EE402 HW2

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Q1

1)

a) $q_i = q_1 + q_2 + q_3$ $\qquad q_o = K\,w(t)$ where $K = \dfrac{q_{o,max} - q_{o,min}}{w_{max} - w_{min}}$

$$\underbrace{\frac{dw}{dt}}_{\text{accumulation}} = q_i(t) - q_o(t) \implies \boxed{\frac{dq_o(t)}{dt}\frac{1}{K} = q_i(t) - q_o(t)}$$

in s domain: $\boxed{\dfrac{Q_o(s)}{Q_i(s)} = \dfrac{K}{s+K}}$ assuming zero initial conditions.

b) from part a)

$\dfrac{Q_o(s)}{Q_i(s)} = \dfrac{K}{s+K}$ $\qquad q_i = u(t) \xrightarrow{\mathcal{L}\{\}} Q_i(s) = \dfrac{1}{s}$

$\implies Q_o(s) = \dfrac{K}{s(s+K)}$, taking the inverse Laplace transform:

$Q_o(s) = \dfrac{C_1}{s} + \dfrac{C_2}{s+K}$ $\qquad C_2 = \lim\limits_{s\to -K} (s+K)\,Q_o(s) = -1$

$C_1 = \lim\limits_{s\to 0} s\,Q_o(s) = 1$

$\implies Q_o(s) = \dfrac{+1}{s} - \dfrac{1}{s+K} \xrightarrow{\mathcal{L}^{-1}\{\}} \boxed{(1 - e^{-Kt})\,u(t)}$

$\implies$

c) $\frac{dw}{dt} = q_i(t) - q_o(t) \implies sW(s) - w_{min} = Q_i(s) - KW(s)$

$\implies W(s)(s+K) = Q_i(s) + w_{min}$, where $Q_i(s) = BKw_{max}\, u(t)$

$\implies W(s)(s+K) = \frac{BKw_{max}}{s} + w_{min}$

$\implies W(s) = \frac{sw_{min} + BKw_{max}}{(K+s)s} = -\frac{Bw_{max} - w_{min}}{K+s} + \frac{Bw_{max}}{s}$

Taking inverse Laplace Transform:

$w(t) = \left(Bw_{max} - (Bw_{max} - w_{min})e^{-Kt}\right) u(t)$

$5\tau = 24.5$ where $\tau = \frac{1}{K} \implies K = \frac{1}{24}$, $\tau = 24$

$\implies \frac{q_{o\gamma}}{w_\gamma} = \frac{1}{24} \implies w_\gamma = 24 q_{o\gamma}$

Q2)

a)

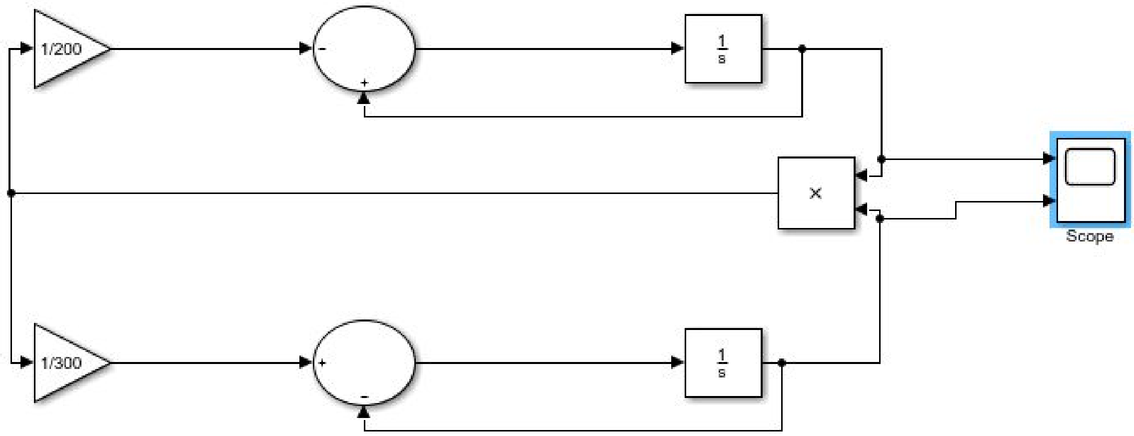

Figure 1: Model of the system

b)

