Machine Learning Nanodegree
Capstone Report

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1 Definition

1.1 Project Overview

Reinforcement learning is the area of machine learning concerned with the idea of learning by interacting with an environment (Sutton and Barto, 1998). It has a long and notable history in games. The checkers program written by Samuel (1959) was the first remarkable application of temporal difference learning, and also the first significant learning program of any kind. It had the principles of temporal difference learning decades before it was described and analyzed. However, it was another game where reinforcement learning reached its first big success, when TD-Gammon (Tesauro, 1995) reached world-class gameplay in Backgammon by training a neural network-based evaluation function through self-play.

Deep Learning (LeCun et al., 2015) is another branch of machine learning that allows computational models that are composed of multiple processing layers to learn representations of data with multiple levels of abstraction. Deep learning techniques have dramatically improved the state-of-the-art in areas such as speech recognition (Hinton et al., 2012), image recognition (Krizhevsky et al., 2012) and natural language processing (Colbert et al., 2011).

Reecently, there have been several breakthroughs when combining reinforcement learning and deep learning. Mni et al. (2015) used a convolutional neural network trained with a variant of Q-learning and learned to play Atari 2600 games at a human-level. Last year, one of the biggest challenges for artificial intelligence was solved, when Google's DeepMind (Silver et al., 2016) created AlphaGo to beat the world's greatest Go player. AlphaGo used deep neural networks trained by a combination of supervised learning from human expert games, and reinforcement learning from games of self-play.

Developing strong game-playing programs for classic two-player games (e.g. Chess, Checkers, Go) is important in two respects: first, for humans that play the games looking for an intellectual challenge, and second, for AI researchers that use the games as testbeds for artificial intelligence. In both cases, the task for game programmers and AI researchers of writing strong game AI is a hard and tedious task that requires hours of trial and error adjustments and human expertise. Therefore, when a strong game-playing algorithm is created to play a specific game, it is rarely useful for creating an algorithm to play another game, since the domain knowledge is non-transferable. For this reason, there is enormous value in using machine learning to learn to play these games without using any domain knowledge.

1.2 Problem Statement

In this project we use deep reinforcement learning to create an agent that learns to play the game of Connect 4 by playing games against itself. We define the machine learning problem as follows:

- **Task:** Playing Connect 4.
- **Performance:** Winning percentage when playing against other agents, and accuracy of the predicted outcomes on the UCI Connect 4 dataset.

- **Experience:** Games played against itself.
- **Target function:** $Q^{\pi} : \mathcal{S} \times \mathcal{A} \rightarrow \mathbb{R}$, where $\mathcal{S}$ is the set of *states* (board positions) and $\mathcal{A}$ is the set of *actions* (moves), and $\mathbb{R}$ represents the value of being in a state $s \in \mathcal{S}$, applying a action $a \in \mathcal{A}$, and following policy π thereafter.
- **Target function representation:** Deep neural network.

More specifically, we seek to build an agent that uses Q-learning via self-play to train a deep convolutional neural network to approximate the optimal action-value function:

$$Q^*(s, a) = \max_{\pi} Q^{\pi}(s, a), \forall s \in \mathcal{S}, a \in \mathcal{A} \tag{1}$$

which is the maximum sum of rewards achievable by a behavior policy π.

1.3 Metrics

- **Winning percentage.** Consists in playing a high number of games (e.g. 100,000) against other agents (e.g. Alpha-Beta player) using the trained deep neural network as the action-value function to take greedy actions.
- **Prediction accuracy.** The learned action-value function is used to predict the game-theoretic outcomes (win, loss or draw) of the board positions in the Connect 4 Data Set.

2 Analysis

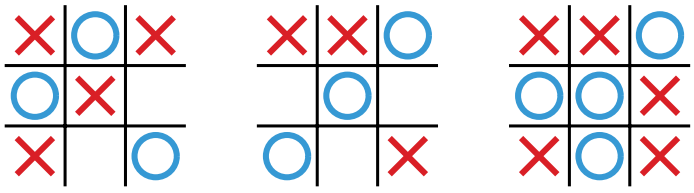
2.1 Data Exploration and Visualization

In this project we use two environments and one dataset. The environments are the games of Tic-Tac-Toe and Connect 4, and the dataset is the UCI Connect 4 dataset.

2.1.1 Tic-Tac-Toe Environment

Tic-Tac-Toe [source] is a paper-and-pencil game for two players, X and O, who take turns marking the spaces in a 3x3 grid. The player who succeeds in placing three of their marks in a horizontal, vertical, or diagonal row wins the game. Figure 1 shows three Tic-Tac-Toe game positions: a win, a draw and a loss.

