obtain improved performance in the presence of disturbances, such as wind gusts in aircraft control systems. One can also select a suitable compensator to obtain good steady-state tracking by studying the steady-state error.

When the true plant parameters are unknown, the controller parameters are either estimated directly (direct schemes) or computed by solving the same design equations using plant parameter estimates (indirect schemes). Figures 5-7 show three of the existing controller topologies. We will mostly add a communication channel between the sensor and the controller, see Figure 8. This channel can add some problems, e.g. the transmitted data may be delayed, lost or otherwise corrupted. In research papers, instead of Figure 8, you shall see Figure 9. The latter, designs both controller and information transmission blocks and transmit maximal control relevant information with respect to constraints due to the channel.

The use of feedback in control systems has a lot of advantages, including increased accuracy (e.g. gets to the desired final position more accurately because small errors will get corrected on subsequent measurement cycles), less sensitivity to nonlinearities (e.g., hysteresis) and reduced sensitivity to noise in the input signal. But if the designer selects a high control gain then it can cause instability, it can couple noise from sensors into the dynamics of a system and finally it can increase the overall complexity of a system.

3.2.2 Continuous-Time and Discrete-Time Control Systems

The dynamical systems can be classified as discrete time and continuous time.

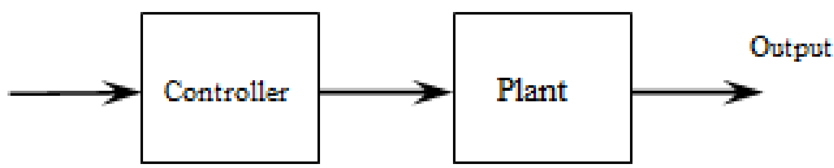

Figure 3: Open-loop control system.

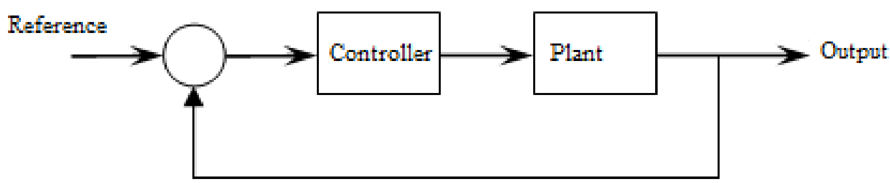

Figure 4: Closed-loop control system.

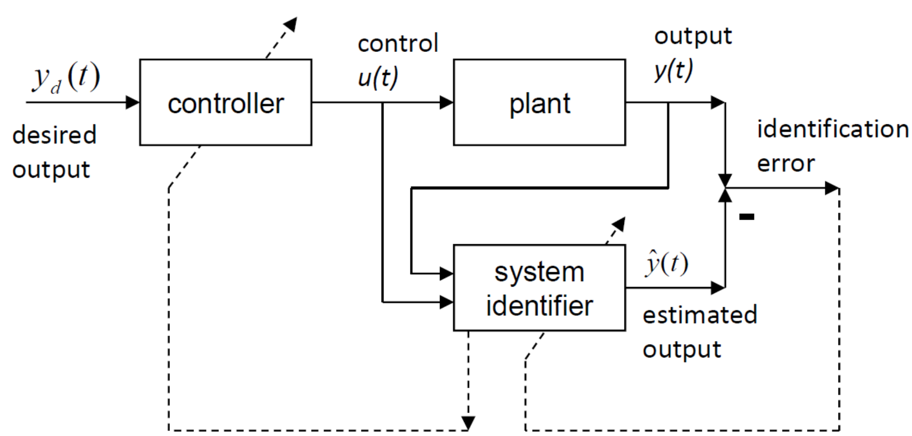

Figure 5: Indirect control scheme.

For a discrete time nonlinear dynamical system [18], we denote time by $k \in \mathbb{N}_{>0}$, and the system is specified by the equations

$$x(k+1) = f(x(k)) + g(x(k))u(k), \; x(0) = x_0$$

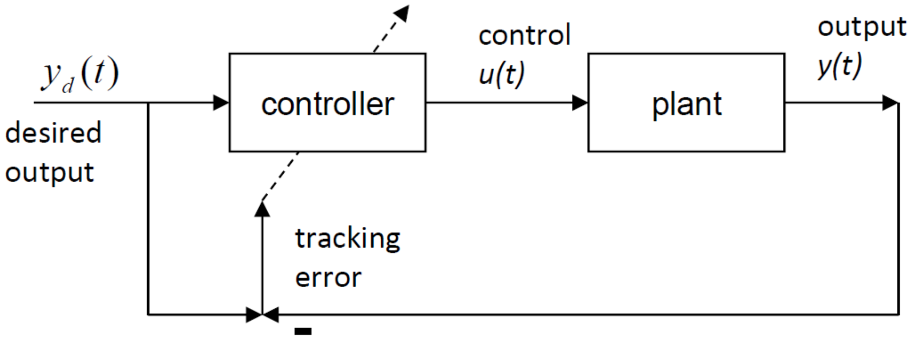

Figure 6: Direct control scheme.

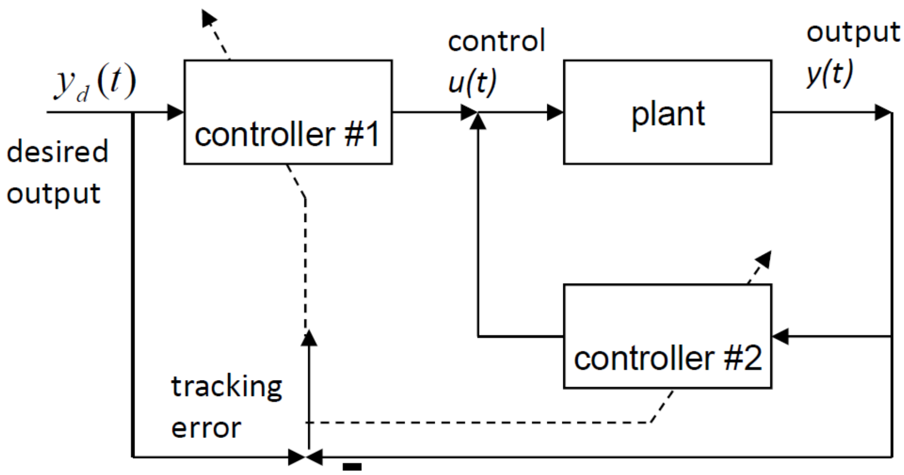

Figure 7: Feedback/feedforward control scheme.

where $x \in \mathbb{R}^n$ is the state of the system, $u \in \mathbb{R}^m$ is the control input, $f : \mathbb{R}^n \to \mathbb{R}^n$, and $g : \mathbb{R}^n \to \mathbb{R}^{n \times m}$ are continuous functions of x.

