CH2061

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ASSIGNMENT - 4

CH19B072

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1:

(a) Least square Fitting

Fitted line : $\boxed{y = a_0 + a_1 x}$

In least square fitting,

$$S_r = \sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} (y_{i,\,measured} - y_{i,\,model})^2$$

$$S_r = \sum_{i=1}^{n} (y_i - a_0 - a_1 x_i)^2$$

Condition to minimise the errors :

$$\frac{dS_r}{da_0} = 0.0 \quad \text{and} \quad \frac{dS_r}{da_1} = 0.0$$

$$\Rightarrow a_1 = \frac{n\sum x_i y_i - \sum x_i \sum y_i}{n\sum x_i^2 - (\sum x_i)^2}$$

$$a_1 = \frac{9 \times 683 - 76 \times 70}{9 \times 916 - (76)^2} = \underline{\underline{0.335089}}$$

$$a_0 = \bar{y} - a_1\bar{x} = \frac{\sum y_i}{n} - a_1 \frac{\sum x_i}{n}$$

$$= \frac{70}{9} - (0.335089)\frac{(76)}{9}$$

$$= \underline{\underline{4.948137}}$$

$\therefore$ slope of fitted line $= a_1 = 0.335089$

and intercept $= a_0 = 4.948137$

(b) $$S_t = \sum_{i=1}^{n} (y_i - \bar{y})^2$$

$n = 9$

$$= 39.55556$$

$$S_r = \sum_{i=1}^{n} (y_i - a_0 - a_1 x_i)^2$$

$$= 8.7658671$$

~~standard deviation~~ $\cancel{S_y = \sqrt{\frac{S_t}{n-1}} = \sqrt{\frac{8.7658671}{8}}}$

$\cancel{= 1.04677}$

~~Standard error~~ $\cancel{S_{y|x} = \sqrt{\frac{S_r}{n-2}} = \sqrt{\frac{39.55556}{7}}}$

standard deviation $$S_y = \sqrt{\frac{S_t}{n-1}} = \sqrt{\frac{39.55556}{8}}$$

$$= 2.22361$$

standard error $$S_{y|x} = \sqrt{\frac{S_r}{n-2}} = \sqrt{\frac{8.7658671}{7}}$$

$$= 1.11905$$

Coefficient of determination

$$r^2 = \frac{S_t - S_r}{S_t} = 0.778391$$

It is somewhat close to 1 $\Rightarrow$ fitted line is correct

(c) Calculations and plots $\rightarrow$ attached

2: Gauss - Newton method

$$y = a_0 \left(1 - e^{-a_1 x}\right)$$

Here, $a_0 = a$
$a_1 = b$

In Gauss-Newton method, we use iterative techniques. ~~(i → no. of iteration)~~

$$\underbrace{y_i - f(x_i)}_{D} = \frac{\partial f}{\partial a_0} \Delta a_0 + \frac{\partial f}{\partial a_1} \Delta a_1 + e_i$$

$y_i \rightarrow$ y value from data

$f(x_i) \rightarrow$ calculated y value

$e_i \rightarrow$ error

Converting the above equation to matrix form, we have,

$$[D] = [Z][\Delta A] + [E]$$

n = no. of data points

where,

$$D = \begin{bmatrix} y_1 - f(x_1) \\ y_2 - f(x_2) \\ \vdots \\ y_n - f(x_n) \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_n \end{bmatrix}$$

$$Z = \begin{bmatrix} \frac{\partial f_1}{\partial a_0} & \frac{\partial f_1}{\partial a_1} \\ \vdots & \vdots \\ \frac{\partial f_n}{\partial a_0} & \frac{\partial f_n}{\partial a_1} \end{bmatrix} \qquad \Delta A = \begin{bmatrix} \Delta a_0 \\ \Delta a_1 \end{bmatrix}$$

$$E = \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_n \end{bmatrix}$$

Condition in least square method : $\frac{\partial S_n}{\partial a_j} = 0$

$$S_n = \sum_{i=1}^{n} e_i^2$$

$$S_n = \sum_{i=1}^{n} \left(d_i - \sum_{j=1}^{m} \Delta a_j z_{ji} \right)$$

Applying the condition and solving,

we have,

$$\boxed{[\Delta A] = \left([Z]^T [Z]\right)^{-1} [Z]^T [D]} \rightarrow (1)$$

$[Z]$ and $[D]$ can be found easily ~~are~~ ~~we~~

using a values for that iteration.