Tic-Tac-Toe is an extremely simple game, nonetheless, is very useful to analyze simple concepts and to verify that the implementation of the learning algorithms are behaving as expected before moving into the Connect 4 environment, which is more complex because of its large state space.

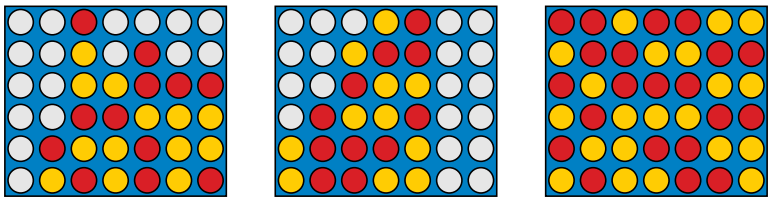


(a) Win for X **(b)** Win for O **(c)** Draw

Figure 1: Examples of a win, a loss and a draw in Tic-Tac-Toe.

2.1.2 Connect 4 Environment

Connect 4 [source] is a two-player board game of perfect information where pieces are dropped into the columns of a vertical 6×7 grid with the goal of forming a straight line of 4 connected pieces. There are at most seven actions per state, since placing a piece in a column is a legal action only if that column has at least one empty location. In this project we use pieces of two colors: YELLOW for the first player, and RED for the second player. Figure 2 shows three Connect 4 game positions: a win, a draw and a loss:



(a) Win for YELLOW **(b)** Win for RED **(c)** Draw

Figure 2: Examples of a win, a loss and a draw in Connect Four.

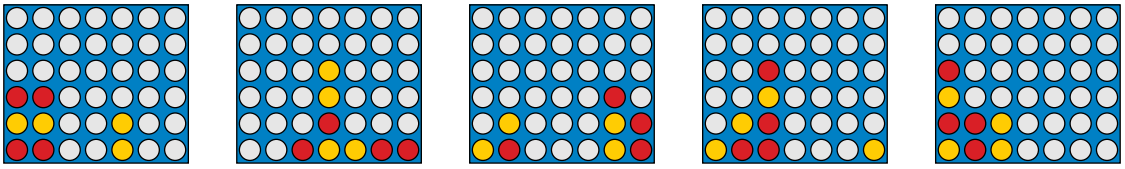
2.1.3 UCI Connect 4 Data Set

As part of the testing phase, we will use the *Connect 4 Data Set*[1] that is available from the UCI Machine Learning Repository (Hettich and Merz, 1998). A partial view of the dataset is presented in Table 1. The dataset has a total of 67,557 instances, representing all legal 8-ply positions in Connect 4 in which neither player has won yet, and which the next move is not forced. Each instance is described by 42 features, one for each space in the 6×7 board, and can belong to one of the classes $\{\texttt{x}, \texttt{o}, \texttt{b}\}$, where `x` is the first player, `o` is the second player, and `b` is empty. The outcome class is the game theoretical value for the first player, and can belong to one of the classes $\{\texttt{win}, \texttt{loss}, \texttt{draw}\}$. There are 44,473 wins, 16,635 losses and 6,449 draws [source]. Figure 3 shows a visual representation of five randomly selected instances of the data set. As we can see, all game positions have eight pieces in the board, representing all legal 8-ply positions [source].

[1] https://archive.ics.uci.edu/ml/datasets/Connect-4

	Features																	Target
No.	**a1**	**a2**	**a3**	**a4**	**a5**	**a6**	**b1**	**b2**	**...**	**f5**	**f6**	**g1**	**g2**	**g3**	**g4**	**g5**	**g6**	**outcome**
0	b	b	b	b	b	b	b	b	...	b	b	b	b	b	b	b	b	win
1	b	b	b	b	b	b	b	b	...	b	b	b	b	b	b	b	b	win
2	b	b	b	b	b	b	o	b	...	b	b	b	b	b	b	b	b	win
3	b	b	b	b	b	b	b	b	...	b	b	b	b	b	b	b	b	win
4	o	b	b	b	b	b	b	b	...	b	b	b	b	b	b	b	b	win
...	..	..	..	..	..	..	..	..	...	..	..	..	..	..	..	..	..	...
67552	x	x	b	b	b	b	o	x	...	b	b	o	o	x	b	b	b	loss
67553	x	x	b	b	b	b	o	b	...	b	b	o	x	o	o	x	b	draw
67554	x	x	b	b	b	b	o	o	...	b	b	o	x	x	o	b	b	loss
67555	x	o	b	b	b	b	o	b	...	b	b	o	x	o	x	x	b	draw
67556	x	o	o	o	x	b	o	b	...	b	b	x	b	b	b	b	b	draw

Table 1: UCI Connect 4 Dataset. Each row represents a different board position, and each feature represents a specific cell in the board. The target is the outcome of the game for the first player, assuming perfect play.