For a continuous time nonlinear dynamical system, we denote time by $k \in \mathbb{R}_{>0}$, and the system is specified by the equations

$$\dot{x} = f(x(t)) + g(x(t))u(t), \; x(0) = x_0$$

where $x \in \mathbb{R}^n$ is the state of the system, $u \in \mathbb{R}^m$ is the control input, $f : \mathbb{R}^n \to \mathbb{R}^n$, and $g : \mathbb{R}^n \to \mathbb{R}^{n \times m}$ are continuous functions of x.

Linearization If the system is nonlinear of the general form $\dot{x} = f(x, u), \; y = h(x, u)$, we can approximate it with a linearized system by computing the Jacobian matrices, $A(x, u) = \frac{\partial f}{\partial x}, \; B(x, u) = \frac{\partial f}{\partial u}, \; C(x, u) = \frac{\partial h}{\partial x}, \; D(x, u) = \frac{\partial h}{\partial u}$ which are evaluated at a nominal set point (x, u) to obtain $A, \; B, \; C, \; D$ yielding a linear time-invariant state description which is approximately

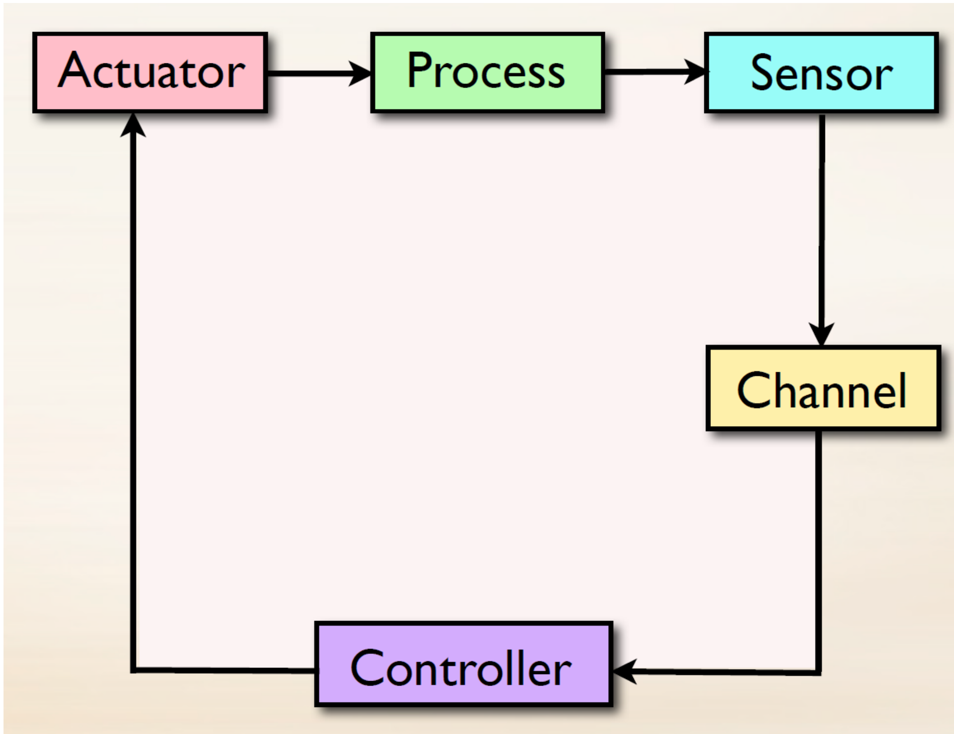

Figure 8: Communication channels in control loops.

valid for small excursions about the nominal point. Note that The Jacobian resulting from differentiation of a p-vector function h with respect to an m-vector variable $u = \begin{bmatrix} u_1 & u_2 & \cdots & u_m \end{bmatrix}^{\mathrm{T}}$ is a $p \times m$ matrix.

Note that in linear systems [19] we make the assignment $A := f(\cdot)$ and $B := g(\cdot)$ where A, B are matrices of appropriate dimensions.

A control systems engineer needs to pick appropriately the control input u such that the system is operating under desired properties in terms of stability and performance.

3.2.3 Time-Invariant/Time-Varying Linear Control Systems and the Notions of Controllability and Observability

We define as linear time varying linear systems, the systems that evolve in continuous time $t \in \mathbb{R}_{>0}$ through linear state space equations as,

$$\begin{aligned} \dot{x}(t) &= A(t)x(t) + B(t)u(t), \\ y(t) &= C(t)x(t) + D(t)u(t), \end{aligned}$$

where $x(t) \in \mathbb{R}^n$ denotes the system state, $u(t) \in \mathbb{R}^m$ denotes the system inputs, $y(t) \in \mathbb{R}^p$ denotes the system outputs, $A(t) \in \mathbb{R}^{n \times n}$, $B(t) \in \mathbb{R}^{n \times m}$, $C(t) \in \mathbb{R}^{p \times n}$ and $D(t) \in \mathbb{R}^{p \times m}$ are matrices of appropriate dimensions.

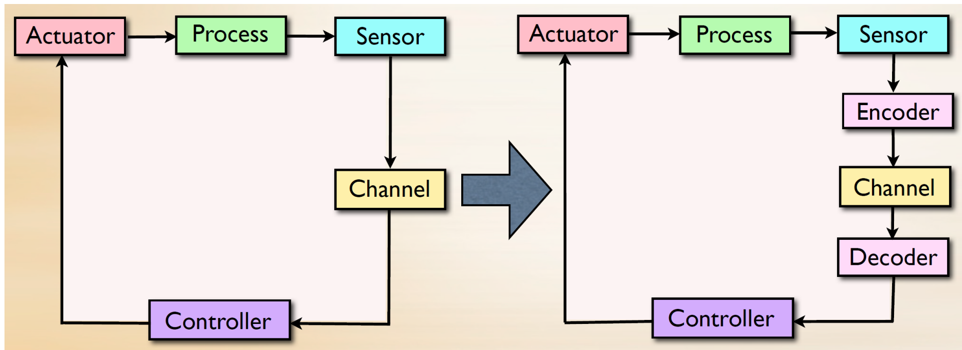

Figure 9: From single to two blocks.

Time varying linear systems are useful in many application areas. They frequently arise as models of mechanical or electrical systems whose parameters (for example, the stiffness of a spring or the inductance of a coil) change in time. As we will see, time varying linear systems also arise when one linearizes a non-linear system around a trajectory, which is very common approach in practice. Faced with a nonlinear system one often uses the full nonlinear dynamics to design an optimal trajectory to guide the system from its initial state to a desired final state. However, ensuring that the system will actually track this trajectory in the presence of disturbances is not an easy task. One solution is to linearize the nonlinear system (i.e. approximate it by a linear system) around the optimal trajectory; the approximation is accurate as long as the nonlinear system does not drift too far away from the optimal trajectory. The result of the linearization is a time varying linear system, which can be controlled using several methods. If the control design is done well, the state of the nonlinear system will always stay close to the optimal trajectory, hence ensuring that the linear approximation remains valid.

Now, a special class of linear time varying systems are linear time invariant systems, usually referred to by the acronym LTI with a description shown in Figure 10.

