TOKENOMICS OF THE NOETHER PROTOCOL

INTRODUCTION

This document gives a detailed explanation about how the NOETH token is produced, distributed within their users, and how one can spend tokens to buy computing power within the system.

1. THE DISTRIBUTION OF THE TOKEN BY EPOCHS

The token is distributed in epochs which will coincide with the ones from Cardano blockchain by design plus an initial emission when the project is launched. We then enumerate the epochs $n = 1, 2, \ldots$ with natural numbers, where epoch 1 is exactly the one where the project is launched. Initially, there are $\kappa_0 = 10000$ tokens distributed before the automatic emission system takes place. These are distributed in the following way:

- 4000 tokens are given to scientific institutions, including universities, research centers, etc.
- 3000 tokens are given in an *airdrop* to early adopters that follow the project in social media.
- 2000 tokens for developers.
- 500 tokens are saved as treasury.
- 500 tokens for future collaborations.

Afterwards, for the epoch $n \in \mathbb{N}$ we distribute a certain quantity of tokens κ_n. Hence we have a function $\kappa \colon \mathbb{N} \longrightarrow \mathbb{R}$ called *the emission function*. Clearly κ depends of the epoch n, but it also depends of the demand of the token. To measure the demand of the token we need the following definition.

Definition 1.1. For each epoch n, we define the *heat of the network* h_n as how many users participated giving their computing power in this epoch.

Now, let κ_n is defined recursively as follows:

$$\kappa_{n+1} = \kappa_n \times \mathcal{H}_n + \frac{100 \times (1 + h_n)}{n},$$

where

$$\mathcal{H}_n = \begin{cases} \frac{1}{2} & \text{if } 72 \mid n \\ 1 & \text{otherwise.} \end{cases}$$

The number 72 is because that is the number of Cardano epochs needed to cover one year. Colloquially, this means that there is a one per year halving implementation on the emission rate. The tokens κ_n are distributed between the users as follows:

- 50% for processing power lending users.

- 20% tokens are given to scientific institutions, including universities, research centers, etc.
- 20% for developers.
- 5% tokens are saved as treasury.
- 4% tokens for future collaborations.
- 1% for inner airdrops.

1.1. **Analytic properties of the Emission function.** In the emission function there is a variable that is not predictable, the heat of the network h_n. We can assume, without loss of generality, that the accessible population to the network will eventually reach an equilibrium state for some time scale. This means that at some point the size of the system will remain constant and therefore it will behave as a closed dynamical system, such as [KM, PS, V]. This scenario will serve ass an upper bound limit for our model asymptotic analysis. At the equilibrium state we can assume that $h_n = h$ is constant. The emission function has a halving term $\mathcal{H}_n$. We show first what happens if we ignore this term and why is it necessary. Without this term and in the equilibrium state the emission rule is

$$\kappa_{n+1} = \kappa_n + \frac{100 \times (1+h)}{n},$$

In this context, we can solve for κ_n exactly and we get

$$\kappa_n = 100 \times (1+h)\mathbf{H}_n,$$

where $\mathbf{H}_n$ is the n-th Harmonic number. From classical theory we know that

$$\mathbf{H}_n \approx \ln n + \gamma + \frac{1}{2n} - \sum_{k=1}^{\infty} \frac{B_{2k}}{2kn^{2k}} \approx \ln n + \gamma + \frac{1}{2n} + \mathcal{O}\left(\frac{1}{n^2}\right),$$

where γ is the Euler-Mascheroni constant ($\gamma \approx 0.57721\ldots$) and B_k are the Bernoulli numbers. We can see that $\mathbf{H}_n \xrightarrow[n\to\infty]{} \infty$, but $\mathbf{H}_n - \mathbf{H}_{n-1} \xrightarrow[n\to\infty]{} 0$. So there is no upper limit to the grow of the emission function, but it does in a way that it rate of change is slower as time passes. The problem appears when we consider the total tokens in circulation. If we call this $\mathbf{K}_n$ we get that

$$\mathbf{K}_n = \sum_{j=1}^{n} \kappa_j \approx n \ln n.$$

Hence $\mathbf{K}_n$ behaves as a linear function with a logarithmic modulation. This is clearly inflationary, hence we need to change this simple model to make it deflationary. This is achieved with the $\mathcal{H}_n$ term that models a halving. In the same stationary approximation as before, and since $\frac{100\times(1+h_n)}{n} \xrightarrow[n\to\infty]{} 0$ the emission equation is

$$\kappa_{n+1} \approx \kappa_n \times \mathcal{H}_n,$$

therefore $\kappa_n \approx C(1 - 2^{\lfloor \frac{n}{72} \rfloor})$, for some constant C. Hence there is a theoretical maximum supply in this model, but the emission is always positive. This guarantee a deflationary token in the long term, even if locally behaves as inflationary in the initial stages to stimulate the spending.

1.2. Some simulations of the Emission function and quantitative overview.

2. A credit system for measuring computing power

How to spend the token

References

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