Hamming(7,4)-Code

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Introduction

Question

Could you please raise your hand if you never heard about Hamming Code?

質問

ハミングコードについて聞いたことがない場合は、手を挙げていただけますか?

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Let's Play a Game (Rules)

1. Please Pick a positive integer from 1 to 15

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Let's Play a Game (Rules)

1. Please Pick a positive integer from 1 to 15
2. I will give you 7 questions. You can answer either YES or NO each question.
3. Here is the fun thing. You may **lie at most one time** when answering all questions in total.

Question 1: : Do you see your chosen number?

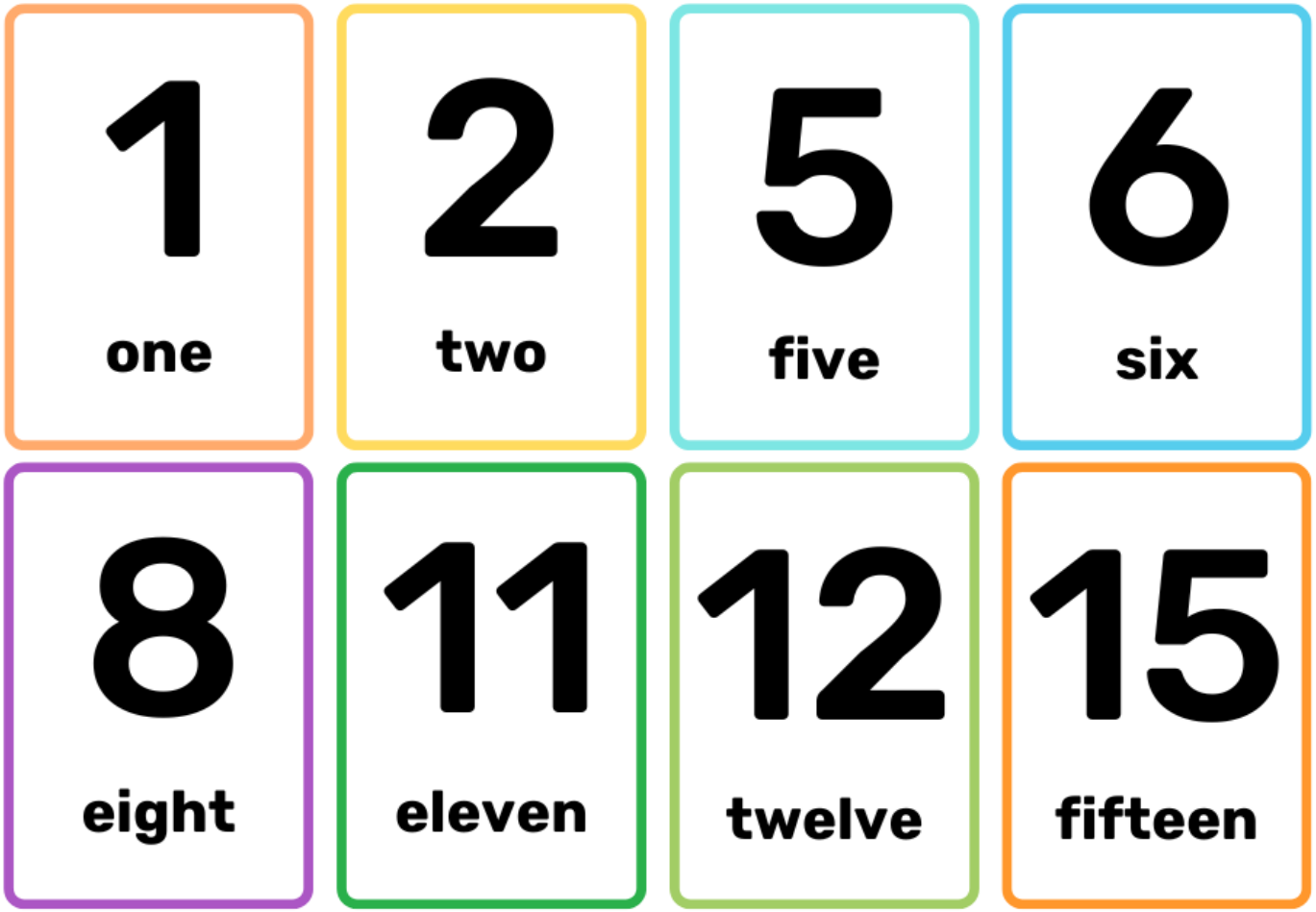

Figure: List 1

Question 2: : Do you see your chosen number?

