Discrete Math Homework 2

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1 Answer_1

a)

p	q	$\neg p$	$\neg q$	$\neg p \vee q$	$\neg q \vee p$	$(\neg p \vee q) \wedge (\neg q \vee p)$
T	T	F	F	T	T	T
T	F	F	T	F	T	F
F	T	T	F	T	F	F
F	F	T	T	T	T	T

b)

p	q	$\neg p$	$\neg q$	$\neg p \vee q$	$\neg q \vee p$	$(\neg p \vee q) \vee (\neg q \vee p)$
T	T	F	F	T	T	T
T	F	F	T	F	T	T
F	T	T	F	T	F	T
F	F	T	T	T	T	T

c)

p	q	r	$q \vee r$	$p \wedge (q \vee r)$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	F	F
F	T	T	T	F
F	T	F	T	F
F	F	T	T	F
F	F	F	F	F

d)

p	q	r	$p \wedge q$	$p \wedge r$	$(p \wedge q) \vee (p \wedge r)$
T	T	T	T	T	T
T	T	F	T	F	T
T	F	T	F	T	T
T	F	F	F	F	F
F	T	T	F	F	F
F	T	F	F	F	F
F	F	T	F	F	F
F	F	F	F	F	F

e)

p	q	$p \wedge q$	$\neg(p \wedge q)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

f)

p	q	$\neg p$	$\neg q$	$\neg p \vee \neg q$
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

g)

p	q	$p \wedge q$	$\neg p$	$\neg q$	$\neg p \wedge \neg q$	$(p \wedge q) \vee (\neg p \wedge \neg q)$
T	T	T	F	F	F	T
T	F	F	F	T	F	F
F	T	F	T	F	F	F
F	F	F	T	T	T	T

Obviously, $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$, $\neg(p \wedge q) \equiv \neg p \vee \neg q$

2 Answer_2

Proof. Writing the truth table:

p	q	r	$\neg p$	$\neg q$	$p \vee q$	$\neg p \vee r$	$\neg q \vee r$
T	T	T	F	F	T	T	T
T	T	F	F	F	T	F	F
T	F	T	F	T	T	T	T
T	F	F	F	T	T	F	T
F	T	T	T	F	T	T	T
F	T	F	T	F	T	T	F
F	F	T	T	T	F	T	T
F	F	F	T	T	F	T	T

Noting that with under truth assignment $\mathcal{J}$ in line 1 and line 5, for any $\phi \in \Phi$, $[\![\phi]\!]_{\mathcal{J}} = \mathrm{T}$. and $\mathcal{J}(r) = \mathrm{T}$. That means $\Phi \models r$. □

3 Answer_3

a) *Proof.* $\phi \models \psi$ means that under any trhth assignmeny $\mathcal{J}$, where $[\![\phi]\!]_{\mathcal{J}} = \mathrm{T}$,
we have $[\![\psi]\!]_{\mathcal{J}} = \mathrm{T}$.
Assuming a truth $\mathcal{J}$ s.t. $[\![\phi]\!]_{\mathcal{J}} = \mathrm{T}$, then $[\![\psi]\!]_{\mathcal{J}} = \mathrm{T}$, so

$$[\![\phi \wedge \psi]\!]_{\mathcal{J}} = [\![\wedge]\!]_{\mathcal{J}}([\![\phi]\!]_{\mathcal{J}}, [\![\psi]\!]_{\mathcal{J}}) = \mathrm{T} = [\![\phi]\!]_{\mathcal{J}}$$

Assuming $[\![\phi]\!]_{\mathcal{J}} = \mathrm{F}$, then

$$[\![\phi \wedge \psi]\!]_{\mathcal{J}} = [\![\wedge]\!]_{\mathcal{J}}([\![\phi]\!]_{\mathcal{J}}, [\![\psi]\!]_{\mathcal{J}}) = \mathrm{F} = [\![\phi]\!]_{\mathcal{J}}$$

That means $\phi \wedge \psi \equiv \phi$.
Similarly, Assuming $[\![\phi]\!]_{\mathcal{J}} = \mathrm{T}$, then $[\![\psi]\!]_{\mathcal{J}} = \mathrm{T}$

$$[\![\phi \vee \psi]\!]_{\mathcal{J}} = [\![\vee]\!]_{\mathcal{J}}([\![\phi]\!]_{\mathcal{J}}, [\![\psi]\!]_{\mathcal{J}}) = \mathrm{T} = [\![\psi]\!]_{\mathcal{J}}$$

Assuming $[\![\phi]\!]_{\mathcal{J}} = \mathrm{F}$, then

$$[\![\phi \vee \psi]\!]_{\mathcal{J}} = [\![\vee]\!]_{\mathcal{J}}([\![\phi]\!]_{\mathcal{J}}, [\![\psi]\!]_{\mathcal{J}}) = [\![\psi]\!]_{\mathcal{J}}$$

That means $\phi \vee \psi = \psi$. □

b) We will now prove the absorption rule.

Proof. For all $\mathcal{J}$ s.t. $[\![\phi \wedge \psi]\!]_{\mathcal{J}} = \mathrm{T}$, we know that $[\![\phi]\!]_{\mathcal{J}} = \mathrm{T}$, which means that $\phi \wedge \psi \models \phi$. So

$$\phi \vee (\phi \wedge \psi) \equiv (\phi \wedge \psi) \vee \phi \equiv \phi$$

□

Proof. For all $\mathcal{J}$ s.t. $[\![\phi]\!]_{\mathcal{J}} = \mathrm{T}$, we know that $[\![\phi \vee \psi]\!]_{\mathcal{J}} = \mathrm{T}$, which means that $\phi \models \phi \vee \psi$. So

$$\phi \wedge (\phi \vee \psi) \equiv \phi$$

□