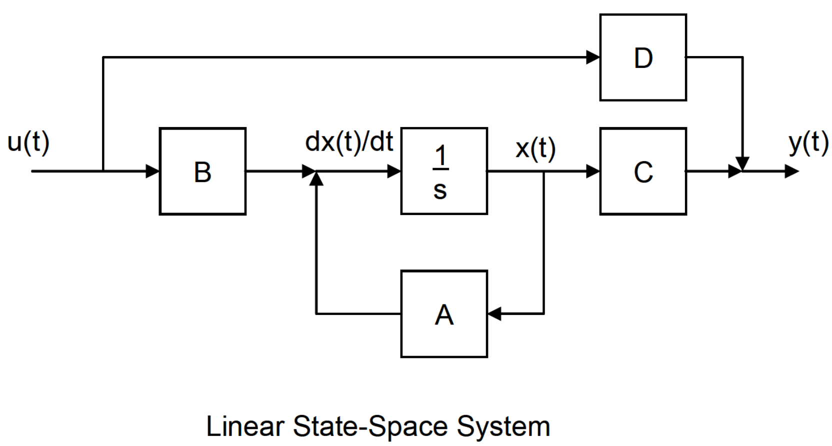

Figure 10: Block diagram.

These systems are described by state equations of the form

$$\dot{x}(t) = Ax(t) + Bu(t),$$
$$y(t) = Cx(t) + Du(t),$$

where the matrices $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{p\times n}$ and $D \in \mathbb{R}^{p\times m}$ are constant matrices $\forall t \in \mathbb{R}_{>0}$ of appropriate dimensions. LTI systems are somewhat easier to deal with and will be treated as a special case of the more general linear time varying systems. Linear systems are ideally suited for research on control systems theory due to their simplicity to derive clean formulas. Although most of the systems in nature are nonlinear. The main point of linear systems theory is to exploit the algebraic structure to develop tractable "algorithms" that allow us to answer analysis questions which appear intractable by themselves.

For example, let us consider the LTI system,

$$\dot{x}(t) = Ax(t) + Bu(t). \tag{1}$$

Given $x_0 \in \mathbb{R}^n,\ T > 0$ and a continuous function (input trajectory) $u(\cdot) : [0, T] \to \mathbb{R}^m$ we can show that there exists a unique function $x(\cdot) : [0, T] \to \mathbb{R}^n$ such that,

$$x(0) = x_0;\ \text{and}\ \dot{x}(t) = Ax(t) + Bu(t),\ \forall t \in [0, T].$$

This function is called the state trajectory with initial condition x_0 under the input $u(\cdot)$. it is obvious that $u(\cdot)$ does not even need to be continuous, provided one appropriately qualifies the statement $\forall t \in [0, T]$. The LTI system is called controllable, if and only if $x_0 \in \mathbb{R}^n,\ \forall \hat{x} \in \mathbb{R}^n$, and $\forall T > 0$, there exists $u(\cdot) : [0, T] \to \mathbb{R}^m$ such that the solution of the system with initial condition x_0 under the input $u(\cdot)$ is such that $x(T) = \hat{x}$. Controllability is clearly an interesting property for a system to have. If the system is controllable then we can guide it from any initial state to any final state by selecting an appropriate input. If not, there may be some desirable parts of the state space that we cannot reach from some initial states. Unfortunately, determining whether a system is controllable directly from the definition is impossible. This would require calculating all trajectories that start at all initial conditions. Except for trivial cases (like the linear system $\dot{x}(t) = u(t),\ t \geqslant 0$) this calculation is intractable, since the initial states, x_0, the times T of interest, and the possible input trajectories $u(\cdot) : [0, T] \to \mathbb{R}^m$ are all infinite.

On the other side, observability (mathematical dual to controllability) is a measure for how well internal states of a system can be inferred by knowledge of its external outputs. The concept of observability was firstly introduced by R. E. Kalman for linear dynamic systems.

The controllability property is summarized in the following Theorem.

Definition 1. Consider a system of the form

$$\dot{x}(t) = Ax(t) + Bu(t).$$

The pair (A, B) is controllable if, given a duration $T > 0$ and two arbitrary points $x_0,\ x_T \in \mathbb{R}^n$, there exists a piecewise continuous function $t \to \bar{u}(t)$ from $[0, T]$ to $\mathbb{R}^m$, such that the integral curve $\bar{x}(t)$ generated by $\bar{u}$ with $\bar{x}(0) = x_0$, satisfies $\bar{x}(T) = x_T$. □

Theorem 1. *A system of the form*

$$\dot{x}(t) = Ax(t) + Bu(t),$$

is controllable if and only if the matrix $\begin{bmatrix} B & AB & \cdots & A^{n-1}B \end{bmatrix} \in \mathbb{R}^{n \times nm}$ *has rank* n. ■

Similarly we can define the notion of observability.

Definition 2. A system of the form

$$\begin{aligned} &\dot{x}(t) = Ax(t) + Bu(t), \\ &y = Cx(t) \end{aligned}$$

is said to be observable if, for any possible sequence of state and control vectors, the current state can be determined in finite time using only the outputs. Less formally, this means that from the system's outputs it is possible to determine the behavior of the entire system. If a system is not observable, this means the current values of some of its states cannot be determined through output sensors. This implies that their value is unknown to the controller. □

Now we can state the following theorem,

Theorem 2. *A system of the form*

$$\begin{aligned} &\dot{x}(t) = Ax(t) + Bu(t), \\ &y = Cx(t) \end{aligned}$$

is observable if and only if the matrix $\begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix} \in \mathbb{R}^{n \times p \times n}$ *has rank* n. ■

Note that the previous theorems of controllability and observability were defined for LTI systems. One has to be very careful when extending those theorems to time varying linear systems.

3.2.4 Basic Concepts of a Control System

There are seven basic concepts of a control system (see Figure 11) as shown below,

- **Plant:** a physical object to be controlled such as a mechanical device, a heating furnace, a chemical reactor or a spacecraft.
- **Controlled variable:** the variable controlled, generally refers to the system output.

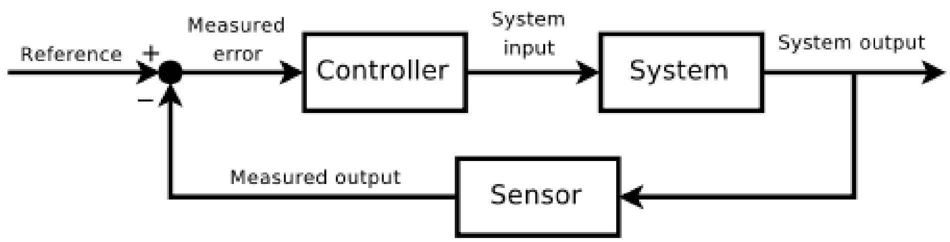

Figure 11: Basic components of a control system.

