Time Series : a sequence data from a natural or social process observed over time.

example: temperature measurement on every day at a school forms a time series

Definition 1: An ordered sequence of random variables $\{X_t\}$ is called Time Series where t is time index and $t \in T = \{ \cdots, -n, \cdots, -1, 0, 1, \cdots, n, \cdots \}$. The time series values which have been recorded are called a realization (or sample or observation) of the time series.

Objective : To use descriptive statistics method to analyze time-series data. Its plot is especially useful.
- ① has deterministic trend ?
- ② has deterministic seasonality
- ③ has dramatic change in its behaviour pattern
- ④ has outliers / anomalies

Application :
- • Forecast
- • signal detection
- • estimation $\Big\}$ signal processing
- • clustering
- • classification $\Big\}$ machine learning, data mining, pattern recognition
- • anomaly detection

TS Stationarity

Definition 2: (1) The mean function (1st moment) of a T.S.

$$\mu_t = E(X_t)$$

i.e. the expected value of the T.S. at time point t.

(2) The Variance function (2nd moment) of a T.S.

$$\sigma_t^2 = Var(X_t) = E[(X_t - \mu_t)^2]$$

Definition 3: (1) The Autocovariance function of a T.S.

$$\gamma(s,t) = Cov(X_s, X_t) = E[(X_s - \mu_s)(X_t - \mu_t)]$$

(2) The Autocorrelation function of a T.S.

$$\rho(s,t) = Corr(X_s, X_t) = \frac{\gamma(s,t)}{\sigma_s \sigma_t}$$

For any integer s and t,

$$\gamma(t,t) = \sigma_t^2 \;;\; \gamma(s,t) = \gamma(t,s) \;;\; |\gamma(s,t)| \leq \sigma_s \sigma_t$$

$$\rho(t,t) \equiv 1 \;;\; \rho(s,t) = \rho(t,s) \;;\; |\rho(s,t)| \leq 1$$

✳ Stationarity

Definition 4: A T.S. $\{X_t\}$ is <mark>strictly stationary</mark> if $\{X_1, \ldots, X_n\}$ and $\{X_{1+k}, \ldots, X_{n+k}\}$ possess the same joint distribution for any integer $n \geq 1$ and any integer k.

Definition 5: A T.S. $\{X_t\}$ is weakly stationary if

(1) $E(X_t) = \mu$ is a constant

(2) for any t, $E(X_t^2) < \infty$

and $Cov(X_t, X_{t+k}) = \gamma(k)$ is independent of t