(a) Win **(b)** Loss **(c)** Win **(d)** Loss **(e)** Draw

Figure 3: Five randomly selected instances of the UCI Connect 4 Data Set. The outcome of each board position is from the point of view of the firs player (Yellow discs).

2.2 Algorithms and Techniques

2.2.1 Markov Decision Process

A *Markov decision process* (MDP) consist of four elements:

- $\mathcal{S}$ is the set of *states* (state space).
- $\mathcal{A}$ is the set of *actions* (action space). The set of actions that are available in some particular state $s_t \in \mathcal{S}$ is denoted $\mathcal{A}(s_t)$.
- $T : \mathcal{S} \times \mathcal{A} \times \mathcal{S} \to \mathbb{R}$ is the *transition function*, which is the probability given we are in state $s_t \in \mathcal{S}$, take action $a_t \in \mathcal{A}(s_t)$ and we will transition to state $s_{t+1} \in \mathcal{S}$.
- $R : \mathcal{S} \times \mathcal{A} \times \mathcal{S} \to \mathbb{R}$ is the *reward function*, which is the immediate reward received when in state $s_t \in \mathcal{S}$ action $a_t \in \mathcal{A}$ is taken and the MDP transitions to state $s_{t+1} \in \mathcal{S}$. However, it is also possible to define it either as $R : \mathcal{S} \times \mathcal{A} \to \mathbb{R}$ or $R : \mathcal{S} \to \mathbb{R}$. The first one gives rewards for performing an action a_t in a particular state s_t, and the second gives rewards when transitioning to state s_{t+1}.

2.2.2 Environment

In the reinforcement learning problem an agent does not have access to the dynamics (reward and transition function) of the MDP. However, it interacts with an *environment* by way of three signals: a *state*, which describes the state of the environment, an *action*, which allows the agent to have some impact on the environment, and a *reward*, which provides the agent with feedback on its immediate performance.

2.2.3 Policy

In an MDP, the agent acts according to a policy π, which maps each state $s \in \mathcal{S}$ to an action $a \in \mathcal{A}(s)$. A policy that specifies a unique action to be performed is called a *deterministic* policy, and is defined as $\pi : \mathcal{S} \to \mathcal{A}$.

The interaction between the policy used by the agent and the environment works as follows. First, it starts at an *initial state* s_0. Then, the policy π selects an action $a_0 = \pi(s_0)$ from the set of available actions $\mathcal{A}(s_0)$, and the action is executed. The environment transitions to a new state s_1 based on the transition function T with probability $T(s_0, a_0, s_1)$, and a reward $r_0 = R(s_0, a_0, s_1)$ is received. This process continues, producing a *trajectory* of experience $s_0, a_0, s_1, r_1, a_1, s_2, r_2, a_2, \ldots$, and the process ends in a *terminal state* s_T and is restarted in the initial state.

We use three types of policies in this project:

- **Random.** Selects actions uniformly at random.
- **Greedy.** Selects the *max action*, which is the greedy action with the highest value,

$$\pi_{\text{greedy}}(s) = \underset{a \in \mathcal{A}(s)}{\text{argmax}}\, Q(s, a) \tag{2}$$

- **ϵ-greedy.** Selects the best action for a proportion $1 - \epsilon$ of the trials, and another action is randomly selected (with uniform probability) for a proportion,

$$\pi_\epsilon(s) = \begin{cases} \pi_{\text{rand}}(s, a) & \text{if } rand() < \epsilon \\ \pi_{\text{greedy}}(s, a) & \text{otherwise} \end{cases} \tag{3}$$

  where $\epsilon \in [0, 1]$ and $rand()$ returns a random number from a uniform distribution $\in [0, 1]$.

2.2.4 Value Functions

Most of the algorithms for solving MDPs (computing optimal policies) do it by learning a *value function*. A value function estimates what is good for an agent over the long run. It estimates the expected outcome from any given state, by summarising the total amount of reward that an agent can expect to accumulate into a single number. Value functions are defined for particular policies.

Two types of value functions exist: state value functions and action value functions. In this project, however, we only focus in the latter.

