作业八

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1

$$\|\boldsymbol{x}\|_A = \sqrt{\boldsymbol{x}^T A \boldsymbol{x}} = \sqrt{(1, -1, -1, 1)\begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix}} = 2$$

$$B^T \boldsymbol{x} = \begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix}$$
$$\|\boldsymbol{x}\|_C = \sqrt{\boldsymbol{x}^T B B^T \boldsymbol{x}} = \sqrt{26}$$

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证明. (1) 显然

$$\|\boldsymbol{x}\|_\infty = \max_{1\le i\le n} |x_i| \le \sqrt{\max_{1\le i\le n} x_i^2} \le \sqrt{\sum_{i=1}^n x_i^2} = \|\boldsymbol{x}\|_2$$
$$\|\boldsymbol{x}\|_2 = \sqrt{\sum_{i=1}^n x_i^2} \le \sqrt{n \max_{1\le i\le n} x_i^2} = \sqrt{n}\,\|\boldsymbol{x}\|_\infty$$

(2) 显然

$$\|\boldsymbol{x}\|_\infty = \max_{1\le i\le n} |x_i| \le \sum_{i=1}^n |x_i| \le n \max_{1\le i\le n} |x_i| = n\,\|\boldsymbol{x}\|_\infty$$

□

3

$$\|A\|_1 = 7$$
$$\|A\|_\infty = 7$$
$$\|A\|_F = \sqrt{73}$$

而

$$A^T A = \begin{pmatrix} 25 & 5 & 5 \\ 5 & 11 & 7 \\ 5 & 7 & 37 \end{pmatrix}$$

其最大特征值为 $\lambda = 36$. 故 $\|A\|_2 = 6$

4

$$A = \begin{pmatrix} 0.6 & 0.6 \\ 0.6 & -0.6 \end{pmatrix}$$
$$B = \begin{pmatrix} 0.5 & 0.5 & 0.5 \\ 0.5 & -0.5 & 0.5 \\ 0.5 & 0.5 & -0.5 \end{pmatrix}$$

5

$$\frac{\mathrm{d}f}{\mathrm{d}\boldsymbol{x}} = \cos\ln\boldsymbol{x}^T\boldsymbol{x} \cdot \frac{1}{\boldsymbol{x}^T\boldsymbol{x}}\frac{\mathrm{d}\boldsymbol{x}^T\boldsymbol{x}}{\mathrm{d}\boldsymbol{x}} = \cos\ln\boldsymbol{x}^T\boldsymbol{x} \cdot \frac{1}{\boldsymbol{x}^T\boldsymbol{x}} \cdot 2\boldsymbol{x}$$

6

$$\frac{\mathrm{d}f}{\mathrm{d}X} = \boldsymbol{a}\boldsymbol{b}^T$$

7

$$\mathrm{d}\mathrm{tr}(AXB) = \mathrm{tr}(A\mathrm{d}XB) = \mathrm{tr}(BA\mathrm{d}X)$$

故 $\nabla_X f = B^T A^T$

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由于

$$\mathrm{d}\mathrm{tr}(XAX^TB) = \mathrm{tr}(\mathrm{d}XAX^TB + XA\mathrm{d}X^TB) = \mathrm{tr}(AX^TB\mathrm{d}X + A^TX^TB^T\mathrm{d}X)$$

故 $\nabla_X f = B^TXA^T + BXA$

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$$\mathrm{d}f = \mathrm{tr}(Xba^T\mathrm{d}X^T + X^T\mathrm{d}Xba^T)$$

故 $\dfrac{\mathrm{d}f}{\mathrm{d}X}X(ab^T + ba^T)$

10

$$\varepsilon = \mathrm{tr}((A - CB)^T(A - CB))$$

同理求导得 $\dfrac{\partial \varepsilon}{\partial C} = -2(A - CB)B^T$, $\dfrac{\partial \varepsilon}{\partial B} = -2C^T(A - CB)$

11

$$\frac{\mathrm{d}AX}{\mathrm{d}X} = E \otimes A^T$$
$$\frac{\mathrm{d}XA}{\mathrm{d}X} = A \otimes E$$

12

证明.

$$XX^{-1} = E \Rightarrow \mathrm{d}X \cdot X^{-1} + X \cdot \mathrm{d}X^{-1} \Rightarrow \mathrm{d}(X^{-1}) = -X^{-1}\mathrm{d}XX^{-1}$$

梯度矩阵为 $-\left(X \otimes X^{-1}\right)$ □