大物作业二

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1 - 12

连接 OQ, 有 $\angle QOP' = 2\angle QPP' = 2\theta$. 故物体在做角速度为 2ω 的匀速圆周运动. $a = (2\omega)^2 r = 4\omega^2 r$

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由图像易得 $a_r = g\cos\dfrac{\pi}{3} = \dfrac{1}{2}g$

因此由 $a_n = \dfrac{v^2}{\rho} = \dfrac{\sqrt{3}}{2}g$ 有 $\rho = \dfrac{2\sqrt{3}v_0^2}{3g}$

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由图，物体速度与 x 轴夹角为 $\dfrac{\pi}{3}$ 斜向下. y 方向，有 $v_y = \sqrt{3}v_0 = 10\sqrt{3}\text{m/s}$. 故 $\delta y = \dfrac{v_y^2 - 0}{2g} = 15\text{m}$

此时 $v = 2v_0 = 20\text{m/s}$, $\rho = \dfrac{v^2}{a_n} = 80\text{m}$

1 - 22

(1) $v = \dfrac{\mathrm{d}y}{\mathrm{d}t} = 6t \text{ m/s}$; $a = \dfrac{\mathrm{d}v}{\mathrm{d}t} = 6\text{m/s}^2$

(2) $v_P = \dfrac{1}{2}v = 3t \text{ m/s}$; $a_n = v_P^2/0.5R = 180t^2 \text{ m/s}^2$, $a_t = \dfrac{1}{2}a = 3 \text{ m/s}^2$

1 - 23

(1) 在位置 x 处，$v_y = \dfrac{\mathrm{d}y}{\mathrm{d}t} = -2x\dfrac{\mathrm{d}x}{\mathrm{d}t} = -2xv$. 故速度大小为 $v = \sqrt{v_x^2 + v_y^2} = \sqrt{1+4x^2}v$, 与 x 轴正方向夹角为 $\theta = \arccos\dfrac{1}{\sqrt{1+4x^2}}$

(2) $a = a_y = \dfrac{\mathrm{d}v_y}{\mathrm{d}t} = -2v\dfrac{\mathrm{d}x}{\mathrm{d}t} = -2v^2$. 故 $a_n = a\cos\theta = \dfrac{-2v^2}{\sqrt{1+4x^2}}$, $a_t = \sqrt{a_y^2 - a_n^2} = \dfrac{-4vx}{\sqrt{1+4x^2}}$

1 - 26

由几何图形和正弦定理，有

$$\frac{v_0}{\sin 45^\circ} = \frac{v}{\sin 105^\circ}$$

故 $v_0 = 35\left(\sqrt{3}-1\right)\ \mathrm{m/s}$

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正方形边长为 $a = \dfrac{vT}{4}$. 后来时，迎风时间 $t_1 = \dfrac{T}{4(1-k)}$, 顺风时间 $t_2 = \dfrac{T}{4(1+k)}$, 两边时间各为 $t_3 = \dfrac{T}{4\sqrt{1-k^2}}$, 因此时间差为 $T\left(\dfrac{1}{4(1-k)} + \dfrac{1}{4(1+k)} + \dfrac{1}{2\sqrt{1-k^2}} - 1\right)$

1 - 31

设离出发河岸的距离为 x, 有水速 $u(x) = -\dfrac{4u_0}{l^2}$, 又实际速度 $v_x = v_0 \sin\dfrac{\pi}{4}, v_y = v_0\cos\dfrac{\pi}{4} + u(x)$. 故时间 t 时，

$$\begin{aligned} x &= \frac{\sqrt{2}}{2}v_0 t \\ y &= \frac{\sqrt{2}}{2}v_0 t - \frac{\sqrt{2}v_0u_0t}{l} + \frac{2v_0^2u_0t^3}{3l^2} \end{aligned}$$

消去 t 得轨迹

$$y = x - \frac{2\sqrt{2}u_0}{v_0 l}x^2 + \frac{4\sqrt{2}u_0}{3v_0l^2}x^3$$

令 $x = l$, 有 $y = \left(1 - \dfrac{2\sqrt{2}u_0}{3v_0}\right)l$

1 - 33

由速度分解，有 $v_{实际} = v\tan\theta$.

P 相对圆柱的切向、法向加速度分别为 a_t, a_n. 有 $a_n = \dfrac{(v/\cos\theta)^2}{R} = \dfrac{v^2}{R\cos^2\theta}$. 又由于实际加速度为竖直方向，有

$$a_n\sin\theta + a_t\cos\theta = a$$

$$a_{实际} = a_t\sin\theta - a_n\cos\theta$$

解得 $a_{实际} = a\tan\theta - \dfrac{v^2}{R\cos^3\theta}$