- **Expected value:** the desired value of controlled variable based on requirement, often it is used as the reference input
- **Controller:** an agent that can calculate the required control signal.
- **Actuator:** a mechanical device that takes energy, usually created by air, electricity, or liquid, and converts that into some kind of motion.
- **Sensor:** a device that measures a physical quantity and converts it into a signal which can be read by an observer or by an instrument.
- **Disturbance:** the unexpected factors disturbing the normal functional relationship between the controlling and controlled parameter variations.

3.3 Control Techniques

3.3.1 Optimal Control

Optimal control [20, 21, 22, 23, 24, 25] has emerged as one of the fundamental design philosophies of modern control systems design. Optimal control policies satisfy the specified system performance while minimizing a structured cost index which describes the balance between desired performance and available control resources. From a mathematical point of view the solution of the optimal control problem is based on the solution of the underlying Hamilton-Jacobi-Bellman (HJB) equation. Until recently, due to the intractability of this nonlinear differential equation for continuous-time (CT) systems, which form the object of interest in this work, only particular solutions were available (e.g. for the linear time-invariant case, the HJB becomes the Riccati equation).

Optimal control design algorithms using the Riccati equation are well known. Good design algorithms for the Linear Quadratic Regulator (LQR) [20, 21, 22, 23] are available in MATLAB Control systems Toolbox and elsewhere. The structure and design procedure for the LQR is shown in Figure 12. This figure shows an LQR design for system (1) with the performance to be minimized given as,

$$J_{\mathrm{LQR}} = \frac{1}{2}\int_0^{\infty} (x^{\mathrm{T}} Q x + u^{\mathrm{T}} R u)\mathrm{d}t \tag{2}$$

where the user defined matrices are $Q \geq 0$ and $R \succ 0$ of appropriate dimensions.

Depending on how these design parameters are selected, the closed-loop system will exhibit a different response. Generally speaking, selecting Q large means that, to keep J_{LQR} small, the state $x(t)$ must be smaller. On the other hand selecting R large means that the control input $u(t)$ must be smaller to keep J_{LQR} small. This means that larger values of Q generally result in the poles of the closed-loop system matrix being further left in the frequency domain-plane so that the state decays faster to zero. On the other hand, larger R means that less control effort is used, so that the poles are generally slower, resulting in larger values of the state $x(t)$. In terms of eigenvalues, the eigenvalues of Q should be non-negative, while those of R should be positive. If both matrices are selected diagonal, this means that all the entries of R must be positive while those of Q should be positive, with possibly some zeros on its diagonal. Note that then R is invertible. Since the plant is linear and the performance index is quadratic, the problem of determining the state variable feedback control to minimize J_{LQR} is called LQR. The word 'regulator' refers to the fact that the function of this feedback is to regulate the states to zero. This is in contrast to tracker problems, where the objective is to make the output follow a prescribed (usually nonzero) reference command.

To find the optimal feedback control K we proceed as follows. Suppose there exists a constant matrix P and $u = -Kx$ such that,

$$\frac{\mathrm{d}}{\mathrm{d}t}(x^{\mathrm{T}}Px) = -x^{\mathrm{T}}(Q + K^{\mathrm{T}}RK)x. \tag{3}$$

Then substituting in (2) yields,

$$J_{\text{LQR}} = -\frac{1}{2}\int_0^\infty \frac{\mathrm{d}}{\mathrm{d}t}(x^{\mathrm{T}}Px)\mathrm{d}t = \frac{1}{2}x^{\mathrm{T}}(0)Px(0), \tag{4}$$

where we assumed that the closed-loop system is stable so that $x(t)$ goes to zero as time t goes to infinity. Equation (4) means that J is now independent of K. It is a constant that depends only on the auxiliary matrix P and the initial conditions.

Now, we can find K so that assumption (3) does indeed hold. To accomplish this, differentiate (3) and then substitute from the closed-loop state equation to see that (3) is equivalent to,

$$x^{\mathrm{T}}\big((A - BK)^{\mathrm{T}}P + P(A - BK) + Q + K^{\mathrm{T}}RK\big)x = 0.$$

Now note that the last equation has to hold for every $x(t)$. Therefore, the term in brackets must be identically equal to zero. Thus, proceeding one sees that,

$$(A - BK)^{\mathrm{T}}P + P(A - BK) + Q + K^{\mathrm{T}}RK = 0.$$

This is a matrix quadratic equation. Exactly as for the scalar case, one may complete the squares. Though this procedure is a bit complicated for matrices, suppose we select $K = R^{-1}B^{\mathrm{T}}P$ then we have,

$$(A - BR^{-1}B^{\mathrm{T}}P)^{\mathrm{T}}P + P(A - BR^{-1}B^{\mathrm{T}}P) + Q + (R^{-1}B^{\mathrm{T}}P)^{\mathrm{T}}R(R^{-1}B^{\mathrm{T}}P) = 0. \tag{5}$$

This result is of extreme importance in modern control theory. Equation (5) is known as the algebraic Riccati equation (ARE). It is named after Riccati, an Italian who lived in the 19th century and used a similar equation in the study of heat flow. It is a matrix quadratic equation that can be solved for the auxiliary matrix P given (A, B, Q, R). Then, the optimal K gain is given by $K = R^{-1}B^{\mathrm{T}}P$. The minimal value of the performance index using this gain is given by $\frac{1}{2}x^{\mathrm{T}}(0)Px(0)$, which only depends on the initial condition. This mean that the cost of using the selected K can be computed from the initial conditions before the control is ever applied to the system.

The LQR has many good properties, but these limitations:

- It requires full state feedback from all the states. These can be hard to measure.
- Its function is regulation, that is, to guarantee stability, that is, to regulate all the states to zero.
- It has no structure. All the inputs are linear combinations of all the states.
- It relies on offline design methods that require full knowledge of the systems dynamics. However, finding the dynamics model requires a great deal of expensive effort in system identification techniques. Also, system models are not usually completely accurate
- It only minimizes quadratic performance indices, e.g. energies. What about minimum fuel, minimum time, etc.?
- It is an offline design procedure, and so does not allow real-time changes in system performance index requirements.

3.3.2 Adaptive Control

The history of adaptive control began from the early 1950's. With the passing of the years a lot of papers and books have been published. These research activities have proposed solutions for basic problems and for broader classes of systems. Especially the interest for nonlinear adaptive control began from the mid-1980's. Adaptive control [26, 27, 28, 29, 30, 31] is a powerful tool that deals with modeling uncertainties in nonlinear (and linear) systems by on line tuning of parameters. Very important research activities include on-line identification and pattern recognition inside the feedback control loop. It is well known that global stability properties of model reference adaptive systems are guaranteed under the "matching assumption" that the model order is not lower than that of the unknown plant. This restrictive assumption is likely to be violated in applications. Hence, it is important to determine stability and robustness properties of adaptive schemes with respect to modeling errors.

In order to see how adaptive control works, we will define the control error $r(t)$ with dynamics given by,

$$\dot{r} = f(x) - \tau,$$

where τ is a control input and $f(x)$ are some unknown nonlinearities.