$\Downarrow$

From that, we find $[\Delta A]$ matrix $= \begin{bmatrix} \Delta a_0 \\ \Delta a_1 \end{bmatrix}$

and update a_0 and a_1 for next iteration as

$$(a_0)_{j+1} = (a_0)_j + \Delta a_0$$

$$(a_1)_{j+1} = (a_1)_j + \Delta a_1$$

Therefore,

→ We use an initial guess for a_0 and a_1,

→ use those values to find $[\Delta A]$ from (1)

→ Update the value of ~~ΔA~~ a_0 and a_1 using $[\Delta A]$ obtained

Repeat the above process till convergence

[Criteria can be taken as :

$$\frac{(a_0)_{j+1} - (a_0)_j}{(a_0)_{j+1}} < 0.001$$

$$\frac{(a_1)_{j+1} - (a_1)_j}{(a_1)_{j+1}} < 0.001 \quad]$$

We have,

$$\frac{\partial f(x)}{\partial a_0} = 1 - e^{-a_1 x}$$

$$\frac{\partial f(x)}{\partial a_1} = a_0 x e^{-a_1 x}$$

(a) iteration - 1 [Initial guess : $a_0 = 30$, $a_1 = 0.1$]

$$Z = \begin{bmatrix} \frac{\partial f_1}{\partial a_0} & \frac{\partial f_1}{\partial a_1} \\ \vdots & \vdots \\ \frac{\partial f_{10}}{\partial a_0} & \frac{\partial f_{10}}{\partial a_1} \end{bmatrix} = \begin{bmatrix} 0.39347 & 90.97960 \\ 0.63212 & 110.36383 \\ 0.77689 & \cancel{81.20117}\ 100.40857 \\ 0.86466 & 81.20117 \\ 0.91791 & 61.56374 \\ 0.95021 & 44.80836 \\ 0.96980 & 31.70725 \\ 0.98168 & 21.97877 \\ 0.98889 & 14.99714 \\ 0.99326 & 10.10692 \end{bmatrix}$$

$$D = \begin{bmatrix} 5.19592 \\ 5.03638 \\ 7.69390 \\ 7.0600585 \\ 9.46254 \\ 8.49361 \\ 10.90592 \\ 10.54947 \\ 12.33327 \\ 11.20214 \end{bmatrix}$$

$$\Delta A = ([Z]^T[Z])^{-1}[Z]^T[D]$$

$$= \begin{bmatrix} 11.17035 \\ \cancel{0.08680} \\ -0.01320 \end{bmatrix}$$

$$\Rightarrow a_{0_{new}} = 30 + 11.17035 = \underline{41.17035}$$

$$a_{1_{new}} = 0.1 - 0.01320 = \underline{0.08680}$$

$$\left|\frac{a_{0_{new}} - a_{0_{initial}}}{a_{0_{new}}}\right| = \left|\frac{41.17035 - 30}{41.17035}\right| = 0.27132$$

$$\left|\frac{a_{1_{new}} - a_{1_{initial}}}{a_{1_{new}}}\right| = \left|\frac{0.08680 - 0.1}{0.08680}\right| = 0.15207$$

iteration - 2 $[a_0 = 41.17035 \quad a_1 = 0.08680]$

$$Z = \begin{bmatrix} 0.35209 & 133.37267 \\ 0.58022 & 172.82600 \\ 0.72802 & 167.96259 \\ 0.82378 & 145.09871 \\ 0.88582 & 117.51297 \\ 0.92603 & 91.36488 \\ 0.95207 & 69.06187 \\ 0.96895 & 51.13785 \\ 0.97988 & 37.27411 \\ 0.98696 & 26.83348 \end{bmatrix}$$

$$D = \begin{bmatrix} 2.50418 \\ 0.11224 \\ 1.02715 \\ -0.91542 \\ 0.53016 \\ -1.12486 \\ 0.80284 \\ 0.10809 \\ 1.65796 \\ 0.36631 \end{bmatrix}$$

$$\Delta A = \begin{bmatrix} 0.14006 \\ 0.00295 \end{bmatrix}$$

$$a_{0_{new}} = 41.17035 + 0.14006 = \underline{\underline{41.31041}}$$

$$a_{1_{new}} = 0.08680 + 0.00295 = \underline{\underline{0.08975}}$$

$$\left| \frac{a_{0_{new}} - a_{0_{prev}}}{a_{0_{new}}} \right| = \left| \frac{41.31041 - 41.17035}{41.31041} \right| = 3.39 \times 10^{-3}$$

$$\left| \frac{a_{1_{new}} - a_{1_{prev}}}{a_{1_{new}}} \right| = \left| \frac{0.08975 - 0.08680}{0.08975} \right| = 0.0329$$

iteration - 3 $\left[a_0 = 41.31041 \quad a_1 = 0.08975\right]$

$$Z = \begin{bmatrix} 0.36158 & 131.86681 \\ 0.59242 & 168.37260 \\ 0.73980 & 161.23844 \\ 0.83388 & 137.25028 \\ 0.89394 & 109.52901 \\ 0.93229 & 83.91050 \\ 0.95677 & 62.49842 \\ 0.97240 & 45.60021 \\ 0.98238 & 32.75106 \\ 0.98875 & 23.23211 \end{bmatrix}$$

