Exercise 01

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1 Answer_1

- It is below freezing and snowing: $p \wedge q$
- It is below freezing but not snowing: $p \wedge \neg q$
- It is not below freezing and it is not snowing: $\neg p \wedge \neg q$
- It is either snowing or below freezing (or both): $p \vee q$

2 Answer_2

2.1 a)

Proof. Let a truth assignment $\mathcal{J}$ s.t.

$$\mathcal{J}(p) = \mathrm{T}, \mathcal{J}(q) = \mathrm{T}, \mathcal{J}(r) = \mathrm{T}$$

Then

$$[\![(p \vee \neg q)]\!]_{\mathcal{J}} = \mathrm{T}, [\![(q \vee \neg r)]\!]_{\mathcal{J}} = \mathrm{T}, [\![(r \vee \neg p)]\!]_{\mathcal{J}} = \mathrm{T},$$

So

$$[\![(p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p)]\!]_{\mathcal{J}} = \mathrm{T}$$

That means the proposition is satisfiable. □

2.2 b)

Proof. Let a truth assignment $\mathcal{J}$ s.t.

$$\mathcal{J}(p) = \mathrm{T}, \mathcal{J}(q) = \mathrm{T}, \mathcal{J}(r) = \mathrm{F}$$

Then

$$[\![(p \vee \neg q)]\!]_{\mathcal{J}} = \mathrm{T}, [\![(q \vee \neg r)]\!]_{\mathcal{J}} = \mathrm{T}, [\![(r \vee \neg p)]\!]_{\mathcal{J}} = \mathrm{F},$$

So

$$[\![(p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p)]\!]_{\mathcal{J}} = \mathrm{F}$$

And

$$[\![\neg(p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p)]\!]_{\mathcal{J}} = \mathrm{T}$$

That means the proposition is satisfiable. □