The *action value function* (or Q-function), is the expected return after selecting action a in state s and then following policy π,

$$Q^\pi(s, a) = \mathbb{E}_\pi \left[R_t | s_t = s, a_t = a \right] \tag{4}$$

The *optimal value function* is the unique value function that maximises the value of every state, or state-action pair,

$$Q^*(s, a) \quad = \quad \max_\pi Q^\pi(s, a), \forall s \in \mathcal{S}, a \in \mathcal{A} \tag{5}$$

An *optimal policy* $\pi^*(s, a)$ is a policy that maximises the action value function from every state in the MDP,

$$\pi^*(s, a) = \underset{\pi}{\text{argmax}}\, Q^\pi(s, a) \tag{6}$$

2.2.5 Q-learning

One of the most basic and popular methods to estimate action-value functions is the *Q-learning* algorithm (Watkins, 1989) [source]. It is model-free online off-policy algorithm, whose main strength is that it is able to compare the expected utility of the available actions without requiring a model of the environment. Q-learning works by learning an action-value function that gives the expected utility of taking a given action in a given state and following a fixed policy thereafter. The update rule uses action-values and a built-in max-operator over the action-values of the next state in order to update $Q(s_t, a_t)$ as follows,

$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha[r_{t+1} + \gamma \max_a Q(s_{t+1}, a) - Q(s_t, a_t)] \tag{7}$$

The agent makes a step in the environment from state s_t to s_{t+1} using action a_t while receiving reward r_t. The update takes place on the action-value a_t in the state s_t from which this action was executed. This version of Q-learning works well for tasks with a small a state-space, since it uses arrays or tables with one entry for each state-action pair. In many cases in which there are far more states than could possibly be entries in a table we need to use function approximation.

2.2.6 Approximate Q-learning

Approximate Q-learning consists in parameterizing an approximate action-value function, $Q(s, a; \theta_i) \approx Q(s, a)$, in which θ_i are the parameters (weights) of the action-value function at iteration i.

2.2.7 Experience Replay

Reinforcement learning is known to be unstable or even to diverge when a nonlinear function approximator such as a neural network is used to represent the action-value function. One trick is to make it work is to use *experience replay*, which consists in storing the experiences (s_t, a_t, r_t, s_{t+1}). During the training of approximate Q-learning random minibatches from the replay memory are used instead of the most recent transition. This breaks the similarity of subsequent training samples, which otherwise might drive the network into a local minimum.

2.2.8 Self-play

Self-play is by far the most popular training method. It is a single policy $\pi(s, a)$ that is used by both players in a two-player game, $\pi_1(s, a) = \pi_2(s, a) = \pi(s, a)$. The first reason for its popularity is that training is quickest if the learner's opponent is roughly equally strong, and that definitely holds for self-play. As a second reason for popularity, there is no need to implement or access a different agent with roughly equal playing strength. However, self-play has several drawbacks, with the main one being that a single opponent does not provide sufficient exploration (Szita, 2011).

2.3 Benchmark

- **Random agent**. This benchmark consists in playing against an agent that takes uniformly random moves. This is the most basic benchmark, but first we have to be sure that our learned evaluation function can play better than a random agent before moving into a harder benchmark. Also, this will help us to detect bugs in the code and algorithms: if a learned value function does not play significantly better than a random agent, is not learning. The idea is to test against this benchmark using Alpha-beta pruning at 1, 2 and 4-ply search.

- **Connect 4 Data Set**. This dataset will be used as the main benchmark. The learned value function will be used to predict the game-theoretic outcomes (win, loss or draw) for the first player in the 67,557 instances of the dataset.

3 Methodology

3.1 Data Preprocessing

In this project data preprocessing is not necessary because all the learning came from two reinforcement learning simulators.

3.2 Implementation

3.2.1 Game

The first part of the implementation was to implement both two games: Tic-Tac-Toe and Connect 4. The idea was to design them first in a way that is independent of reinforcement learning. The following is the abstract Game interface for both [source].

```
class Game(object):

    def copy(self):
        '''Returns a copy of the game.'''
        pass

    def cur_player(self):
        '''
        Returns the index of the player in turn, starting with 0:
        0 (Player 1), 1 (Player 2), etc.
        '''
        pass

    def is_over(self):
        '''Returns True if the game is over.'''
        return len(self.legal_moves()) == 0

    def legal_moves(self):
        '''Returns a list of legal moves for the player in turn.'''
        pass

    def make_move(self, move):
        '''Makes one move for the player in turn.'''
        pass

    def outcomes(self):
        '''Returns a list of outcomes for each player at the end of the game.'''
        pass

    def reset(self):
        '''Restarts the game.'''
        pass
```

Once we had the Game interface well defined we implemented both Tic-Tac-toe [source] and Connect 4 [source].

