

Waveguide Selection

Lumos

The electrodynamics of a waveguide is readily available in any standard electrodynamics textbook, such as *Introduction to Electrodynamics* by *David J. Griffiths*. The wavenumber k for a rectangular waveguide is given by:

$$k = \sqrt{\left(\frac{\omega^2}{c^2}\right) - \left(\frac{m^2\pi^2}{a^2} + \frac{n^2\pi^2}{b^2}\right)}$$

where:

- k is the wavenumber,
- ω is the angular frequency,
- c is the speed of light in vacuum,
- a and b are the dimensions of the waveguide,
- m and n are the mode indices (with $m = 1, 2, 3, \dots$ and $n = 1, 2, 3, \dots$).

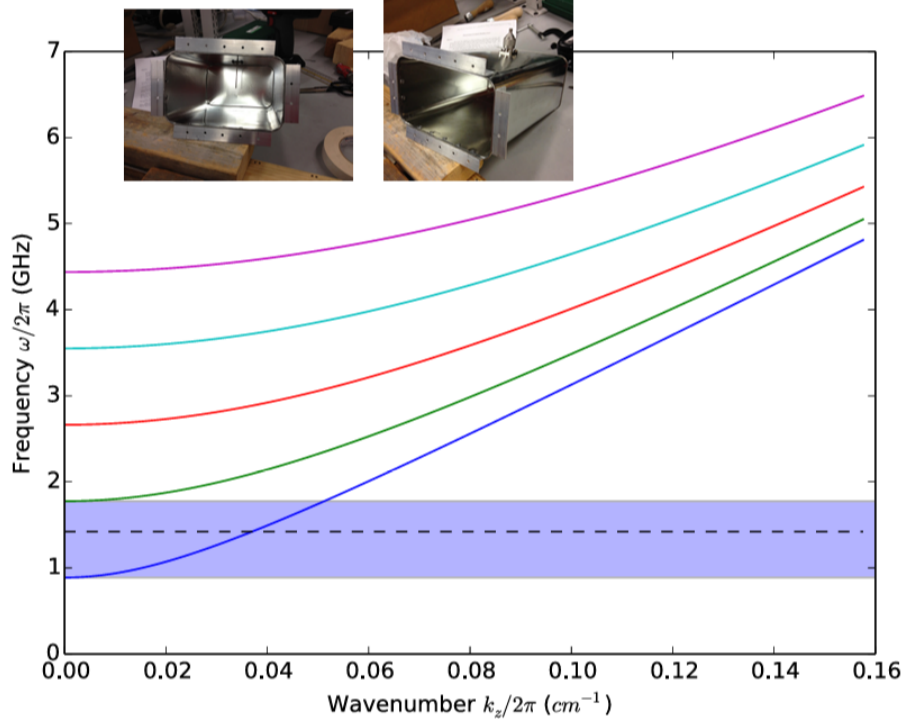
The cutoff frequency f_c for a rectangular waveguide is:

$$f_c = \frac{c}{2} \sqrt{\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)}$$

where:

- f_c is the cutoff frequency,
- m and n are the mode indices,
- a and b are the dimensions of the waveguide,
- c is the speed of light in vacuum.

To achieve the single-mode operation of the waveguide, the dimensions were chosen as $a = 16.9$ cm and $b = 10.5$ cm.



The figure shows the dispersion relations for the first five transverse electric (TE) modes of the rectangular waveguide used for our system. TE modes 1,2,3,4,5 correspond to blue, green, red, cyan, and purple solid lines, respectively. The dotted black line shows the center of the frequency band of interest, 1.42 GHz. The intersection of this line with the solid blue line shows the operating point of the waveguide corresponding to a guided wavelength of approximately 27 cm. The output feed should thus be selected as one-fourth of this or 6.75 cm from the back wall of the waveguide. The blue band corresponds to the range of input frequencies to the antenna which results in the single-mode operation of the guide.

The plot was taken from the referenced poster.¹

Operating a waveguide in a single mode, rather than allowing multiple modes, is crucial for several reasons, particularly in the context of high-frequency applications like communication systems and signal processing:

- **Minimized Signal Distortion:** When multiple modes propagate simultaneously in a waveguide, they can travel at different speeds, leading to signal distortion and dispersion. This can degrade the signal quality, particularly for high-speed or high-frequency applications.
- **Controlled Propagation:** In a single-mode waveguide, only one propagation mode exists, which ensures a controlled and predictable transmission of signals. This is critical for maintaining signal integrity over long distances.

¹Nimesh Patel, *Design and Development of a Horn Antenna*, AAS Poster, available at <https://lweb.cfa.harvard.edu/~npatel/hornAntennaAASposterPDF2.pdf>.

- **Reduced Crosstalk:** In multi-mode waveguides, multiple modes can interfere with each other, leading to crosstalk. This unwanted interaction can introduce noise and reduce the efficiency of the system. A single-mode waveguide eliminates this issue.
- **Efficient Power Transmission:** In single-mode operation, the energy is confined to a single mode, leading to more efficient power transfer and lower losses compared to multi-mode waveguides where energy is spread across multiple modes.
- **Avoiding Mode Coupling:** In multi-mode systems, energy from one mode can be coupled to other modes, leading to unpredictability in the signal's behavior. Single-mode operation prevents this coupling, ensuring stable and reliable transmission.
- **Design Simplicity:** It is easier to design systems and components for single-mode waveguides because their behavior is more predictable and straightforward. This leads to simpler and more efficient designs, especially in systems requiring precision, such as fiber optics or microwave circuits.

Thus, for applications requiring high performance and reliability, such as in optical fibers, communication systems, and high-frequency radar, single-mode operation is essential.