

AE 4361 – Assignment 3

1) a)

$$P = 2\pi \sqrt{\frac{a^3}{\mu}}$$

$$\sqrt[3]{\left(\frac{P}{2\pi}\right)^2} \times \mu = a$$

$$a = \sqrt[3]{\left(\frac{12 \times 3600}{2\pi}\right)^2} \times 3.986 \times 10^5 = \mathbf{26610km}$$

b)

$$r_p = a(1 - e)$$

$$e = -\frac{r_p}{a} + 1 = 0.549$$

Using the provided 'Kepler_solver' code with $t = 9 \times 3600 = 32400s$, $e=0.549$, $M_0 = 0$, $a=26610km$

$$E = 4.2269 \text{ rad}$$

$$r = a(1 - e \cos E) = 26610(1 - 0.549 \cos 4.2269) = \mathbf{33427km}$$

c)

$$V = \sqrt{\mu \left(\frac{2}{r} - \frac{1}{a} \right)} = \sqrt{3.986 \times 10^5 \times \left(\frac{2}{33427} - \frac{1}{26610} \right)} = \mathbf{2.978km/s}$$

d)

$$\tan \frac{v}{2} = \sqrt{\frac{1+e}{1-e}} \tan \frac{E}{2} = \sqrt{\frac{1+0.549}{1-0.549}} \tan \frac{4.2269}{2} = -3.073$$

$$v = 2 \arctan -3.073 = -2.51 \text{ rad}$$

$$h = \sqrt{\mu a (1 - e^2)} = 86080.6$$

$$\gamma = \arcsin \left(\frac{\mu}{h v} (1 + e \cos(v)) \right) = 1.047 \text{ rad}$$

$$V_r = V \times \cos \gamma = 2.978 \times \sin 2.51 = \mathbf{1.49km/s}$$

2)

Semi-major axis (a) <i>km</i>	6877.3
Eccentricity (e)	0.1582 or [0.0379, -0.1444, -0.0524]^T
Inclination (i) <i>deg</i>	20.000
Right ascension of ascending node (Ω) <i>deg</i>	30.000
Argument of perigee (ω) <i>deg</i>	255.581
True Anomaly (ν) <i>deg</i>	104.419

See “AE4361 HW3” starting at Q2 comment at the end of document for code.

Workspace variables for Q2:

a_2	6.8773e+03
e_2	[0.0379;-0.1444;-0.0524;0.1582]
i_2	19.9987
mu	398600
nu_2	104.4190
omega...	255.5810
r_vec	[6045;3490;0]
RAAN_2	60.0006
v_vec	[-2.4570;6.6180;2.5330]

3)

$$\text{a) } \vec{r}_{\text{Vanguard-1}} = \begin{bmatrix} 6.7758 \\ 0.3813 \\ 2.0544 \end{bmatrix} \times 10^6 \text{m}$$

$$\text{b) } \vec{r}_{\text{Hubble}} = \begin{bmatrix} -5.5368 \\ -3.8525 \\ -1.4916 \end{bmatrix} \times 10^6 \text{m}$$

$$\text{c) } \vec{r}_{\text{Molniya comms sat}} = \begin{bmatrix} 0.6497 \\ 1.3425 \\ 3.0208 \end{bmatrix} \times 10^7 \text{m}$$

$$\text{d) } \vec{r}_{\text{Starlink 3327}} = \begin{bmatrix} -3.7498 \\ 2.3053 \\ 5.0795 \end{bmatrix} \times 10^6 \text{m}$$

See code and function on next pages.