Discrete Math Homework 4

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1 Answer_1

These are first order logic propositions: a), f), i), j).

2 Anewer_2

a), b), d), e), f).

3 Answer_3

a).

4 Answer_4

c), d), e)

5 Anewer_5

a) *Proof.* $[\![\forall x \exists y R(x,y)]\!]_{\mathcal{J}} = \mathbf{T}$ iff. for all $a \in \mathbb{N}$, exists $a+1 \in \mathbb{N}$, Let $\mathcal{J}' = \mathcal{J}_1[x \mapsto a][y \mapsto a+1]$

$$[\![R(x,y)]\!]_{\mathcal{J}_1[x \mapsto a][y \mapsto a+1]} = \mathcal{J}'(R)(\mathcal{J}'(x), \mathcal{J}'(y)) = \mathcal{J}'(R)(a, a+1)$$

i.e. $a < a+1$ and it's obviously true. □

b) *Proof.* $[\![\exists y R(x,y)]\!]_{\mathcal{J}_2} = \mathbf{T}$ iff. exists $b \in \mathbb{N}$,

$$[\![\exists y R(x,y)]\!]_{\mathcal{J}_2[y \mapsto b]} = \mathcal{J}_2[y \mapsto b](R)(\mathcal{J}_2[y \mapsto b](x), \mathcal{J}_2[y \mapsto b](y)) = \mathcal{J}_2[y \mapsto b](R)(0,b) = \mathbf{T}$$

But it's obvious that for all $b \in N$, $b \geqslant 0$, $\mathcal{J}_2[y \mapsto b](R)(0,b) = \mathbf{F}$. So $[\![\exists y R(x,y)]\!]_{\mathcal{J}_2} = \mathbf{F}$ □

c) *Proof.* To proof $[\![\forall x \exists y R(x,y)]\!]_{\mathcal{J}_3} = \mathbb{F}$, we only need to give a counterexample.
Noting that if we let $\mathcal{J}' = \mathcal{J}_3[x \mapsto 0]$,
The proposition $[\![\forall x \exists y R(x,y)]\!]_{\mathcal{J}'}$ is the same as the one in problem b).
So for all $a \in \mathbb{N}$, s.t. $\mathcal{J}' = \mathcal{J}_3[y \mapsto a]$,

$$[\![\forall x \exists y R(x,y)]\!]_{\mathcal{J}'} = \mathcal{J}'(R)(\mathcal{J}'(x), \mathcal{J}'(y)) = \mathcal{J}'(R)(0,a) = \mathbf{F}$$

That indicates that $[\![\forall x \exists y R(x,y)]\!]_{\mathcal{J}_3} = \mathbf{F}$ □