$$D = \begin{bmatrix} 2.06294 \\ -0.47315 \\ 0.43881 \\ -1.44190 \\ 0.07074 \\ -1.51340 \\ 0.47525 \\ -0.17041 \\ 1.41738 \\ 0.15422 \end{bmatrix} \qquad \Delta A = \begin{bmatrix} -0.01257 \\ 0.00022 \end{bmatrix}$$

$$a_{0_{new}} = 41.31041 - 0.01257$$
$$= \underline{\underline{41.29784}}$$

$$a_{1_{new}} = 0.08975 + 0.00022$$
$$= \underline{\underline{0.08997}}$$

$$\left|\frac{a_{0new} - a_{0prev}}{a_{0new}}\right| = 3.044 \times 10^{-4} \quad < 0.001$$

$$\left|\frac{a_{1new} - a_{1prev}}{a_{1new}}\right| = 2.445 \times 10^{-3} \quad \text{close to } 0.001$$

$\therefore$ It is safe to stop iteration

$$a = a_0 = \underline{\underline{41.29784}}$$

$$b = a_1 = \underline{\underline{0.08997}}$$

(b) $S_t = \sum_{i=1}^{10} (y_i - \bar{y})^2 = 601.6$

$[\bar{y} = 34.2]$

$S_r = \sum_{i=1}^{10} (y_i - y_{calculated})^2$

$= \sum_{i=1}^{10} (y_i - a_0(1 - e^{-a_1 x_i}))^2$

$= 11.34857$

coefficient of determination,

$r^2 = \frac{S_t - S_r}{S_t} = \underline{\underline{0.981}}$

r^2 is close to 1

$\Rightarrow$ It is nearly a perfect fit

(c) Plots and calculations : $\hookrightarrow$ attached

3:

(a) Newton's divided difference interpolating polynomial

$$f_n(x) = b_0 + b_1(x-x_0) + \cdots\cdots b_n(x-x_0)(x-x_1)\cdots\cdots(x-x_{n-1})$$

$$b_0 = f(x_0)$$

$$b_1 = f(x_1, x_0)$$

$$b_2 = f(x_2, x_1, x_0)$$

$$\vdots$$

$$b_n = f(x_n, x_{n-1} \cdots\cdots x_0)$$

where,

$$f(x_i, x_j) = \frac{f(x_i) - f(x_j)}{x_i - x_j}$$

$$f(x_i, x_j, x_k) = \frac{f(x_1, x_j) - f(x_j, x_k)}{x_i - x_k}$$

$$\vdots$$

$$f(x_n, x_{n-1} \cdots x_0) = \frac{f(x_n, x_{n-1} \cdots\cdots x_1) - f(x_{n-1}, x_{n-2} \cdots x_0)}{x_n - x_0}$$

From above, we can find ~~the~~ $f_n(x)$ and then using the function f_n, we can find interpolated values at any x (here, $x = 2.8$)

Order - 1

$f(x) = b_0 + b_1 (x - x_0)$

[I take $x_0 = 2.5$ $\quad f(x_0) = 14$

$x_1 = 3.2$ $\quad f(x_1) = 15$

As those are the values closest to 2.8]

$b_0 = f(x_0) = f(2.5) = \underline{14}$

$b_1 = f[x_1, x_0] = \dfrac{f(x_1) - f(x_0)}{x_1 - x_0} = \dfrac{15 - 14}{3.2 - 2.5} = \underline{1.42857}$

$\Rightarrow f(x) = 14 + 1.42857 (x - 2.5)$

$\Rightarrow f(x) = 1.42857 x + 10.42858$

$\Rightarrow f(2.8) = \underline{\underline{14.42858}}$

Order - 2

$f(x) = b_0 + b_1 (x - x_0) + b_2 (x - x_0)(x - x_1)$

[I take $x_0 = 2.5$ $\quad f(x_0) = 14$

$x_1 = 3.2$ $\quad f(x_1) = 15$

$x_2 = 4$ $\quad f(x_2) = 8$

Because these three points are close to 2.8 and also, there is transition from increasing to decreasing part between ~~2.3~~ 2.5 and 4. So, in order to cover both increasing part and decreasing part in the function, I chose these three points]

