

Discrete Math Homework 5

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1

Proof. For all \mathcal{J} is a \mathcal{S} -interpretation, satisfies $\llbracket \exists x \forall y (R(x, y)) \rrbracket_{\mathcal{J}} = \mathbf{T}$. So exists an a in \mathcal{J} 's domain, for all b in \mathcal{J} 's domain, $\llbracket R(x, y) \rrbracket_{\mathcal{J}[x \mapsto a][y \mapsto b]} = \mathbf{T}$.

Then let $a = b$, we have $\llbracket R(x, y) \rrbracket_{\mathcal{J}[x \mapsto a][y \mapsto a]} = \mathcal{J}(R)(a, a) = \mathbf{T}$. That means that $\llbracket R(x, x) \rrbracket_{\mathcal{J}[x \mapsto a]} = \mathbf{T}$. So for all \mathcal{J} , $\llbracket \exists x (R(x, x)) \rrbracket_{\mathcal{J}} = \mathbf{T}$, i.e.

$$\exists x \forall y (R(x, y)) \models \exists x (R(x, x))$$

□