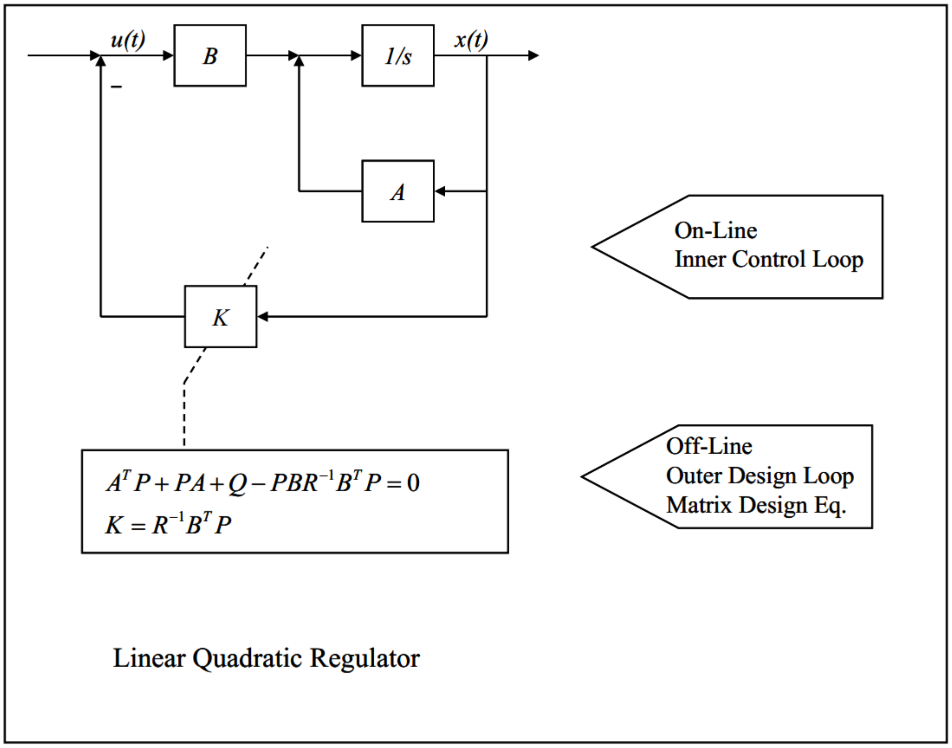

Figure 12: Linear quadratic regulator (LQR).

Now we assume that $f(x)$ is known to be of structure,

$$f(x) = W^{\mathrm{T}}\phi(x)$$

with W an unknown parameter and $\phi(x)$ a known basis set or regression vector. Note that this structure is linear in the parameters W. Also the regression matrix $\phi(x)$ depends on the system and changes for different systems.

Then the error dynamics are,

$$\dot{r} = W^{\mathrm{T}}\phi(x) - \tau.$$

A scheme of an adaptive controller is shown in Figure 13.

Define the estimate for the nonlinear function as $\hat{f}(x) = \hat{W}^{\mathrm{T}}\phi(x)$. Select the controller as,

$$\tau = \hat{f}(x) + K_v r = \hat{W}^{\mathrm{T}}\phi(x) + K_v r,$$

with $\hat{W}(t)$ a time varying estimate of the unknown parameters W and control gain $K_v > 0$ any symmetric positive definite matrix.

Then the closed-loop system becomes,

$$\dot{r} = W^{\mathrm{T}}\phi(x) - \tau = W^{\mathrm{T}}\phi(x) - \hat{W}^{\mathrm{T}}\phi(x) - K_v r := \tilde{W}^{\mathrm{T}}\phi(x) - K_v r$$

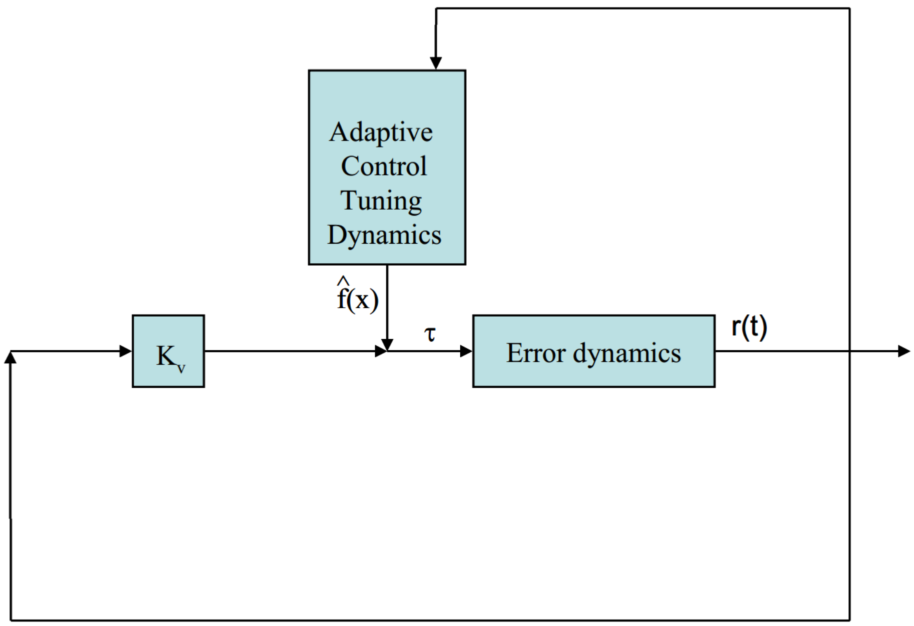

Figure 13: Adaptive controller.

where $\tilde{W} := W - \hat{W}(t)$.

Let the parameter estimate be updated using

$$\dot{\hat{W}} = F\phi(x)r^{\mathrm{T}},$$

where the tuning gain $F > 0$ is any positive definite symmetric matrix. This is the internal (tuning) dynamics of the adaptive controller. The state of the adaptive controller is given by the parameter estimates. Using the adaptive controller, the closed-loop system is asymptotically stable (the equilibrium point of the system rests at the origin), i.e. the control error $r(t)$ goes to zero. If an additional Persistence of Excitation (PE) condition holds, the parameter estimates converge to the actual unknown parameters. The typical behavior of adaptive controllers is to guarantee that the errors go to zero and the parameter estimates converge. This assumes that the parametrization $f(x) = W^{\mathrm{T}}\phi(x)$ holds exactly, and that there are no disturbances in the system.

3.3.3 Robust Control

Robust control [32, 33, 34, 35, 36, 37] deals explicitly with uncertainty in its approach to controller design. Controllers designed using robust control methods tend to be able to cope with small differences between the true system and the nominal model used for design. The early methods of Bode and others were fairly robust; the state-space methods invented in the 1960s and 1970s were sometimes found to lack robustness. Examples of modern robust control techniques include H-infinity loop-shaping developed by Duncan McFarlane and Keith Glover [38] of Cambridge University, UK and sliding mode control (SMC) developed by V. Utkin [39]. Robust methods aim to achieve robust performance and/or stability in the presence of small modeling errors.