$b_0 = f(x_0) = \underline{\underline{14}}$

$b_1 = f(x_1, x_0) = \underline{\underline{1.42857}}$

$$b_2 = f(x_2, x_1, x_0) = \frac{f(x_2, x_1) - f(x_1, x_0)}{x_2 - x_0}$$

$$= \frac{\dfrac{f(x_2) - f(x_1)}{x_2 - x_1} - 1.42857}{4 - 2.5}$$

$$= \frac{\dfrac{8 - 15}{0.8} - 1.42857}{1.5}$$

$$= \underline{\underline{-6.78571}}$$

$\Rightarrow f(x) = 14 + 1.42857(x - 2.5) - 6.78571(x - 2.5)(x - 3.2)$

$\Rightarrow f(x) = -6.78571x^2 + 40.10712x - 43.85710$

$\Rightarrow f(2.8) = \underline{\underline{15.24287}}$

Order - 3

$f(x) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1) + b_3(x - x_0)(x - x_1)(x - x_2)$

[I take

	$x_0 = 2.5$	$f(x_0) = 14$
	$x_1 = 3.2$	$f(x_1) = 15$
	$x_2 = 4$	$f(x_2) = 8$
	$x_3 = 4.5$	$f(x_3) = 2$

because these are close and also ~~these~~ some of those points we taken before and ~~...~~ calculations can be reused]

$$b_0 = f(x_0) = 14$$

$$b_1 = f[x_1, x_0] = 1.42857$$

$$b_2 = f[x_2, x_1, x_0] = -6.78571$$

$$b_3 = f[x_3, x_2, x_1, x_1]$$

$$= \frac{f[x_3, x_2, x_1] - f[x_2, x_1, x_0]}{x_3 - x_0}$$

$$f(x_3, x_2, x_1) = \frac{f(x_3, x_2) - f(x_2, x_1)}{x_3 - x_1}$$

$$= \frac{\frac{f(x_3) - f(x_2)}{x_3 - x_2} - \frac{(f(x_2) - f(x_1))}{x_2 - x_1}}{x_3 - x_1}$$

$$= \frac{\frac{2 - 8}{0.5} - \frac{(8 - 15)}{0.8}}{1.3} = -2.5$$

$$\Rightarrow b_3 = \frac{-2.5 - -6.78571}{2} = 2.14286$$

$$\Rightarrow f(x) = 14 + 1.42857(x - 2.5) - 6.78571(x - 2.5)(x - 3.2)$$
$$+ 2.14286(x - 2.5)(x - 3.2)(x - 4)$$

$$\Rightarrow f(x) = 2.14286x^3 - 27.57145x^2 + 106.10721x$$
$$- 112.42862$$

$$\Rightarrow f(2.8) = \underline{\underline{15.55146}}$$

(b) Quadratic Spline

We have 5 points and their values.

$\Rightarrow$ We need to fit 4 equations between them

$$f(x) = \begin{cases} a_1 x^2 + b_1 x + c_1 & 1.6 \le x \le 2 \\ a_2 x^2 + b_2 x + c_2 & 2 \le x \le 2.5 \\ a_3 x^2 + b_3 x + c_3 & 2.5 \le x \le 3.2 \\ a_4 x^2 + b_4 x + c_4 & 3.2 \le x \le 4 \end{cases}$$

$\Rightarrow$ We have $3 \times 4 = 12$ variables

$\Rightarrow$ We need 12 conditions:

Conditions:

$\rightarrow$ 1: $a_{i-1} x_{i-1}^2 + b_{i-1} x_{i-1} + c_{i-1} = f(x_{i-1})$ $(x = 2, 2.5, 3.2)$

$a_i x_{i-1}^2 + b_i x_{i-1} + c_i = f(x_{i-1})$

$\Rightarrow a_1 (2)^2 + b_1 (2) + c_1 = f(2) = 8$

$a_2 (2)^2 + b_2 (2) + c_2 = f(2) = 8$

$a_2 (2.5)^2 + b_2 (2.5) + c_2 = f(2.5) = 14$

$a_3 (2.5)^2 + b_3 (2.5) + c_3 = f(2.5) = 14$

$a_3 (3.2)^2 + b_3 (3.2) + c_3 = f(3.2) = 15$

$a_4 (3.2)^2 + b_4 (3.2) + c_4 = f(3.2) = 15$

$\Rightarrow$ 6 conditions

$\rightarrow$ 2: $f(x)$ at end points $(x = 1.6, \; x = 4)$

$\Rightarrow a_1 (1.6)^2 + b_1 (1.6) + c_1 = f(1.6) = 2$

$a_4 (4)^2 + b_4 (4) + c_4 = f(4) = 8$

$\Rightarrow$ 2 conditions

→ 3: First derivative on both sides equal

$(x = 2, 2.5, 3.2)$

$\Rightarrow 2a_1(2) + b_1 = 2a_2(2) + b_2$

$2a_2(2.5) + b_2 = 2a_3(2.5) + b_3$

$2a_3(3.2) + b_3 = 2a_4(3.2) + b_4$

$\Rightarrow$ 3 conditions

→ 4: Second derivative is zero at $x = 1.6$

$\Rightarrow 2a_1 = 0 \Rightarrow a_1 = 0$

$\Rightarrow$ 1 condition

$\therefore$ We have 12 equations and 12 unknowns

[Actually 11 equations and 11 unknowns because $a_1 = 0$]

