Discrete Math Homework 7

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a) $\neg\exists z\forall y\forall x T(x,y,z) = \forall z\exists y\exists x\neg T(x,y,z)$

b) $\neg\left((\exists x\,\exists y\,P(x,y)) \wedge (\forall x\,\forall y\,Q(x,y))\right) = \neg(\exists x\,\exists y\,P(x,y)) \vee \neg(\forall x\,\forall y\,Q(x,y)) = (\forall x\,\forall y\,\neg P(x,y)) \vee (\exists x\,\exists y\,\neg Q(x,y))$

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Proof. Proof by contradiction.
Assuming there exists a truth assignment $\mathcal{J}$ that

$$\begin{cases} [\![\phi[x \mapsto t]]\!]_{\mathcal{J}} = \mathbf{T} \\ [\![\forall x(\phi \to \psi)]\!]_{\mathcal{J}} = \mathbf{T} \\ [\![\psi[x \mapsto t]]\!]_{\mathcal{J}} = \mathbf{F}. \end{cases}$$

So

$$[\![\phi[x \mapsto t] \to \psi[x \mapsto t]]\!]_{\mathcal{J}} = \mathbf{F}$$

i.e.

$$[\![\phi \to \psi]\!]_{\mathcal{J}[x \mapsto [\![t]\!]_{\mathcal{J}}]} = \mathbf{F}$$

So exist $a = [\![t]\!]_{\mathcal{J}} \in \mathcal{J}$'s domain, $[\![\phi \to \psi]\!]_{\mathcal{J}[x \mapsto a]} = \mathbf{F}$, i.e. $[\![\forall x(\phi \to \psi)]\!]_{\mathcal{J}} = \mathbf{F}$, which is contradictive to our assume.
So

$$\varphi[x \mapsto t], \quad \forall x(\varphi \to \psi) \models \psi[x \mapsto t].$$

$\square$