In order to see how robust control works, we will define in a similar manner as in adaptive control technique the control error $r(t)$. Hence the dynamics are given also given by $\dot{r} = f(x) - \tau$. We also know a fixed nominal value or estimate $\hat{f}(x)$ for unknown $f(x)$, and that the estimation error $\tilde{f} := f(x) - \hat{f}(x)$ is bounded like $\left\|\tilde{f}(x)\right\| \leqslant F(x)$ with $F(x)$ a known upper bound function, possibly nonlinear. Note that robust control assumes less information than the adaptive controller, which needs the linear in the parameters structure $f(x) = W^{\mathrm{T}}\phi(x)$ with W an unknown parameter vector and $\phi(x)$ a known basis set or regression vector. A scheme for robust control is shown in Figure 14.

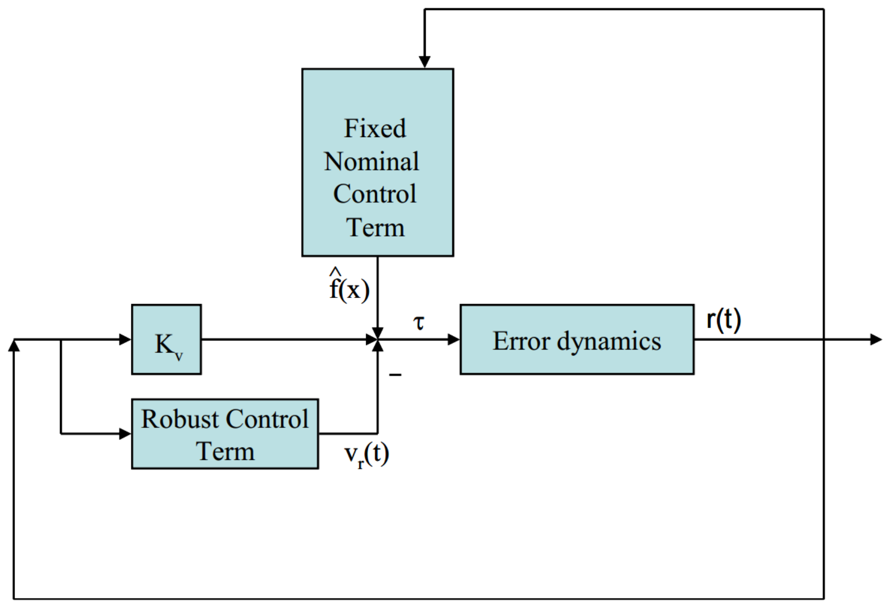

Figure 14: Robust controller.

For robust control, we need to select the controller

$$\tau = \hat{f}(x) + K_v r - v_r,$$

where v_r a robust control term given by,

$$v_r = \begin{cases} -r\frac{F(x)}{\|r\|}, & \|r\| \geqslant \epsilon \\ \\ -r\frac{F(x)}{\epsilon}, & \|r\| < \epsilon, \end{cases}$$

with $\epsilon \in \mathbb{R}^+$ a small design parameter. Then the closed-loop dynamics become,

$$\dot{r} = f(x) - \tau = f(x) - \left(\hat{f}(x) + K_v r - v_r\right) := \tilde{F}(x) - K_v r + v_r.$$

This robust controller is easier to implement than the adaptive controller because the controller does not have any dynamics. As long as the performance of the aforementioned robust controller, the closed-loop system is bounded stable with $\|r\|$ bounded with a magnitude near ϵ. Note that the error does not go to zero but does indeed stay small.

3.3.4 Intelligent Control

Intelligent control, the discipline where control algorithms are developed by emulating certain characteristics of biological systems and being fueled by recent advancements in computing technology and is emerging as a technology that may open avenues for significant technological advances. The characteristics include adaptation and learning, planning under large uncertainty and coping with large amounts of data. Today, the area of intelligent control tends to encompass everything that is not characterized as conventional control; it has, however, shifting boundaries and what is called "intelligent control" today, will probably be called "control" tomorrow.

Intelligent control [40, 41, 42, 43, 44, 45, 46, 47, 48] uses various Artificial Intelligence (AI) computing approaches like neural networks, Bayesian probability, fuzzy logic, machine learning, and self-organizing systems. Neural networks and fuzzy logic systems for feedback control. Discrete event systems, Dempster- Shafer, Petri Nets, and decision-making supervisory control systems, evolutionary computation and genetic algorithms to control a dynamic system.

Intelligent control can also be used in intelligent machinery monitoring, repair in advanced sensor processing including Kalman filtering and sensor fusion.

3.4 Examples of Dynamical Systems

Example: Heat Flow Let us start with a stateless system, where a continuous-time component has two inputs h_+ and h_- and an output h_{net} which denote the heat inflow, heat outflow and net heatflow respectively. We need to map the two input signals to an output signals as,

$$h_{\text{net}}(t) = h_+(t) - h_-(t),\ \forall t.$$

It is obvious that when the inputs are continuous, the output is also continuous. □

Example: Robotic Car Motion For the purpose of designing a cruise controller we need to make some simplifications and/or assumptions. Let us assume that the rotational inertia of the wheels is negligible, and that the friction restricting the motion is proportional to the car's speed. If x denotes the position of the car (measured with respect to the inertial reference) and F denotes the force applied to the car. Then using the Newton's laws for motion we can write the dynamics of the car as,

$$F - k\dot{x} = m\ddot{x},$$

where k is the coefficient of the frictional force and $m \in \mathbb{R}^+$ denotes the mass of the car. To illustrate the behaviors of this model, let us choose the initial position to be x_0 and the initial velocity to be v_0. Now consider the case that the input force $F := kv_0,\ \forall t$. Then the position and the velocity can be obtained by,

$$\begin{aligned}\dot{x} &= v \\ \dot{v} &= k(v_0 - v)/m,\end{aligned}$$

where the initial conditions are given as, $\bar{x}(0) = x_0$ and $\bar{v}(0) = v_0$. These equations have a unique solution. Why? The velocity stays constant at the value v_0 at all times and the distance x increases linearly with time, i.e. $x_0 + tv_0$. Another case is when the input force $F := 0,\ \forall t$, the initial position of the car is 0 and the initial velocity is v_0. Then the position x and the velocity v of the car at all times can be obtained by solving,

$$\begin{aligned}\dot{x} &= v \\ \dot{v} &= -kv/m,\end{aligned}$$

where the initial conditions are given as, $\bar{x}(0) = 0$ and $\bar{v}(0) = v_0$. Using rules of differential calculus, we can solve these equations and find the position and velocity as,

$$\begin{aligned}\bar{v}(t) &= v_0 e^{-kt/m} \\ \bar{x}(t) &= (mv_0/k)[1 - e^{-kt/m}].\end{aligned}$$

Note that the velocity decreases exponentially to zero, where the position increases exponentially to mv_0/k.