3.2.2 Markov Decision Process

Next, the interface for an MDP [source]:

```
class MDP(object):

    def states(self):
        '''Returns a list of all states. Not generally possible for large MDPs.'''
        pass

    def start_state(self):
        '''Returns the initial state.'''
        pass

    def actions(self, state):
        '''Returns a list of possible actions in the given state.'''
        pass

    def transitions(self, state, action):
        '''
        Returns a dict of (next_state: probability) key/values, where 'next_state' is
        reachable from 'state' by taking 'action'. The sum of all probabilities should
        be 1.0. Not available in reinforcement learning.
        '''
        pass

    def reward(self, state, action, next_state):
        '''
        Returns the reward of being in 'state', taking 'action', and ending up
        in 'next_state'. Not available in reinforcement learning.
        '''
        pass

    def is_terminal(self, state):
        '''.Returns True if the given state is terminal.'''
        pass
```

3.2.3 Markov Decision Process for Games

We also created two helper classes:
[source]

```
class GameMDP(MDP):
    '''
    Converts a Game into an MDP. The agent makes moves for both players
    in the game. Used in self-play learning.
    '''
```

```
    def __init__(self, game):
        self._game = game
        self._states = {}

    def actions(self, state):
        return [None] if state.is_over() else state.legal_moves()

    def is_terminal(self, state):
        return state.is_over()

    def reward(self, state, action, next_state):
        '''Returns the utility from the point of view of the first player.'''
        return utility(next_state, 0) if next_state.is_over() else 0

    def start_state(self):
        return self._game.copy()

    def states(self):
        if not self._states:
            def generate_states(game):
                '''Generates all the states for the game'''
                if game not in self._states:
                    self._states[game] = game
                for move in game.legal_moves():
                    new_game = game.copy().make_move(move)
                    generate_states(new_game)
            generate_states(self._game)
        return self._states

    def transitions(self, state, action):
        if state.is_over():
            return [(state, 1.0)]
        new_game = state.copy().make_move(action)
        return [(new_game, 1.0)]
```

And also, `FixedGameMDP` [source], which converts a `Game` into an `MDP` by using a fixed opponent for one of the players in the game:

```
class FixedGameMDP(GameMDP):

    def __init__(self, game, opp_player, opp_idx):
        '''
        opp_player: the opponent player
        opp_idx: the idx of the opponent player in the game
        '''
        super(FixedGameMDP, self).__init__(game)
        self._opp_player = opp_player
        self._opp_idx = opp_idx
        self._agent_idx = opp_idx ^ 1
```

```
    def reward(self, game, move, next_game):
        return utility(next_game, self._agent_idx) if next_game.is_over() else 0

    def start_state(self):
        new_game = self._game.copy()
        if not new_game.is_over() and new_game.cur_player() == self._opp_idx:
            chosen_move = self._opp_player.choose_move(new_game)
            new_game.make_move(chosen_move)
        return new_game

    def transitions(self, game, move):
        if game.is_over():
            return []
        new_game = game.copy().make_move(move)
        if not new_game.is_over() and new_game.cur_player() == self._opp_idx:
            chosen_move = self._opp_player.choose_move(new_game)
            new_game.make_move(chosen_move)
        return [(new_game, 1.0)]
```

3.2.4 Environment

A reinforcement learning environment [source] for interacting with an MDP. The main idea is that it restricts access to the transition and reward functions. It needs to interact with the environment to transition to another state and receive rewards.

```
class Environment(object):

    def __init__(self, mdp):
        self._mdp = mdp
        self._cur_state = self._mdp.start_state()

    def actions(self, state):
        '''Returns the available actions in the given state.'''
        return self._mdp.actions(state)

    def cur_state(self):
        '''Returns the current state.'''
        return self._cur_state.copy()

    def do_action(self, action):
        '''
        Performs the given action in the current state.
        Returns (reward, next_state).
        '''
        prev = self.cur_state()
        transitions = self._mdp.transitions(self.cur_state(), action)
        for next_state, prob in transitions:
            self._cur_state = next_state
```

```
        reward = self._mdp.reward(prev, action, self.cur_state())
        return reward, self.cur_state()

    def is_terminal(self):
        return self._mdp.is_terminal(self.cur_state())

    def reset(self):
        '''Resets the current state to the start state.'''
        self._cur_state = self._mdp.start_state()
```

3.2.5 Learner

We define an episodic learner. It is basically an interface for algorithms that learn a value function by interacting with an environment in an episodic way.

```
class Learner(object):

    def __init__(self, env, n_episodes=1000):
        self.env = env
        self.n_episodes = n_episodes

    def train(self, callbacks=None):
        '''Trains the model for a fixed number of episodes.'''
        callbacks = CallbackList(callbacks)
        callbacks.on_train_begin()
        for episode in range(self.n_episodes):
            callbacks.on_episode_begin(episode)
            self.env.reset()
            self.episode()
            callbacks.on_episode_end(episode, self.qfunction)
        callbacks.on_train_end(self.qfunction)

    @abc.abstractmethod
    def episode(self):
        pass
```