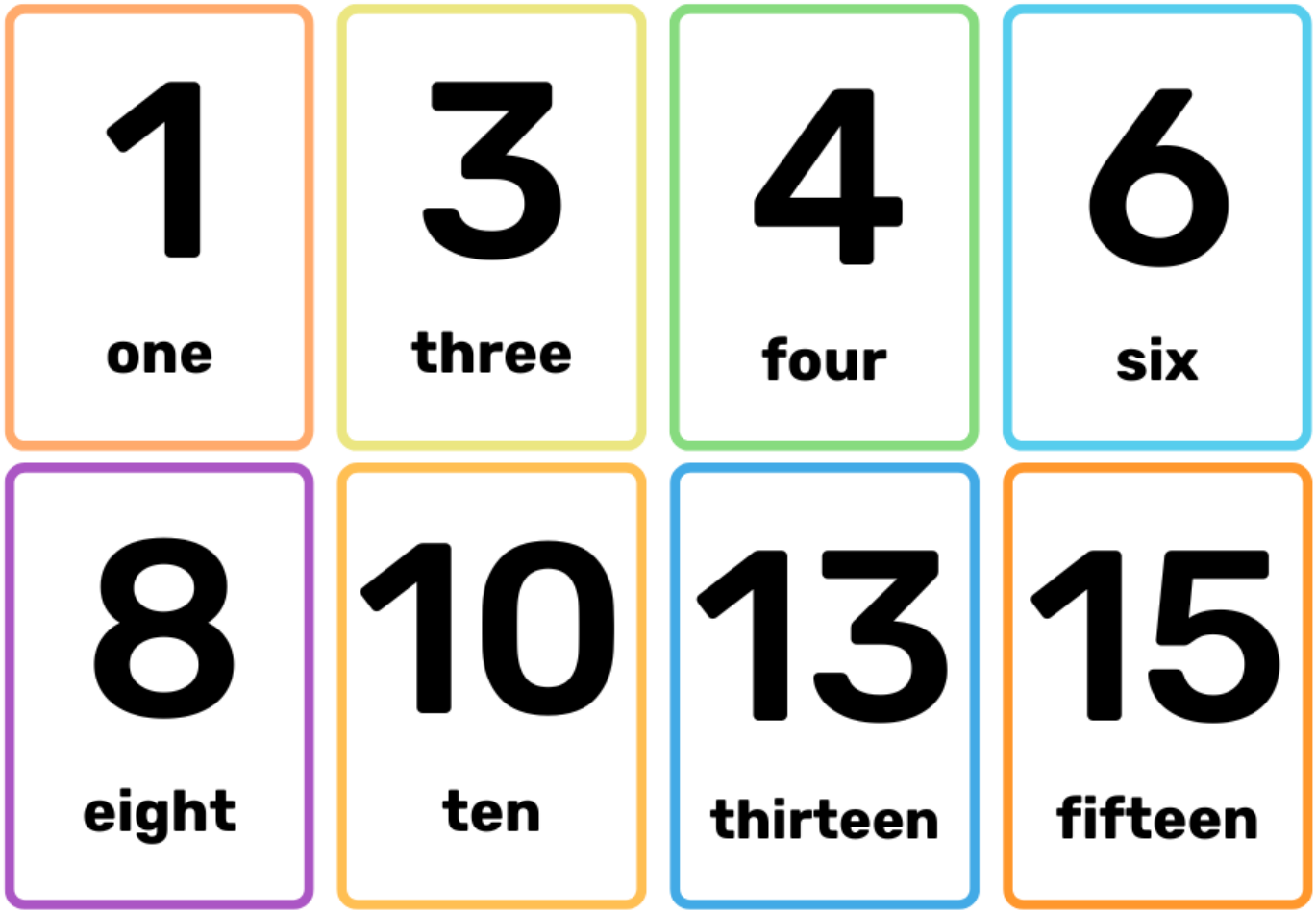

Figure: List 2

Question 3: : Do you see your chosen number?