Another case is when the initial position and velocity are zero and we apply a constant force F_0 to the car. Then we need to solve,

$$\begin{aligned}\dot{x} &= v \\ \dot{v} &= (F_0 - kv)/m,\end{aligned}$$

where the initial conditions are given as, $\bar{x}(0) = 0$ and $\bar{v}(0) = 0$. □

Example: Helicopter Spin A helicopter has 6DOF, three for position and three for velocity. But for simplicity let us assume that the helicopter position is fixed and it remains vertical. Then the only freedom of motion is the angular rotation around the vertical that is the Z-axis. This is called yaw. The friction of the main rotor at the top causes the yaw to change. The tail rotor then needs to apply a torque to counteract this rotational force to keep the helicopter from spinning. In such a setting, the helicopter has a continuous-time input signal T denoting the torque around the Y-axis. The moment of inertia of the helicopter in this setting by a single scalar I. The output of the model is the angular velocity around the vertical axis and is modeled by the spin $s = \dot{\theta}$ where θ gives the yaw. The equation of motion is then given by,

$$\ddot{\theta} = \frac{T}{I}.$$

□

Example: Simple Pendulum The pendulum is a rod with a rotational joint and a mass at the other end as shown in Figure 15. The dynamics are,

$$mL^2\ddot{\theta} + mgL\sin\theta = u.$$

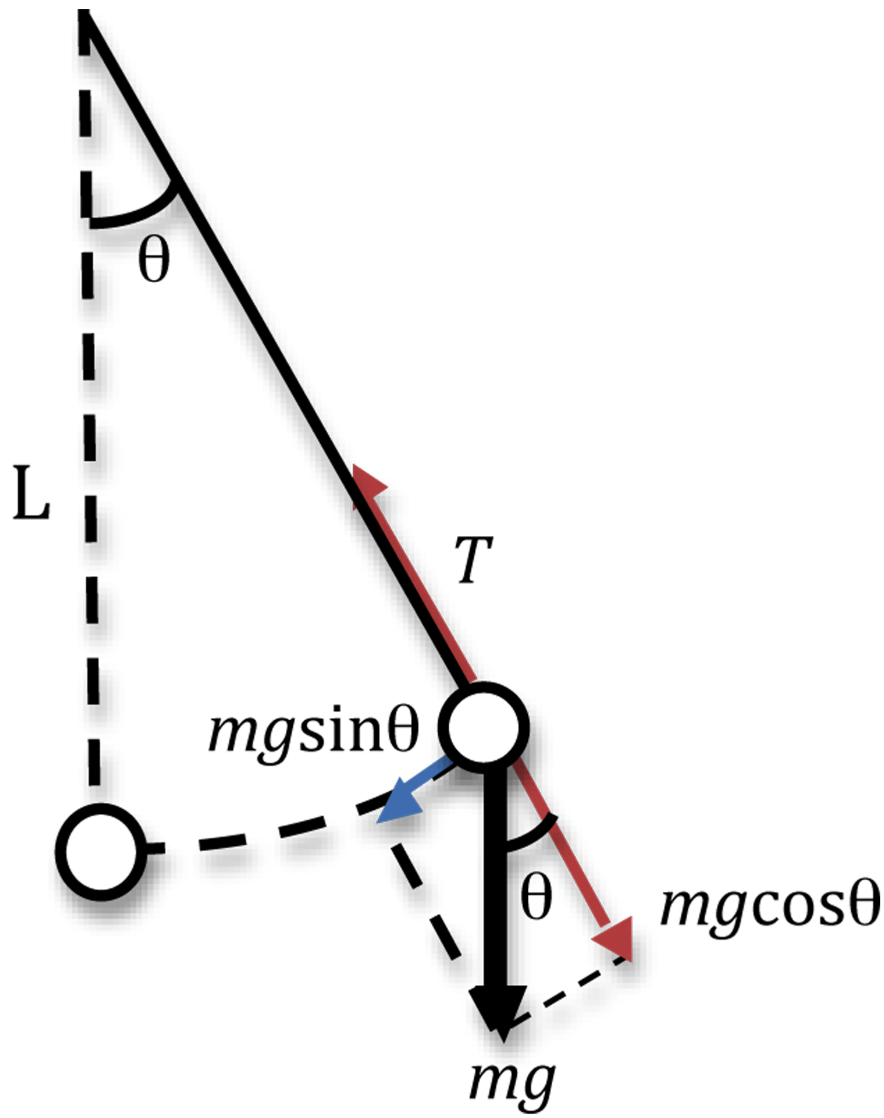

Figure 15: Pendulum.

Example: DC Motor Figure 16 shows the design of a DC motor that converts input voltage to rotational motion and is used in many electromechanical devices. Let us denote the input voltage by v the resistance of the circuit by R and the inductance of the circuit by L. Let i denote the electrical current flowing through the circuit and θ the angular displacement of the wheel. If b

is the electromotive-force (EMF) constant, then the back EMF voltage generated by the rotating shaft equals b times its angular velocity. After using Kirchoff's laws we can obtain the following dynamical equations,

$$Li + Ri + b\dot{\theta} = v.$$

A simple controller is $v = K_p(r - v_o)$ with a task to control the voltage so that the rotational velocity changes from its initial value to a desired speed, r and K_p is a proportional gain. □

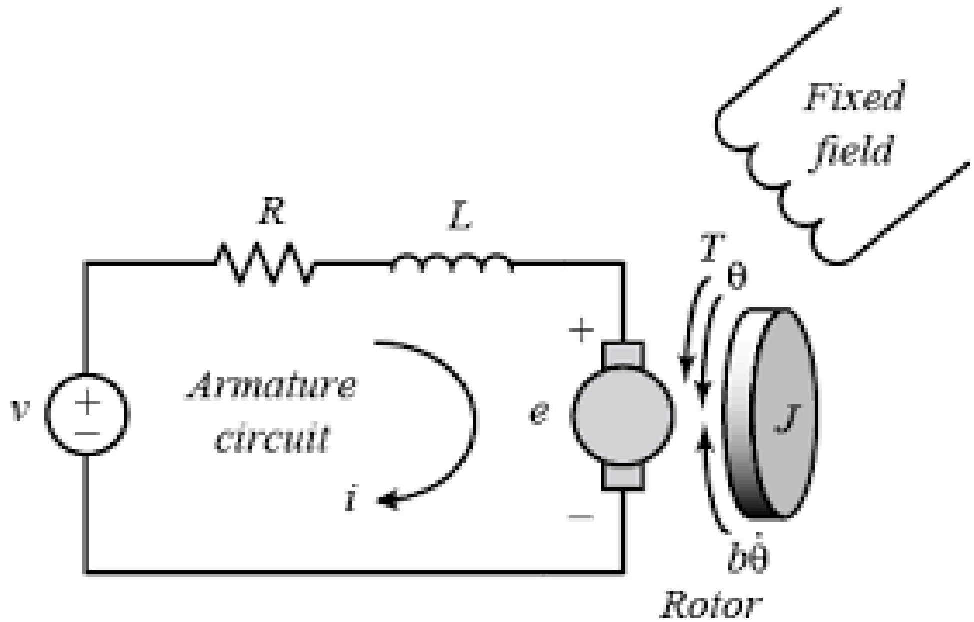

Figure 16: DC motor.