3.2.6 Tabular Q-learning

asdf asd fas fas df a.

```
class QLearning(Learner):
    '''Tabular Q—learning'''

    def __init__(self, env, policy, qfunction, learning_rate=0.1, discount_factor=1.0):
        super(QLearning, self).__init__(env, **kwargs)
        self.policy = policy
        self.qfunction = qfunction
        self.learning_rate = learning_rate
        self.discount_factor = discount_factor

    def best_qvalue(self, state):
        return max_qvalue(state, self.env.actions(state), self.qfunction)

    def episode(self):
        while not self.env.is_terminal():
            state = self.env.cur_state()
            action = self.policy.action(state)
            reward, next_state = self.env.do_action(action)
            best_qvalue = self.best_qvalue(next_state)
            target = reward + (self.discount_factor * best_qvalue)
            td_error = target — self.qfunction[state, action]
            self.qfunction[state, action] += self.learning_rate * td_error
```

3.2.7 Approximate Q-learning

```
class ApproximateQLearning(Learner):
    '''Q—learning with a function approximator'''

    def __init__(self, env, policy, qfunction, discount_factor=1.0,
                 experience_replay=True, **kwargs):
        super(ApproximateQLearning, self).__init__(env, **kwargs)
        self.policy = policy
        self.qfunction = qfunction
        self.discount_factor = discount_factor
        self.experience_replay = experience_replay
        if self.experience_replay:
            self.memory = set()

    def best_qvalue(self, state):
        return max_qvalue(state, self.env.actions(state), self.qfunction)

    def episode(self):
        while not self.env.is_terminal():
```

```
            state = self.env.cur_state()
            action = self.policy.action(state)
            reward, next_state = self.env.do_action(action)
            if self.experience_replay:
                self.memory.add((state, action, reward, next_state))
                state, _, reward, next_state = random.choice(tuple(self.memory))
            best_qvalue = self.best_qvalue(next_state)
            update = reward + (self.discount_factor * best_qvalue)
            self.qfunction.update(state, update)
```

3.3 Experiments

3.3.1 Estimate state-action values using tabular Q-learning

The first step to implement a Q-learning agent with

We estimate the state-action values for two very simple board positions for both Tic-Tac-Toe and Connect 4. We will be using the tabular version of Q-learning. The goal here is simply to verify that our implementation of the Q-learning algorithm is working as expected. We chose very simple positions because the expected q-values are easy to understand.

We converted both games as MDPs by making the opponent a fixed agent that takes optimal moves by using the Alpha-Beta algorithm. This allowed us to verify that we are learning the q-values assuming the worst case scenario. In other situations in which the depth of the game tree is huge Alpha-Beta would need to combined with a good heuristic evaluation function, but in these simple positions it won't be necessary.

The rewards given by the Tic-Tac-Toe environment are $+1$ for a win for the first player, -1 for a win for the first player, and 0 for a draw.

Tic-Tac-toe

For Tic-Tac-Toe we used the board position in Figure 4 as the starting state, s_0. It has five available actions, $\mathcal{A}(s_0) = \{1, 2, 4, 6, 9\}$. By looking at such a simple position we can clearly identify that the best available action is 9, which leads to a win if we keep playing the best possible actions. Any other action (i.e. 1, 2, 4 or 6) would lead to a loss, assuming perfect play from the opponent.

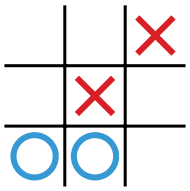
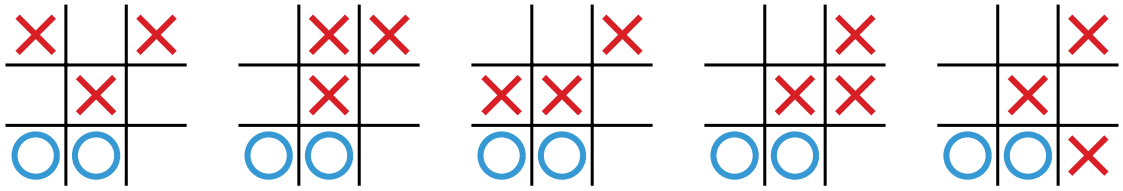


Figure 4: Starting Tic-Tac-Toe state, s_0, with five legal actions, $\mathcal{A}(s_0) = \{1, 2, 4, 6, 9\}$.