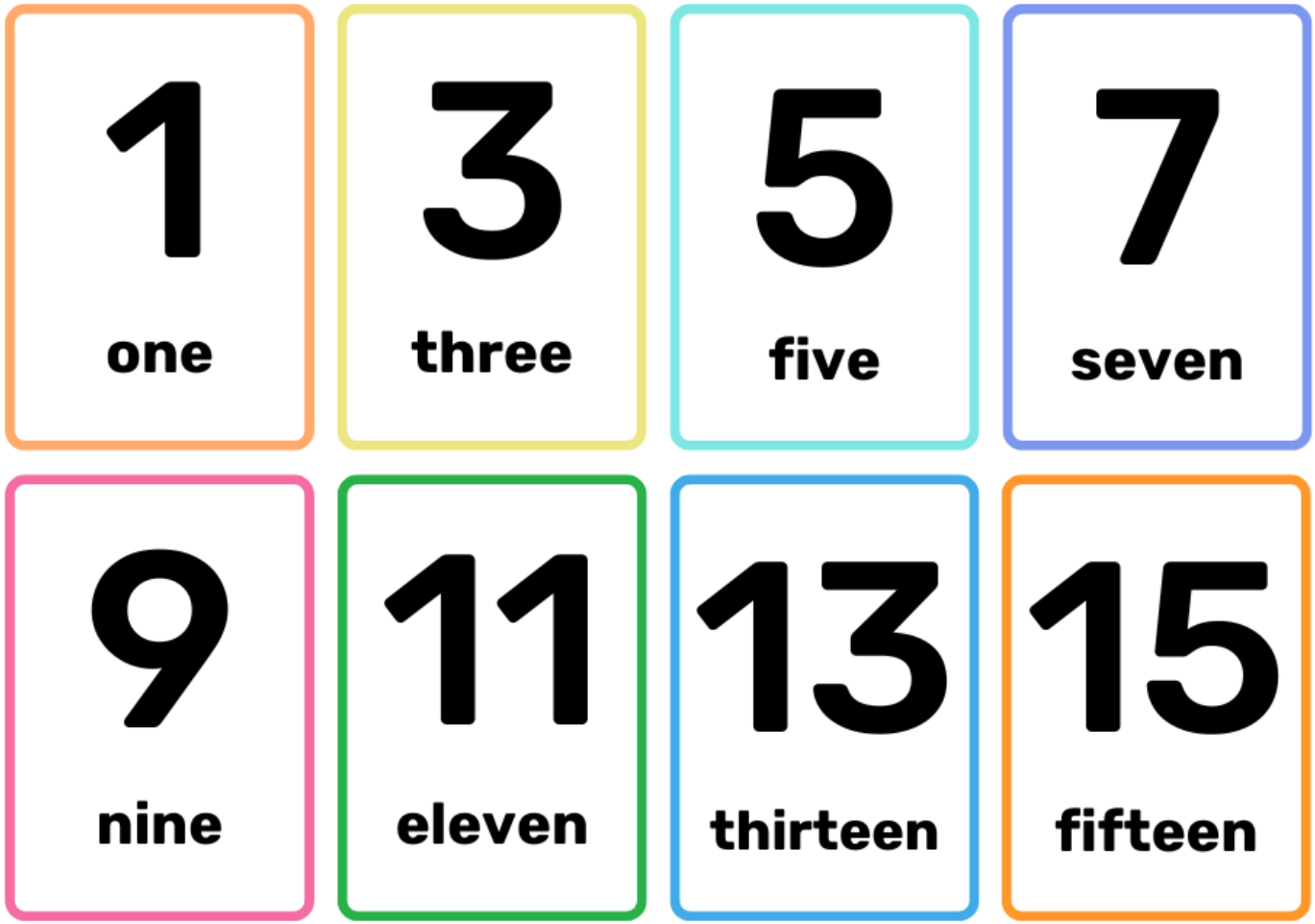

Figure: List 3

Question 4: : Do you see your chosen number?

