Assume $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\in T}, P)$ to be a filtered probability space, and X to be random variable on Ω. Then we have:

$$\mathbb{E}\left[\mathbb{E}\left[X|\mathcal{F}_i\right]|\mathcal{F}_j\right] = \mathbb{E}\left[X|\mathcal{F}_{i\wedge j}\right] \tag{1}$$

The law of iterated expectation says that the conditional expectation of a conditional expectation would equal to the conditional expectation based on the smaller information set.

Proof: If $i < j$, $\mathbb{E}\left[X|\mathcal{F}_i\right]$ itself is $\mathcal{F}_i$ measurable and therefore $\mathcal{F}_j$ measurable, hence $\mathbb{E}\left[\mathbb{E}\left[X|\mathcal{F}_i\right]|\mathcal{F}_j\right] = \mathbb{E}\left[X|\mathcal{F}_i\right]$. If $i \geq j$, we want to show that $\mathbb{E}\left[\mathbb{E}\left[X|\mathcal{F}_i\right]|\mathcal{F}_j\right] = \mathbb{E}\left[X|\mathcal{F}_j\right]$. Equivalently, $\forall A \in \mathcal{F}_j$, we want to show that $\int_A \mathbb{E}\left[X|\mathcal{F}_i\right] \mathrm{d}P = \int_A X \mathrm{d}P$, by definition of conditional expectation, $A \in \mathcal{F}_j \subset \mathcal{F}_i$, hence $\int_A \mathbb{E}\left[X|\mathcal{F}_i\right] \mathrm{d}P = \int_A X \mathrm{d}P$ which proves the result.

A interesting question would be to consider a general higher-order expectation case. Denote the information set of two agent i and j as $\mathcal{I}_i$ and $\mathcal{I}_j$ respectively. Consider agent i's expectation of agent j's expectation of random variable X, we have:

$$\mathbb{E}\left[\mathbb{E}\left[X|\mathcal{I}_j\right]|\mathcal{I}_i\right] = \mathbb{E}\left[X|\mathcal{I}_j \cap \mathcal{I}_i\right] \tag{2}$$

Proof: the statement is equivalent to $\forall A \in \mathcal{I}_j \cap \mathcal{I}_i, \int_A \mathbb{E}\left[\mathbb{E}\left[X|\mathcal{I}_j\right]|\mathcal{I}_i\right] \mathrm{d}P = \int_A X \mathrm{d}P$, since $A \in \mathcal{I}_i$ and $A \in \mathcal{I}_j$, using definition of expectation the equation holds.

This statement shows that higher order expectation equals to the conditional expectation made on common information. However, it's possible that the information set of agent j itself is uncertain.