We ran the tabular version of the Q-learning algorithm with $\alpha = 0.1$ (learning rate), $\gamma = 1.0$ (discount factor), and a uniformly random behavior policy. The Q-values converged to the expected values after around 800 episodes of training, and are shown in Figure 5. As we can see in Figure 5e, the expected future reward of taking action 9 is correctly estimated as 1.0, since that would lead to a win. At the same time, the q-values for all the other actions was -1.0. This makes sense, since any action not being 9 would lead to an immediate loss. Also, we can see in Figure 6 we can see the predicted q-values for each available action. It is interesting to see how the q-value for $Q(s_0, 9)$ takes some time to converge. This is probably because taking action 9 leads to an eventual win, but not immediately. It requires making another move (either 1 or 6) to win.

Connect 4

For Connect 4 we used the board position in Figure 7 as the starting state, s_0. It has three available actions, $\mathcal{A}(s_0) = \{\mathtt{d}, \mathtt{f}, \mathtt{g}\}$. By looking at such a simple position we can clearly identify that the best available



(a) $Q(s_0, 1) = -1.0$ **(b)** $Q(s_0, 2) = -1.0$ **(c)** $Q(s_0, 4) = -1.0$ **(d)** $Q(s_0, 6) = -1.0$ **(e)** $Q(s_0, 9) = 1.0$

Figure 5

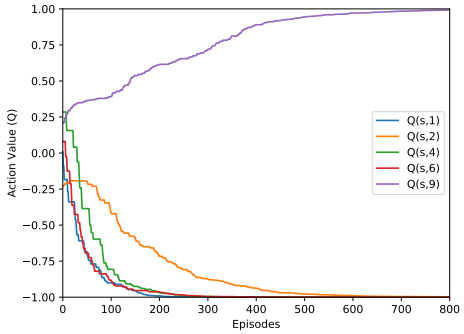


Figure 6: The predicted state-action value during training.

action is `f`, which leads to a win if we keep playing the best possible actions. Any other action (i.e. `d` or `g`) would lead to a loss, assuming perfect play from the opponent.

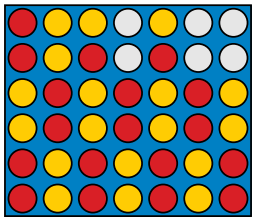
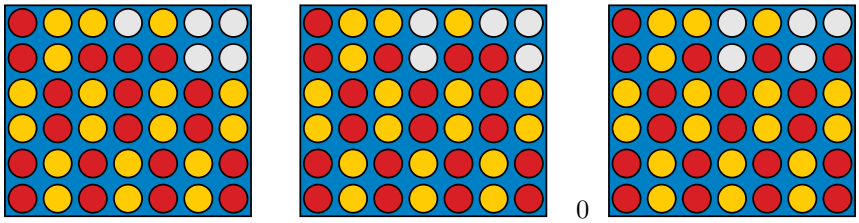


Figure 7: Connect 4 board, s_0, with three legal actions: $\{\texttt{d}, \texttt{f}, \texttt{g}\}$.

We ran the same tabular version of the Q-learning algorithm with the same parameters as the used for Tic-Tac-Toe $\alpha = 0.1$ (learning rate), $\gamma = 1.0$ (discount factor), and a uniformly random behavior policy. The Q-values also converged to the expected values after around 800 episodes of training, and are shown in Figure 8. As we can see in Figure 8b, the expected future reward of taking action `f` is correctly estimated as 1.0, since that would lead to a win. At the same time, the q-values for all the other actions was -1.0. This makes sense, since any action not being `f` would lead to an immediate loss. Also, we can see in Figure 9 we can see the predicted q-values for each available action.



(a) $Q(s_0, \texttt{d}) = -1.0$ **(b)** $Q(s_0, \texttt{f}) = 1.0$ 0 **(c)** $Q(s_0, \texttt{g}) = -1.0$

Figure 8: All the next states and their q values.

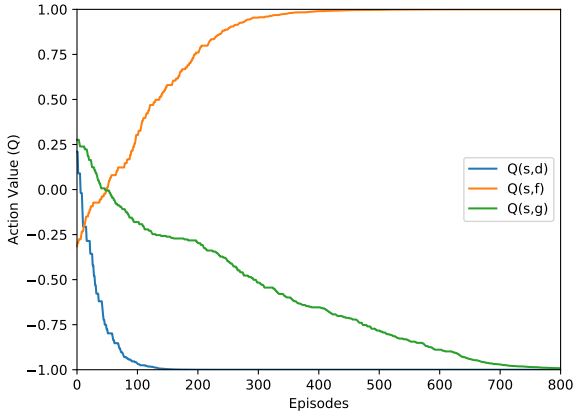


Figure 9: The predicted state-action value during training.

3.4 Refinement

Content.