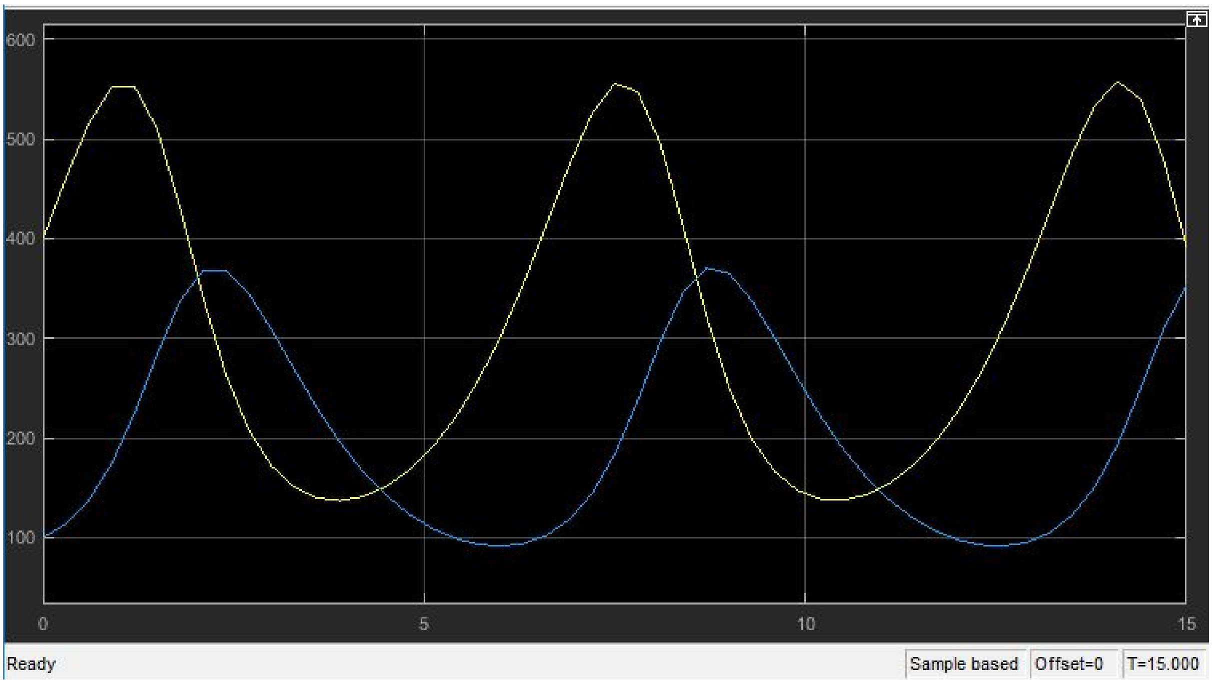

Figure 2: y1 and y2 variations with time

c) The populations exhibit a periodic sinusoidal waveform as a result of the the predator and prey relationship. The remarkability of the results is that the waveforms are periodic with time. As one falls and hits the trough it will eventually rise again and hit the crest. Thus, a sustainable environment for both the prey and the predator.

An example having similar properties can be Photosynthesis in a plant. The conversion of Carbon Dioxide to Oxygen in day and vice versa at night.

e)

(e) $\frac{dy_1}{\partial t} = \left(1 - \frac{y_2}{h_2}\right) y_1$

since equilibrium $\Rightarrow$ $\frac{dy_1}{\partial t} = 0$

$$1 - \frac{y_2}{h_2} = 0$$

$$\boxed{y_2 = h_2} \Rightarrow \boxed{y_2 = 200}$$

$\frac{dy_2}{\partial t} = -\left(1 - \frac{y_1}{h_1}\right) y_2$

since equilibrium $\Rightarrow$ $\frac{\partial y_2}{\partial t} = 0$

$$1 = \frac{y_1}{h_1}$$

$$\boxed{y_1 = h_1} \Rightarrow \boxed{y_1 = 300}$$

f)

