$\boxed{№ 2}$

- $\|x_2\| \le \sqrt{m}\, \|x\|_\infty$

$$\|x_2\|^2 \le m\, \|x\|_\infty$$

$$\|$$

$$\sum_{i=1}^{m} x_i^2 \le m \cdot \max_i |x_i| \text{ - верно}$$

- $\|A\|_\infty \le \sqrt{n}\, \|A\|_2$

$$\frac{\|A\vec{x}\|_\infty}{\|\vec{x}\|_\infty} \le \sqrt{n}\, \frac{\|A\vec{x}\|_2}{\|\vec{x}\|_2} \Rightarrow; \quad \|A\vec{x}\| = \|\vec{y}\|$$

$$\Rightarrow \|\vec{x}\|_2^2\, \|\vec{y}\|_\infty^2 \le n\, \|\vec{y}\|_2^2\, \|\vec{x}\|_\infty^2$$

$$\|\vec{y}\|_\infty^2 \le \|\vec{y}\|_2^2 \text{ - верно}$$

$$\|\vec{x}\|_2^2 \le n\, \|\vec{x}\|_\infty^2 \text{ - верно по п. I}$$

№3

a) $A=\begin{pmatrix}3 & 0\\ 0 & -2\end{pmatrix}$ – т.к. матрица диагональная, то:

$$A = EAE = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}3 & 0\\ 0 & -2\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$

b) $A=\begin{pmatrix}0 & 2\\ 0 & 0\\ 0 & 0\end{pmatrix}$

$$AA^T = \begin{pmatrix}0 & 2\\ 0 & 0\\ 0 & 0\end{pmatrix}\begin{pmatrix}0 & 0 & 0\\ 2 & 0 & 0\end{pmatrix} = \begin{pmatrix}4 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix} \Rightarrow \lambda_1 = 4,\ \lambda_{2,3} = 0 \Rightarrow$$

$$\Rightarrow u_1 = \begin{pmatrix}1\\0\\0\end{pmatrix},\ u_2 = \begin{pmatrix}0\\1\\0\end{pmatrix},\ u_3 = \begin{pmatrix}0\\0\\1\end{pmatrix}$$

$$A^TA = \begin{pmatrix}0 & 0\\ 0 & 4\end{pmatrix} \Rightarrow \lambda_1 = 4,\ \lambda_2 = 0 \Rightarrow v_1 = \begin{pmatrix}1\\0\end{pmatrix},\ v_2 = \begin{pmatrix}0\\1\end{pmatrix}$$

$\sigma_1 = 2$; $\sigma_2 = 0$

$$\Rightarrow A = \begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}2 & 0\\ 0 & 0\\ 0 & 0\end{pmatrix}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$$

c)

$$A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

$$A^T A = A A^T = \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix} \Rightarrow \begin{array}{l} \lambda_1 = 0 \\ \lambda_2 = 4 \end{array} \Rightarrow u_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} -1 \\ 1 \end{pmatrix},$$

$$u_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} -1 \\ -1 \end{pmatrix}, \quad \sigma_1 = 2, \quad \sigma_2 = 0$$

$$\cancel{A = \frac{1}{2}\begin{pmatrix} -1 & -1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} -1 & 1 \\ -1 & -1 \end{pmatrix}}$$

$$A = \frac{1}{2}\begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix}$$