CH2013
Computational Programming and Simulations Lab
Aug-Dec 2021

Problem Sheet #8 (October 13)

1. Write a function "**euler**" which takes h, x_i and y_i as input arguments and yields y_{i+1} as the output, for the solution of this ODE

$$\frac{dy}{dx} = -2x^3 + 12x^2 - 20x + 8.5; y(0) = 10; x \in [0,4]$$

   a) First analytically integrate the equation, and find y(x), and evaluate its value at x=[0,0.5,1,1.5,2,2.5,3,3.5,4]. Let this vector of y values be labelled 'yan'.

   b) Next, use the Euler's method with h=0.5 to find value of y at various x, and put them in a vector 'yEuler1'

   c) With h=0.1, use Euler's method again, and let the vector of y values be 'yEuler2'

   d) Plot yan, yEuler1, yEuler2 vs. x on the same plot using symbols and lines to distinguish.

   e) Solve the above differential equation using the MATLAB built-in function '**ode45**' and plot the resulting y – call it yode45– vs. x, and compare it with the analytical solution.

2. MATLAB's inbuilt function **ode15s** can handle stiff ODEs very well. Recall a problem discussed in class, for which you tried the explicit Euler's method and it failed:

$$\frac{dy}{dt} = 98y + 198z; \frac{dz}{dt} = -99y - 199z; \;\; y(0) = 1; z(0) = 0$$

   The analytical solution is known

$$y(t) = 2e^{-t} - e^{-100t}; z(t) = -e^{-t} + e^{-100t}$$

   Solve the system of ODEs using ode15s and compare it with the analytical solution by plotting them together. Let $t\varepsilon[0,10]$ in intervals of 0.1
   Look at the plot – see how the region at low t has sharp gradients.
   I will check the following: vector yan and zan which represent the analytical solution at t; yode15s and zode15s which are from the ode15s solution.

3. The Finite Difference based method for solution of BVP-ODEs involves the conversion of the original ODEs into a system of linear/non-linear equations, which need to be solved in order to determine the Yi at each ti. *The boundary conditions may be the Dirichlet (function values), Neumannn (derivative values) or mixed (function value at one point, derivative at the other) kind.*

$$\frac{d^2T}{dt^2} = 0.15T; \;\; T(0) = 240 \; and \; T(10) = 150$$

   a) This can be translated to a set of linear equations with h=1, as below.

$$T_0 = 240$$
$$T_{i+1} - 2T_i + T_{i-1} = 0.15(T_i)$$
$$T_{10} = 150$$

(for i=1,9)

Set up the matrix version of the linear equations problem, as $AT = b$ where $T = [T_0;\ T_1;\ \dots..;T_{10}]$.

Solve for T using LU decomposition, Gauss elimination or just the inverse method (using MATLAB functions, no need to write your own code for these). Plot T vs. t.

I plan to examine A, b, and the final solution T.

b) Rework the problem for h=2, and do the same steps for solution as above. Let this equation be represented as $WT2 = c$ where the matrix W replaces A above. And the new temperature is T2. *I will examine W,c and the final solution T2.*

c) Plot the temperatures T and T2 together and think about the result.