CH2061

NOHAN JOEMON

CH19B072

ASSIGNMENT - 6

(a) **Analytically**

$y(0) = 1$

$$\frac{dy}{dx} = (1+2x)\sqrt{y}$$

$$\frac{dy}{\sqrt{y}} = (1+2x)\,dx$$

$$2\sqrt{y} = x + x^2 + c$$

$$y(0) = 1 \Rightarrow 2 = c$$

$$\therefore 2\sqrt{y} = x + x^2 + 2$$

$$\Rightarrow \boxed{y = \frac{(x^2+x+2)^2}{4}}$$ → plot is attached

↓

$y(0) = 1$

$y(0.25) = 1.33691$

$y(0.5) = 1.89062$

$y(0.75) = 2.74316$

$y(1) = 4$

(b) Euler's method

$$\frac{dy}{dx} = f(x, y) \qquad \text{with } y(0) = 1$$

$$\boxed{y_{i+1} = y_i + \phi_i h}$$

$h = 0.25 \rightarrow$ step size

$\phi_i = \frac{dy}{dx} = (1+2x_i)\sqrt{y_i} \rightarrow$ slope

$\underline{\underline{i=1}}$: $x_1 = 0.00 \qquad y_1 = 1 \qquad x_2 = x_1 + h = 0.25$

$\phi_1 = (1+2x_1)\sqrt{y_1} = \underline{1}$

$y_2 = y_1 + \phi_1 h = 1 + 1 \times 0.25 = \underline{\underline{1.25}}$

$\underline{\underline{i=2}}$: $x_2 = 0.25 \qquad y_2 = 1.25 \qquad x_3 = x_2 + h = 0.5$

$\phi_2 = (1+2x_2)\sqrt{y_2} = 1.677$

$y_3 = y_2 + \phi_2 h = 1.25 + 1.677 \times 0.25 = \underline{\underline{1.669}}$

$\underline{\underline{i=3}}$: $x_3 = 0.5 \qquad y_3 = 1.669 \qquad x_4 = 0.75$

$\phi_3 = (1+2x_3)\sqrt{y_3} = 2.5838$

$y_4 = y_3 + \phi_3 h = \underline{\underline{2.315}}$

$\underline{\underline{i=4}}$: $x_4 = 0.75 \qquad y_4 = 2.315 \qquad x_5 = 1$

$\phi_4 = (1+2x_4)\sqrt{y_4} = 3.804$

$y_5 = y_4 + \phi_4 h = \underline{\underline{3.266}}$

$\phi_5 = (1+2x_5)\sqrt{y_5} = \underline{\underline{5.422}}$

x_i	y_i	$\phi_i = \frac{dy}{dx}$
0	1	1
0.25	1.25	1.677
0.5	1.669	2.5838
0.75	2.315	3.804
1	3.266	5.422

(c) <u>Heun's method</u>

$$\frac{dy}{dx} = f(x,y) = (1+2x)\sqrt{y} \quad \text{with} \quad y(0) = 1$$

$h = 0.25$

$$\boxed{y_{i+1} = y_i + \left(\tfrac{1}{2}k_1 + \tfrac{1}{2}k_2\right)h}$$

$$k_1 = f(x_i, y_i)$$

$$k_2 = f(x_i + h, y_i + k_1 h)$$

<u>i=1</u> : $x_1 = 0$, $y_1 = 1$, $x_2 = 0.25$

$$k_1 = f(x_1, y_1) = f(0,1) = 1$$

$$k_2 = f(x_1 + 0.25, y_1 + k_1 \times 0.25) = f(0.25, 1.25)$$

$$= 1.677$$

$$y_2 = y_1 + \left(\tfrac{1}{2}k_1 + \tfrac{1}{2}k_2\right)h$$

$$= 1 + \frac{(1+1.677)}{2}(0.25) = \underline{\underline{1.335}}$$

$\underline{\underline{i=2}}$: $x_2 = 0.25$ $x_3 = 0.5$ $y_2 = 1.335$

$k_1 = f(x_2, y_2) = 1.73313$

$k_2 = f(x_2 + h, y_2 + k_1 h) = f(0.5, 1.76828) = 2.6595$

$y_3 = y_2 + \frac{1}{2}(k_1 + k_2)h = \underline{\underline{1.8841}}$

$\underline{\underline{i=3}}$: $x_3 = 0.5$ $x_4 = 0.75$ $y_3 = 1.8841$

$k_1 = f(x_3, y_3) = 2.7452$

$k_2 = f(x_3 + h, y_3 + k_1 h) = f(0.75, 2.5704) = 4.0081$

$y_4 = y_3 + \frac{1}{2}(k_1 + k_2)h = \underline{\underline{2.7283}}$

$\underline{\underline{i=4}}$: $x_4 = 0.75$ $x_5 = 1.00$ $y_4 = 2.7283$

$k_1 = f(x_4, y_4) = \underline{\underline{4.12939}}$

$k_2 = f(x_4 + h, y_4 + k_1 h) = f(1, 3.76) = \underline{\underline{5.8172}}$

$y_5 = y_4 + \frac{1}{2}(k_1 + k_2)h = \underline{\underline{3.9716}}$

x_i	y_i
0	1
0.25	1.335
0.5	1.8841
0.75	2.7283
1.00	3.9716

(d) Ralston Method

$$\frac{dy}{dx} = f(x,y) = (1+2x)\sqrt{y} \qquad \text{with } y(0) = 1$$

$$h = 0.25$$

$$\boxed{y_{i+1} = y_i + \left(\frac{1}{3}k_1 + \frac{2}{3}k_2\right)h}$$

$$k_1 = f(x_i, y_i)$$

$$k_2 = f\left(x_i + \frac{3}{4}h,\ y_i + \frac{3}{4}k_1 h\right)$$

$i=1$: $x_1 = 0$, $y_1 = 1$, $x_2 = 0.25$

$$k_1 = f(x_1, y_1) = f(0,1) = 1$$

$$k_2 = f\left(x_1 + \frac{3}{4}h,\ y_1 + \frac{3}{4}k_1 h\right) = f(0.1875, 1.1875)$$

$$= 1.49837$$

$$y_2 = y_1 + \left(\frac{1}{3}k_1 + \frac{2}{3}k_2\right)h = \underline{1.33306}$$