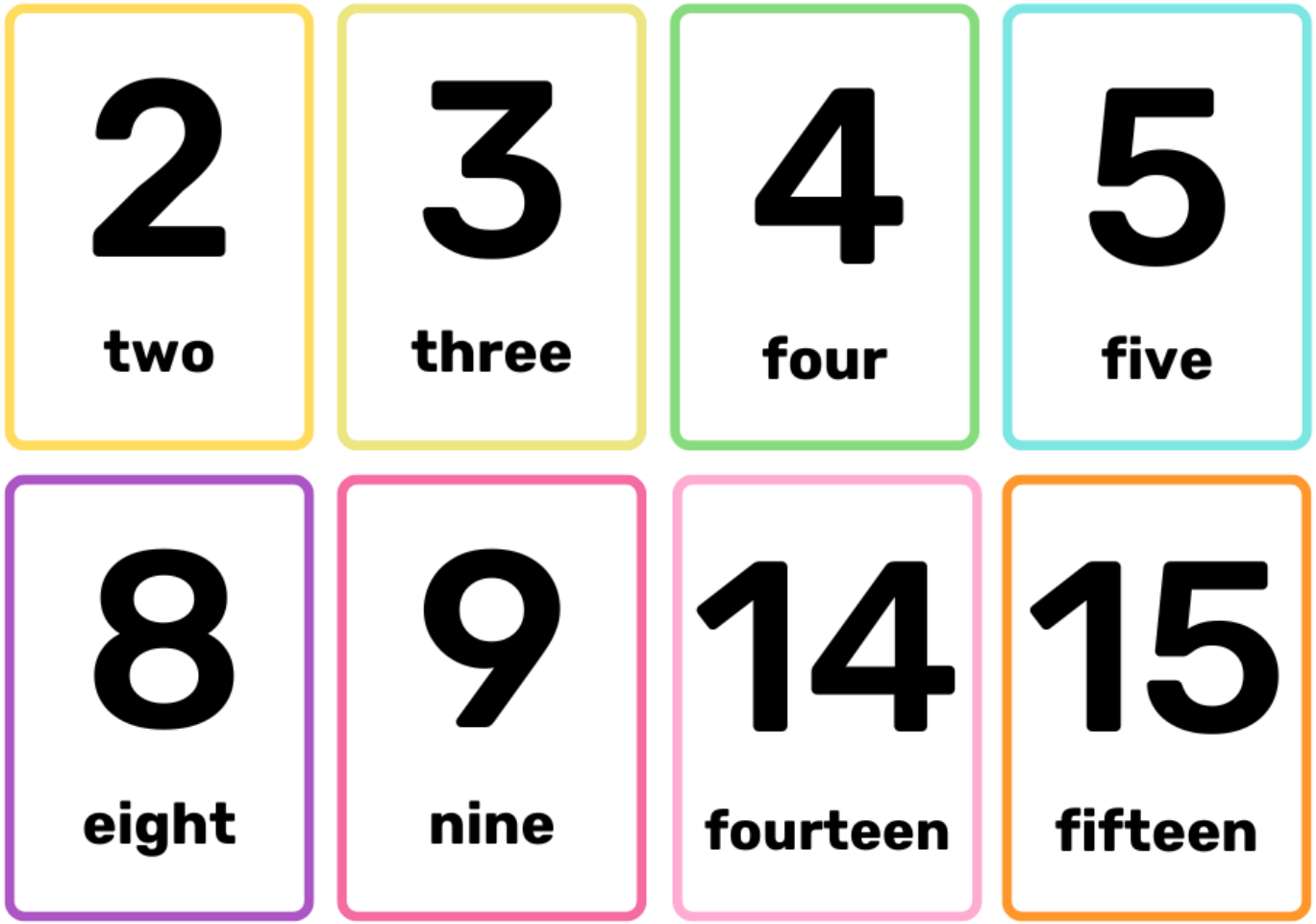

Figure: List 4

Question 5: : Do you see your chosen number?