$\Rightarrow$ We need to solve for:

$$\begin{bmatrix} 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 6.25 & 2.5 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 6.25 & 2.5 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 10.24 & 3.2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 10.24 & 3.2 & 1 \\ 1.6 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 16 & 4 & 1 \\ 1 & 0 & -4 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5 & 1 & 0 & -5 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 6.4 & 1 & 0 & -6.4 & -1 & 0 \end{bmatrix} \begin{bmatrix} b_1 \\ c_1 \\ a_2 \\ b_2 \\ c_2 \\ a_3 \\ b_3 \\ c_3 \\ a_4 \\ b_4 \\ c_4 \end{bmatrix} = \begin{bmatrix} 8 \\ 8 \\ 14 \\ 14 \\ 15 \\ 15 \\ 2 \\ 8 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The above system of equations can be solved using methods like Gauss elimination or LU decomposition

[I am ~~having~~ not mentioning those steps here because it is lengthy and is already covered in module - 3]

$\Rightarrow$ We get

$a_1 = 0$ $\quad b_1 = 15$ $\quad c_1 = -22$

$a_2 = -6$ $\quad b_2 = 39$ $\quad c_2 = -46$

$a_3 = -10.82$ $\quad b_3 = 63.102$ $\quad c_3 = -76.13$

$a_4 = -3.255$ $\quad b_4 = 14.686$ $\quad c_4 = 1.336$

$\therefore$

$$f(3.4) = a_4(3.4)^2 + b_4(3.4) + c_4$$

$$= \underline{\underline{13.641}}$$

$$f(2.2) = a_2(2.2)^2 + b_2(2.2) + c_2$$

$$= \underline{\underline{10.76}}$$

(c) Calculations and Plots

$\rightarrow$ attached

CALCULATIONS AND PLOTS

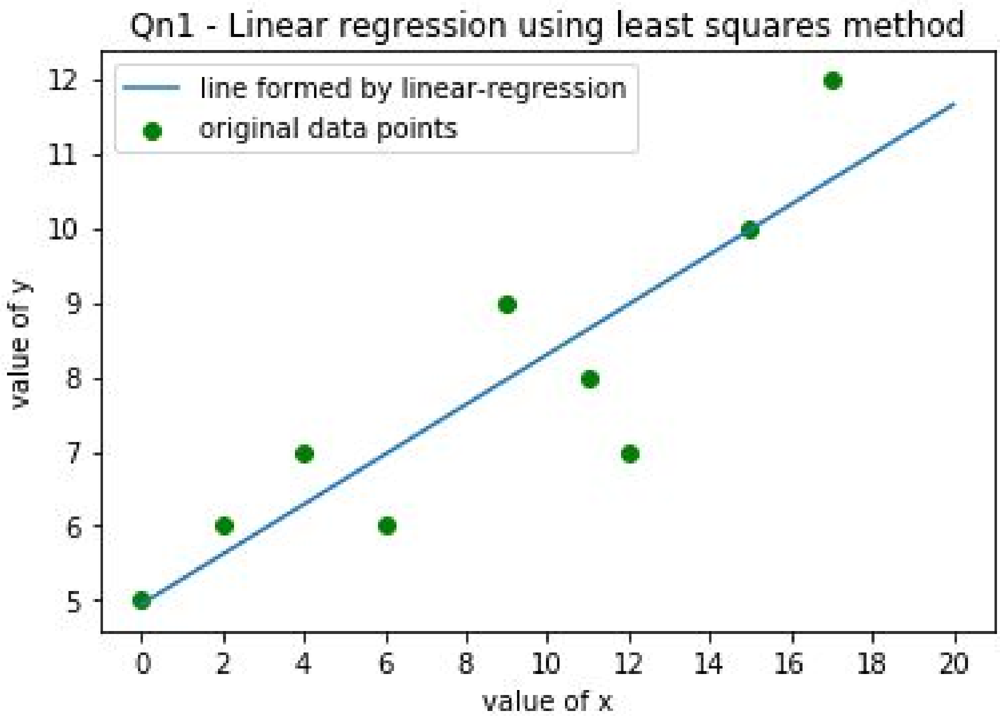
Nohan Joemon

Qn 1:

CALCULATIONS:

					For finding S_t	For finding S_r
i	**x_i (x measured)**	**y_i(y measured)**	**x_i x y_i**	**(x_i)^2**	**(y_i - y_bar)^2**	**(y_i - a0 - a1x_i)^2**
1	0	5	0	0	7.716061728	0.002689770769
2	2	6	12	4	3.160501728	0.1456834392
3	4	7	28	16	0.6049417284	0.506242211
4	6	6	36	36	3.160501728	0.9190500862
5	9	9	81	81	1.493821728	1.073424468
6	11	8	88	121	0.0493817284	0.4021031015
7	12	7	84	144	0.6049417284	3.877768332
8	15	10	150	225	4.938261728	0.000651678784
9	17	12	204	289	17.82714173	1.836973623
n = 9	**Σx_i = 76**	**Σy_i = 70**	**Σ (x_i y_i) = 683**	**Σ((x_i)^2) = 916**	**S_t = Σ(y_i - y_bar)^2) = 39.55556**	**S_r =Σ(y_i - a0 - a1x_i)^2 = 8.7658671**
	a1 = 0.335089	**a0 = 4.948137**				
	y_bar = Σy_i /n = 70/9 = 7.77778					

PLOT:

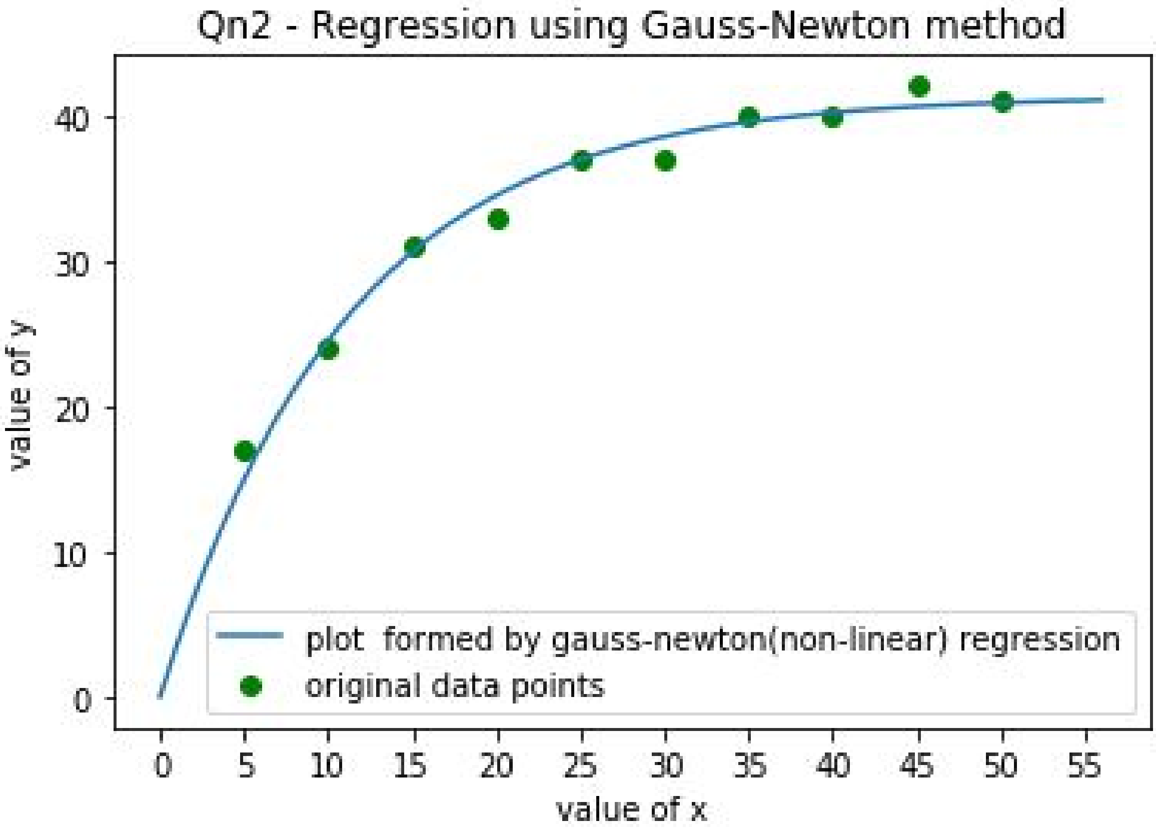

Qn1 - Linear regression using least squares method
line formed by linear-regression
original data points
value of y
value of x
12
11
10
9
8
7
6
5
0
2
4
6
8
10
12
14
16
18
20

Qn 2:

CALCULATIONS:

				For finding S_r	For finding S_t
i	**x**	**y_i (y measured)**	**ycalculated (using a0 = 41.29784 and a1 = 0.08997)**	**(y_i - ycalculated)^2**	**(y_i - y_bar)^2**
1	5	17	14.96122442	4.156605871	295.84
2	10	24	24.50235335	0.2523588912	104.04
3	15	31	30.58695838	0.1706033783	10.24
4	20	33	34.46725584	2.152839702	1.44
5	25	37	36.94181389	0.003385623028	7.84
6	30	37	38.51989847	2.310091344	7.84
7	35	40	39.52628056	0.224410108	33.64
8	40	40	40.16807438	0.02824899604	33.64
9	45	42	40.57736157	2.023900102	60.84
10	50	41	40.83837376	0.02612304285	46.24
	Σx = 275	**Σy_i = 342**		**S_r = Σ (y - ycalculated)^2 = 11.34857**	**S_t = Σ (y_i - y_bar)^2 = 601.6**
				y_bar = 342/10 = 34.2	

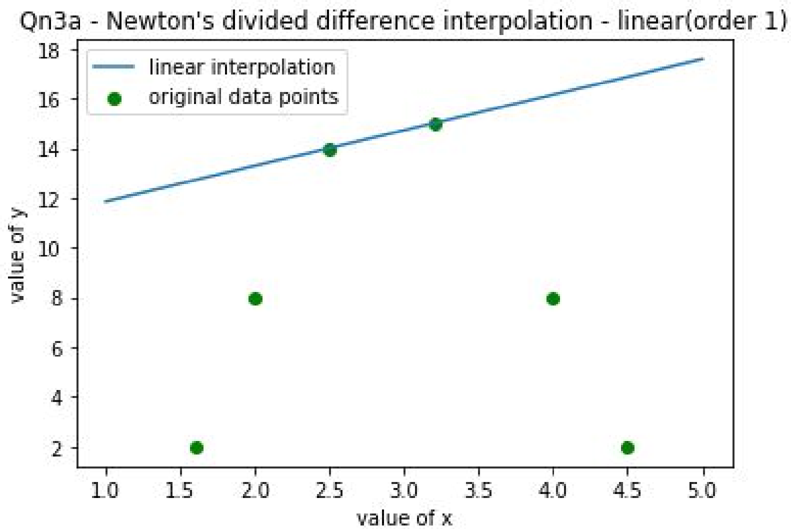
PLOT:



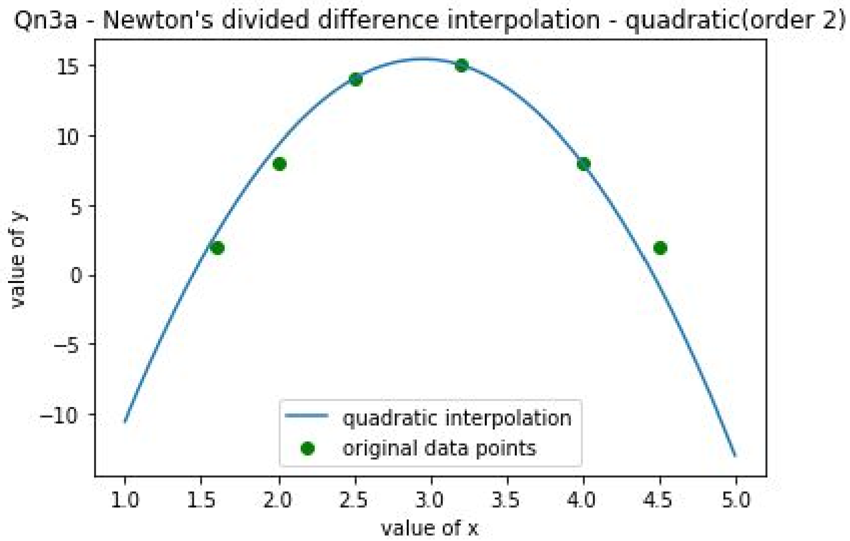
Qn2 - Regression using Gauss-Newton method
value of y
value of x
0
10
20
30
40
0
5
10
15
20
25
30
35
40
45
50
55
plot formed by gauss-newton(non-linear) regression
original data points

Qn 3a:

PLOTS:

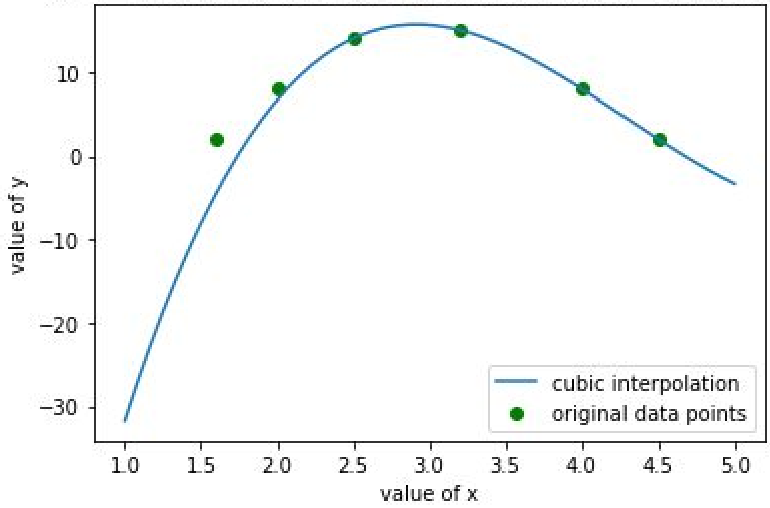


Qn3a - Newton's divided difference interpolation - linear(order 1)
linear interpolation
original data points
value of y
value of x
18
16
14
12
10
8
6
4
2
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0