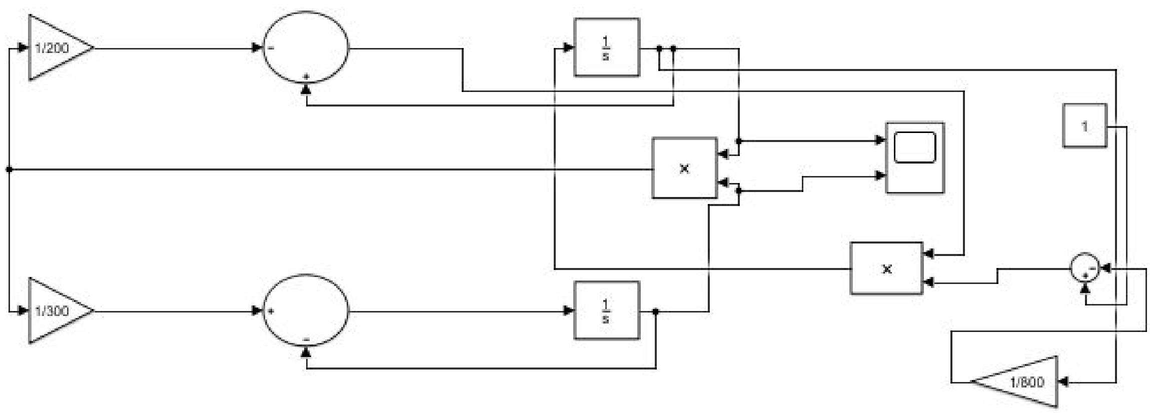

Figure 3: Modified Model of the System

g)

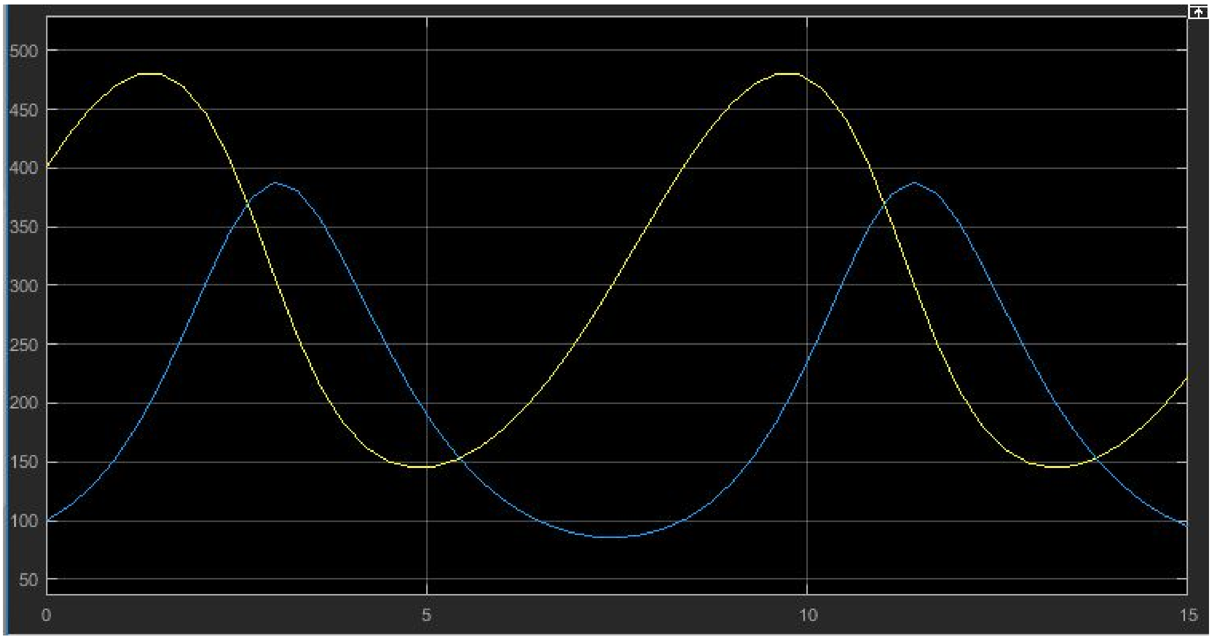

Figure 4: y1 and y2 variations with time

This solution too exhibits a sinusoidal periodic waveform. The length of the priod remains the same, however the amplitudes have decreased.