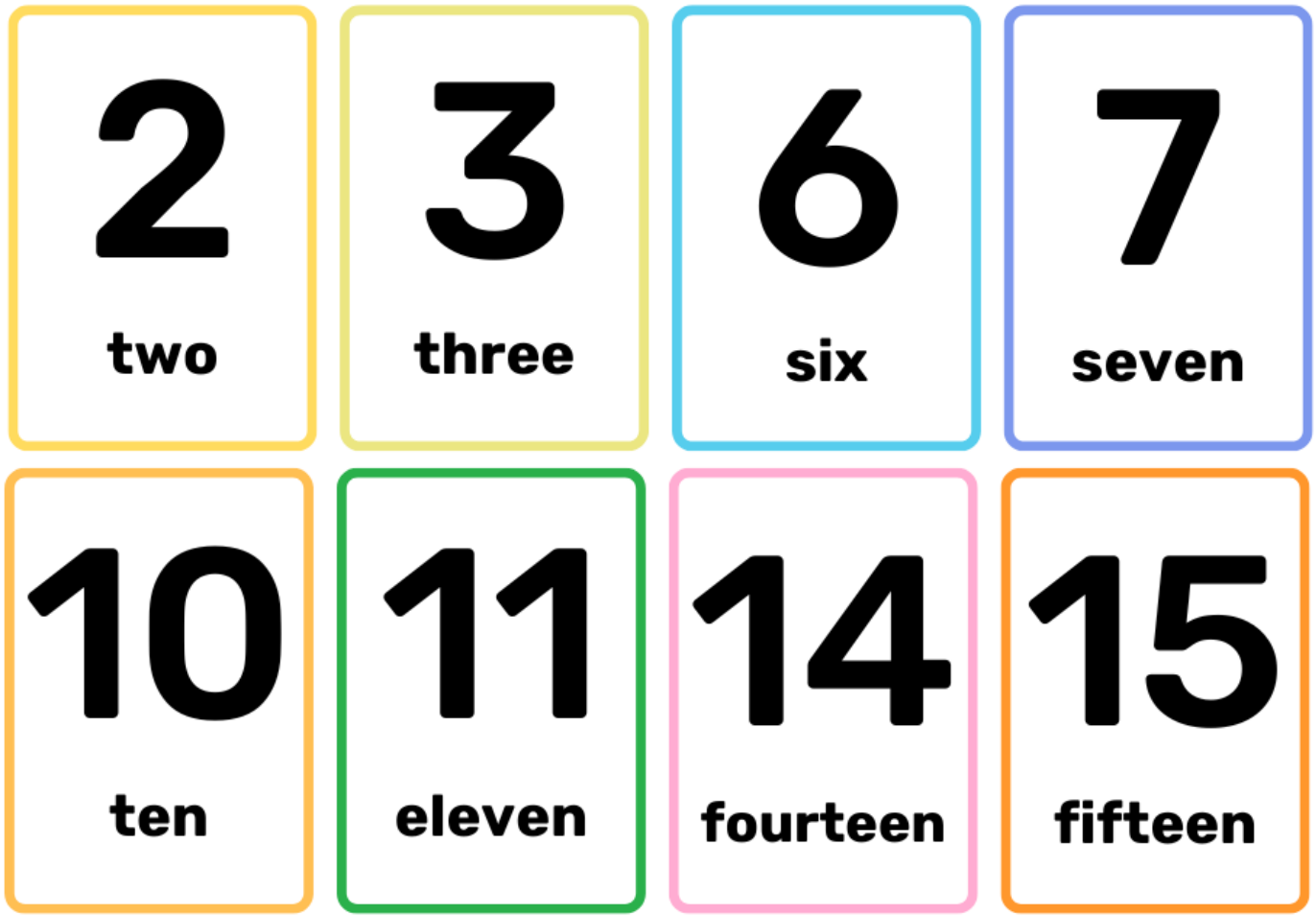

Figure: List 5

Question 6: : Do you see your chosen number?

Figure: List 6

Question 7: : Do you see your chosen number?

