Discrete Math Homework 6

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Proof. Let $\mathcal{J}$, where$\mathcal{J}(p) = \mathbf{T}, \mathcal{J}(q) = \mathbf{F}, \mathcal{J}(r) = \mathbf{F}$, we have

$$[\![(p \land q) \to r]\!]_{\mathcal{J}} = \mathbf{T}, [\![(p \to r) \land (q \to r)]\!]_{\mathcal{J}} = \mathbf{F}$$

That means that they are not equivalent. □

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They are not logically equivalent.

Proof. Let $\mathcal{J}$ is a S-interpretation, where:

- $\mathcal{J}$'s domain is $\mathbb{Z}$,

- $\mathcal{J}(P)(x) = \mathbf{T}$ iff. x is an even number,

- $\mathcal{J}(Q)(x) = \mathbf{T}$ iff. x is an odd number.

Then for all $a \in \mathbb{Z}$, $[\![\forall x(P(x) \to Q(x))]\!]_{\mathcal{J}} = \mathbf{T}$ iff.

$$\mathcal{J}(P)(a) \to \mathcal{J}(Q)(a) = \mathbf{T}$$

(for all $a \in \mathbb{Z}$).
Let $a = 2$, $\mathcal{J}(P)(a) = \mathbf{T}$, and $\mathcal{J}(Q)(a) = \mathbf{F}$, which means that $[\![\forall x(P(x) \to Q(x))]\!]_{\mathcal{J}} = \mathbf{F}$.
However, $[\![\forall x(P(x))]\!]_{\mathcal{J}} = \mathcal{J}(P)(a) = \mathbf{F}$, $[\![\forall x(Q(x))]\!]_{\mathcal{J}} = \mathcal{J}(Q)(a) = \mathbf{T}$, $(a = 2)$.
So $[\![\forall x(P(x)) \to \forall x(Q(x))]\!]_{\mathcal{J}} = \mathbf{T} \neq [\![\forall x(P(x) \to Q(x))]\!]_{\mathcal{J}}$.
So they are not equivalent. □

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Proof. For all $\mathcal{J}$ is a S-interpretation, $\mathcal{J}$'s domain is A.
Then $[\![\exists x(P(x) \to \forall y P(y))]\!]_{\mathcal{J}} = \mathbf{T}$ if forall $a \in A$, $\mathcal{J}' = \mathcal{J}[x \mapsto a][y \mapsto a]$

$$[\![(P(x) \to QPy))]\!]_{\mathcal{J}'} = \mathcal{J}'(P)(a) \to \mathcal{J}'(P)(a) = \mathcal{J}(P)(a) \to \mathcal{J}(P)(a) = \mathbf{T}$$

If $\mathcal{J}(P)(a) = \mathbf{T}$, then $\mathcal{J}(P)(a) \to \mathcal{J}(P)(a) = \mathbf{T}$.
If $\mathcal{J}(P)(a) = \mathbf{F}$, then $\mathcal{J}(P)(a) \to \mathcal{J}(P)(a) = \mathbf{T}$.
So $\exists x(P(x) \to \forall y P(y))$ is true on any interpretation. □