Discrete Math Homework 4

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2024.10.11

1 Answer_1

These are first order logic propositions: a), b), f), i), j).

2 Anewer_2

a), b), d), e), f).

3 Answer_3

a).

4 Answer_4

c), d), e)

5 Anewer_5

a) *Proof.* Let $\mathcal{J}' = \mathcal{J}_1[x \mapsto a][y \mapsto b]$
$[\![\forall x \exists y R(x, y)]\!]_{\mathcal{J}} = \mathbf{T}$ iff. for all $a \in \mathbb{N}$, exists $a + 1 \in \mathbb{N}$,

$$[\![R(x, y)]\!]_{\mathcal{J}_1[x \mapsto a][y \mapsto b]} = \mathcal{J}'(R)(\mathcal{J}'(x), \mathcal{J}'(y)) = \mathcal{J}'(a, a + 1)$$

i.e. $a < a + 1$ and it's obviously true. $\square$

b) *Proof.* Let $\mathcal{J}' = \mathcal{J}_2[y \mapsto 2]$,
Then $[\![\exists y R(x, y)]\!]_{\mathcal{J}_2} = \mathbf{F}$ iff. for all $b \in \mathbb{N}$,

$$\mathcal{J}'(R)(\mathcal{J}'(x), \mathcal{J}'(y)) = \mathcal{J}'(R)(0, b)$$

It's obvious that for all $b \in N$, $b \geqslant 0$. So $[\![\exists y R(x, y)]\!]_{\mathcal{J}_2} = \mathbf{F}$ $\square$

c) *Proof.* Noting that if we let $\mathcal{J}' = \mathcal{J}_3[x \mapsto 0]$,
The proposition $[\![\forall x \exists y R(x, y)]\!]_{\mathcal{J}'}$ is the same as the one in problem b).
So there exist an $a = 0$, s.t. $\mathcal{J}' = \mathcal{J}_3[x \mapsto a]$, $[\![\forall x \exists y R(x, y)]\!]_{\mathcal{J}'} = \mathbf{F}$
That indiates that $[\![\forall x \exists y R(x, y)]\!]_{\mathcal{J}_3} = \mathbf{F}$ $\square$