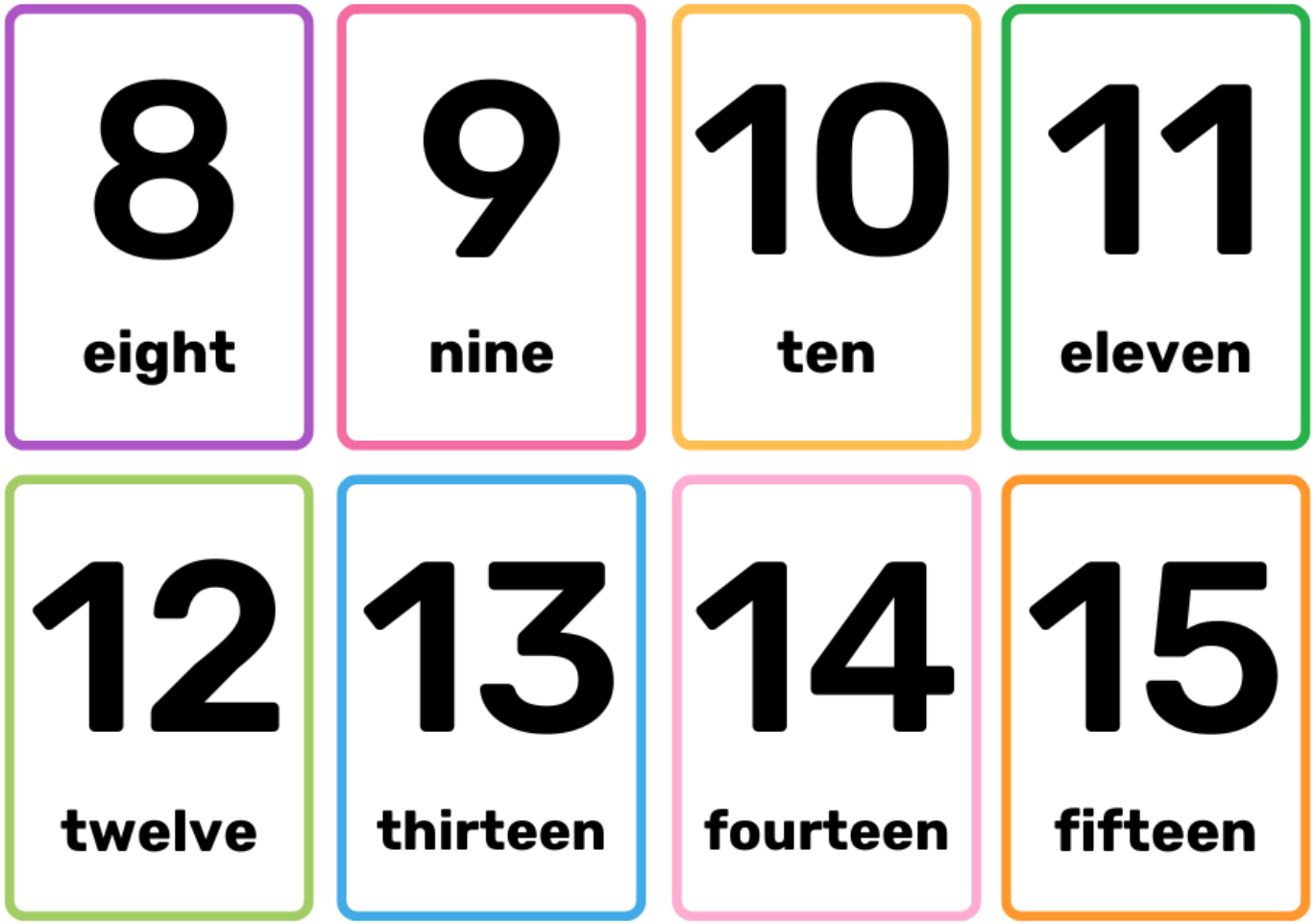

Figure: List 7

Result

Moment of Truth!

Try by Yourself by adding this **Line Account**

Figure: **Mathemagics** Line ID: **@025rlikw**

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Hamming(7,4) Codes

What is Hamming(7,4) Codes?

Hamming(7,4) Codes

What is Hamming(7,4) Codes?

In coding theory, **Hamming(7,4)** is a linear error-correcting code that encodes four bits of data into seven bits by adding three parity bits.