3.5 Games

3.5.1 Tic Tac Toe

3.5.2 GameMDP converter

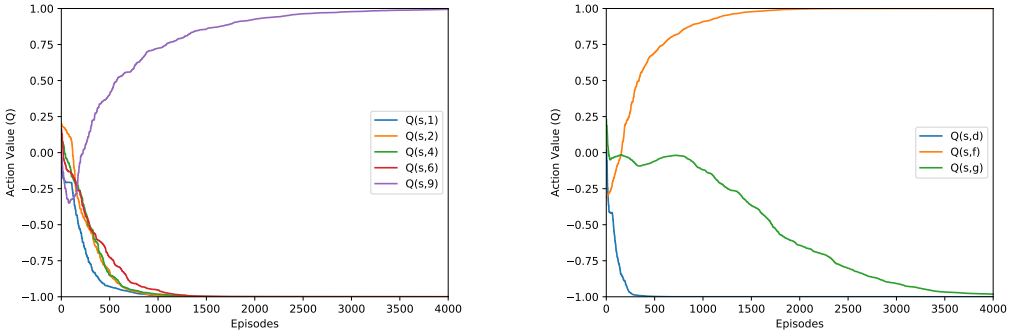
Converting an MDP

4 Examples

4.1 Estimate q-values using Q-learning via self-play

In the previous example we estimated the q-values for Tic-Tac-toe and Connect 4 positions against a fixed Alpha-Beta opponent. While this works very well sometimes we don't have that algorithm. An alternative is to use self-play, which consists in making playing games against itself.

In both examples we can see that it took longer to learn the episodes because it didn't have a perfect guidance on what was good and bad, it had to learn it by itself. This is a powerful technique that will be used in the next sections to learn a value function for Connect 4. The values are the same as the ones shown in Figures 5 and 8.



(a) Tic-Tac-Toe **(b)** Connect 4

Figure 10: All the next states and their q values.

4.2 Estimate all q-values using tabular Q-learning

Now the idea is to learn to play the games in general starting from the same position. This means that we will use Q-learning to estimate the q-values from the empty board of the game. We will do it for Tic-Tac-Toe. We ran the experiment using similar parameters, and after 65,000 episodes the q-values were found correctly.

In Figure 11 we can see how after 65,000 episodes it learned the values.

We were able to do that because the full game-tree of Tic-Tac-Toe is around 888888 game states. And based on the results we can assume that the majority of the states were visited. But the problem now is that Connect 4 has INSERT game states. That makes it impossible to run the Q-learning algorithm with a tabular Q-function to save the exact value for every single state of the game. The solution to this is now to use a function approximator.

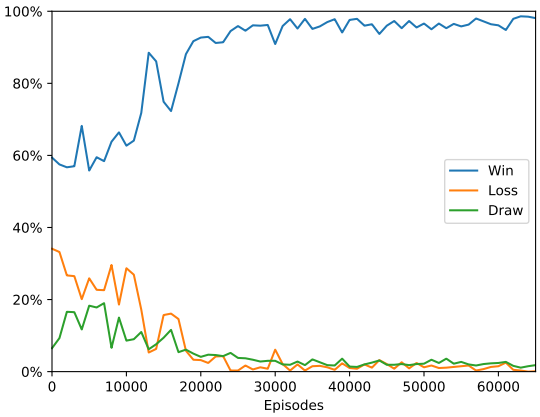


Figure 11: Results after playing 1,000 games every 1,000 episodes against a Random player.

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4.3 Estimate all q-values using tabular Q-learning via Self-Play

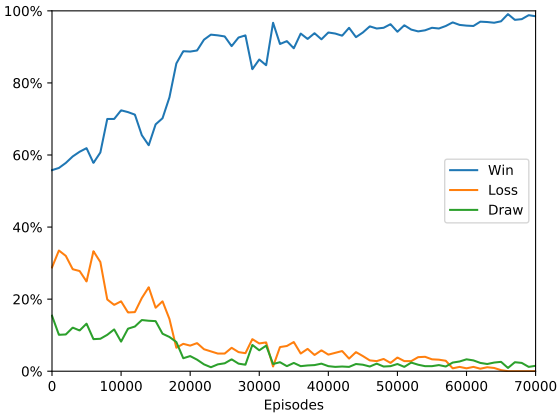

Figure 12: Results after playing 1,000 games every 1,000 episodes against a Random player.

In Figure 12 we can see how after 70,000 episodes it learned the values.

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4.4 Estimate Q-values Using Approximate Q-learning

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5 Results

5.1 Model Evaluation and Validation

Content

5.2 Justification

Content.

6 Conclusion

6.1 Free-Form Visualization

Content.

6.2 Reflection

Content.

6.3 Improvement

Content.

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