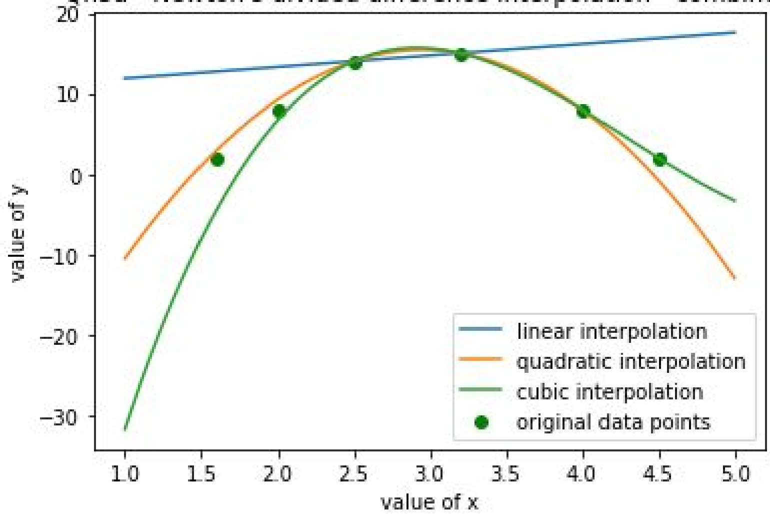


Qn3a - Newton's divided difference interpolation - quadratic(order 2)
quadratic interpolation
original data points
value of y
value of x
15
10
5
0
−5
−10
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0

Qn3a - Newton's divided difference interpolation - cubic(order 3)

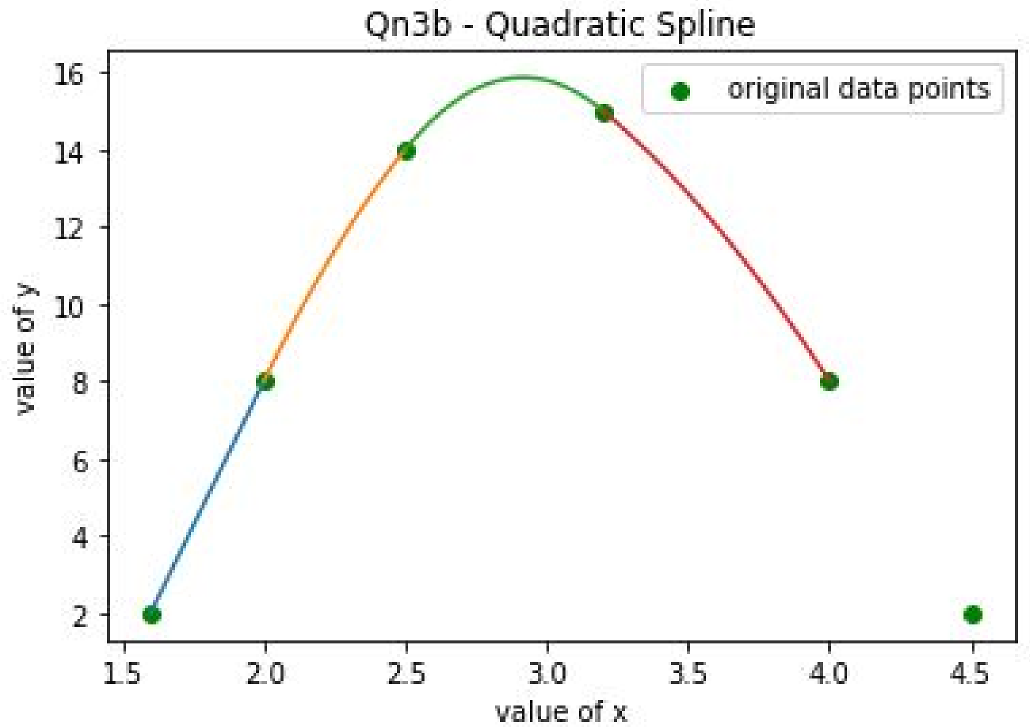


Qn3a - Newton's divided difference interpolation - combined



Qn 3b:

PLOTS:



Qn3b - Quadratic Spline
original data points
16
14
12
10
8
6
4
2
value of y
1.5
2.0
2.5
3.0
3.5
4.0
4.5
value of x