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Digital Communication Channel

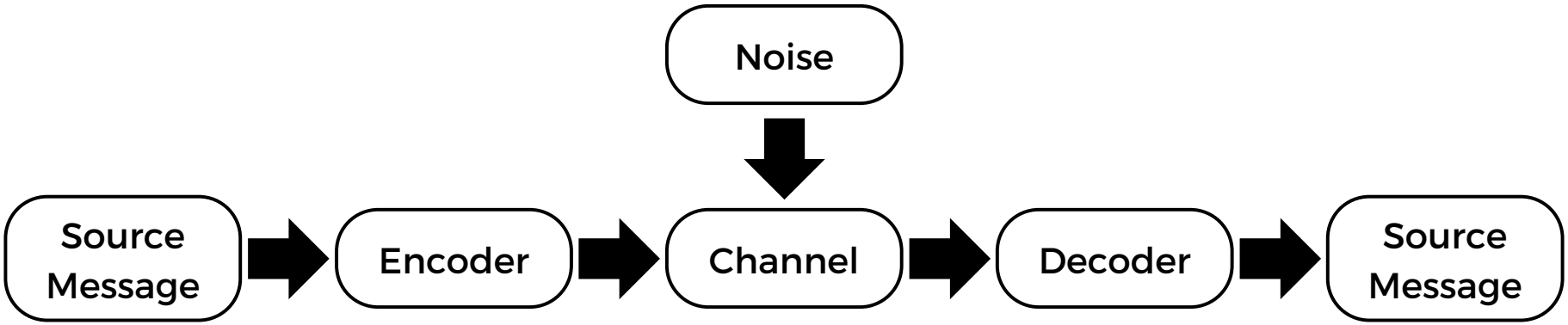

Figure: Flow of Digital Communication Channel

Digital Communication Channel with Example

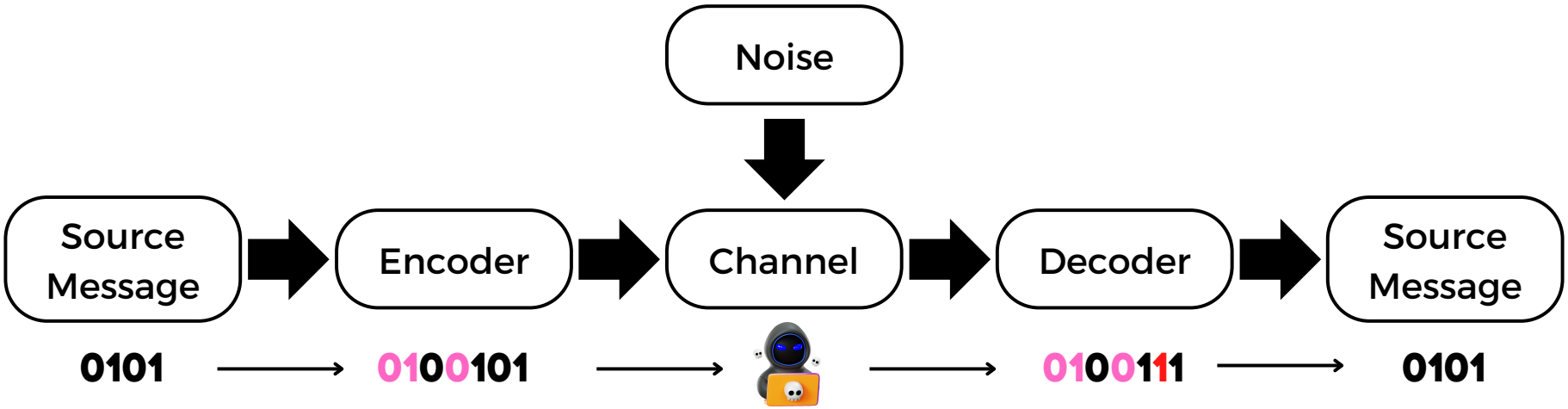

Figure: Flow of Digital Communication Channel with Example

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Generator Matrix and Parity-check Matrix

$$G^T := \begin{pmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

the code generator matrix G

$$H := \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

the parity-check matrix H

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Example

Suppose we want to transmit this data (0111) over a noisy communications channel (specifically, a binary symmetric channel). Let's call it as vector p. Then,

$$x = G^T p = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 0 \\ 3 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

This means that (0001111) would be transmitted instead of transmitting (0111). For example, suppose a bit error occurs on bit 6 while transmitted, it turns to be (0001101) as vector r.

Example (cont.)

Let's do the parity-check,

$$z = Hr = \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

This means that error occurred on bit $(110)_2$, which is bit 6.

Thank You!