3.5 Useful Notions

Lipschitz Continuous Dynamics for Existence and Uniqueness A classical way to ensure uniqueness of the solution to a differential equation to require the right hand side to be Lipschitz continuous. Intuitively, Lipschitz continuity means that there is a constant upper bound on how fast a function changes, i.e. a function $f : \mathbb{R}^n \to \mathbb{R}^n$ is said to be Lipschitz continuous if there exists a constant K such that for all vectors $u, v \in \mathbb{R}^n$ we have,

$$\|f(u) - f(v)\| \leqslant K\|u - v\|.$$

For the robotic car problem, it is obvious that the right hand side is Lipschitz continuous. The function $x^{1/3}$ is not Lipschitz continuous, since its rate of change grows unboundedly as x approaches 0. The function x^2 is not Lipschitz continuous as its rate of change grows unboundedly as x increase unboundedly. But we usually allow the x to lie in a bounded set, e.g. Ω which is typical in dynamical systems. □

Lyapunov Stability Consider a closed continuous-time component H with n state variables S and dynamics given by the equations, $\dot{S} = f(s)$ where $f : \mathbb{R}^n \to \mathbb{R}^n$ is Lipschitz continuous. A state s_e is said to be an equilibrium of the component H if $f(s_e) = 0$. Given an initial state $s_0 \in \mathbb{R}^n$, let $\bar{S}_0 : \mathbb{R} \to \mathbb{R}^n$ be the unique response of the component H from the initial state s_0.

- An equilibrium s_e of H is said to be stable if for every $\epsilon > 0$ there exists a $\delta > 0$ such that for all states s_0, if $\|s_e - s_0\| < \delta$ then $\|\bar{S}_0(t) - s_e\| < \epsilon$ holds at all times $t \geqslant 0$.

- An equilibrium s_e of H is said to be asymptotically stable if it is stable and there exists there exists a $\delta > 0$ such that for all states s_0, if $\|s_e - s_0\| < \delta$ then the limit $\lim_{t\to\infty} \bar{S}_0(t)$ exists and equals to s_e.

$\square$

3.6 How to Simulate Dynamical Systems

Given the state-space description it is very easy to simulate a system on a computer. All one requires is a numerical integration routine such as Runge-Kutta that computes the state derivative using $\dot{x} = f(x, u)$ to determine $x(t)$ over a time interval. MATLAB has an integration routine 'ode23' that can be used to simulate any nonlinear system in state-space form. For linear systems, MATLAB has a variety of simulation routines including 'step' for the step response, etc., however, it is recommended that 'ode23' be used even for linear systems, since it facilitates controller design and simulation.

3.6.1 Second Order System

Consider the following second order differential equation of the sort occurring in robotic systems,

$$m\ddot{q} + mL\dot{q}^2 + mgL\sin q = \tau$$

where $q(t) \in \mathbb{R}$ is an angle and $\tau(t) \in \mathbb{R}$ is the input torque. By defining the state $x = \begin{bmatrix} x_1 & x_2 \end{bmatrix}^{\mathrm{T}}$ as $x_1 = q$ and $x_2 = \dot{q}$ one can write,

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -Lx_2^2 - gL\sin x_1 + \frac{1}{m}u \end{aligned}$$

where the control input is $u = \tau$.

To solve such system we need two initial conditions, e.g. $q(0), \dot{q}(0)$. Let us assume that $L = 1m$ and $m = 10Kg$ while the control input is set to be a sinusoid. The M-file is,

```
function xdot= robot(t,x)
g= 9.8 ;
L= 1 ;
m= 10 ;
```

```
u= sin(2*pi*t);
xdot(1)= x(2) ;
xdot(2)= -g*L*sin(x(1)) - L*x(2)^2 + u/m ;
```

Given this M-file, stored in a file named 'robot.m', let's say, the following command lines compute and plot the time response over the time interval 0 to 20 sec:

```
tint= [0  20] ;  % define time interval [t0  tf]
x0= [0   0.1]' ;  % initial conditions
[t,x]= ode23('robot', tint, x0);
plot(t,x);
```

Using this code, one obtains the Figure 17.

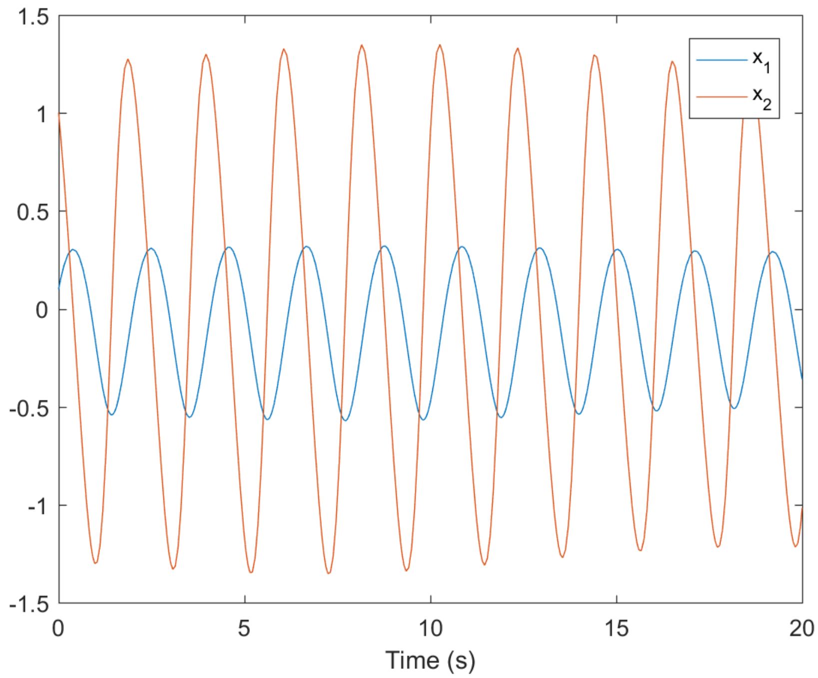

Figure 17: Evolution of the states during time for the simulation example.

4 Conclusion

Technology is part of human activity, but also part of economics and society. There is no doubt that control has brought enormous benefits, enabling autonomous vehicles to carry out missions in hostile environments with great success.

We are moving toward control systems that are able to cope and maintain acceptable performance levels under significant unanticipated uncertainties and failures, systems that exhibit considerable degrees of autonomy. We are moving toward autonomous underwater, land, air and space vehicles; highly automated manufacturing; intelligent robots; highly efficient and fault tolerant voice and data networks; reliable electric power generation and distribution; seismically tolerant structures; and highly efficient fuel control for a cleaner environment. Future developments in feedback control theory for autonomous systems will explore more sophisticated mathematical models with the use of machine learning to solve problems that are not solvable with traditional control techniques.

As a future work in control systems, a lot of control engineers, grant agencies and research universities/institutes, emphasize the need for human-machine interaction, decision support, mixed manned-unmanned teams and the integration of neuroscience in machines.

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