$i=2$: $x_2 = 0.25$, $y_2 = 1.33306$, $x_3 = 0.5$

$$k_1 = f(x_2, y_2) = 1.73187$$

$$k_2 = f\left(x_2 + \frac{3}{4}h,\ y_2 + \frac{3}{4}k_1 h\right) = f(0.4375, 1.65778)$$

$$= 2.4142$$

$$y_3 = y_2 + \left(\frac{1}{3}k_1 + \frac{2}{3}k_2\right)h = \underline{1.87975}$$

$i=3$: $x_3 = 0.5$, $y_3 = 1.87975$, $x_4 = 0.75$

$$k_1 = f(x_3, y_3) = 2.74208$$

$$k_2 = f\left(x_3 + \frac{3}{4}h,\ y_3 + \frac{3}{4}k_1 h\right) = f(0.6875, 2.39389)$$

$$= 3.67465$$

$$y_4 = y_3 + \left(\frac{1}{3}k_1 + \frac{2}{3}k_2\right)h = \underline{2.72070}$$

$i=4: \; x_4 = 0.75, \quad y_4 = 2.72070, \quad a_5 = 1$

$$k_1 = f(x_4, y_4) = 4.12364$$

$$k_2 = f\left(x_4 + \frac{3}{4}h, \; y_4 + \frac{3}{4}k_1 h\right) = f(0.9375, 3.49388)$$

$$= 5.37393$$

$$y_5 = y_4 + \left(\frac{1}{3}k_1 + \frac{2}{3}k_2\right)h = \underline{\underline{3.95999}}$$

x_i	y_i
0	1
0.25	1.33306
0.5	1.87975
0.75	2.72070
1	3.95999

(e) **Fourth-order RK method :**

$$\frac{dy}{dx} = f(x,y) = (1+2x)\sqrt{y} \quad \text{with} \quad y(0) = 1$$

$$\boxed{y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)h} \qquad h = 0.25$$

$$k_1 = f(x_i, y_i)$$

$$k_2 = f\left(x_i + \frac{1}{2}h, \; y_i + \frac{1}{2}k_1 h\right)$$

$$k_3 = f\left(x_i + \frac{1}{2}h, \; y_i + \frac{1}{2}k_2 h\right)$$

$$k_4 = f(x_i + h, \; y_i + k_3 h)$$

$\underline{\underline{i=1}}$: $x_1 = 0$, $y_1 = 1$, $x_2 = 0.25$

$k_1 = f(x_1, y_1) = 1$

$k_2 = f\left(x_1 + \frac{h}{2}, y_1 + \frac{k_1 h}{2}\right) = 1.32582$

$k_3 = f\left(x_1 + \frac{h}{2}, y_1 + \frac{k_2 h}{2}\right) = 1.34961$

$k_4 = f(x_1 + h, y_1 + k_3 h) = 1.73469$

$y_2 = y_1 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)h$

$y_2 = \underline{\underline{1.336898}}$

$\underline{\underline{i=2}}$: $x_2 = 0.25$, $y_2 = 1.336898$, $x_3 = 0.5$

$k_1 = f(x_2, y_2) = 1.734365$

$k_2 = f\left(x_2 + \frac{h}{2}, y_2 + \frac{k_2 h}{2}\right) = 2.18133$

$k_3 = f\left(x_2 + \frac{h}{2}, y_2 + \frac{k_2 h}{2}\right) = 2.22020$

$k_4 = f(x_2 + h, y_2 + k_3 h) = 2.75096$

$y_3 = y_2 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)h = \underline{\underline{1.89058}}$

$\underline{\underline{i=3}}$: $x_3 = 0.5$, $y_3 = 1.89058$, $x_4 = 0.75$

$k_1 = f(x_3, y_3) = 2.74997$

$k_2 = f\left(x_3 + \frac{h}{2}, y_3 + \frac{k_1 h}{2}\right) = 3.36322$

$k_3 = f\left(x_3 + \frac{h}{2}, y_3 + \frac{k_2 h}{2}\right) = 3.42043$

$k_4 = f(x_3 + h, y_3 + k_3 h) = 4.14253$

$y_4 = y_3 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)h = \underline{\underline{2.74307}}$

$\underline{i=4}$: $x_4 = 0.75$, $y_4 = 2.74307$, $x_5 = 1.00$

$k_1 = f(x_4, y_4) = 4.14056$

$k_2 = f\left(x_4 + \frac{h}{2}, y_4 + \frac{k_1 h}{2}\right) = 4.96574$

$k_3 = f\left(x_4 + \frac{h}{2}, y_4 + \frac{k_2 h}{2}\right) = 5.04368$

$k_4 = f(x_4 + h, y_4 + k_3 h) = 6.00299$

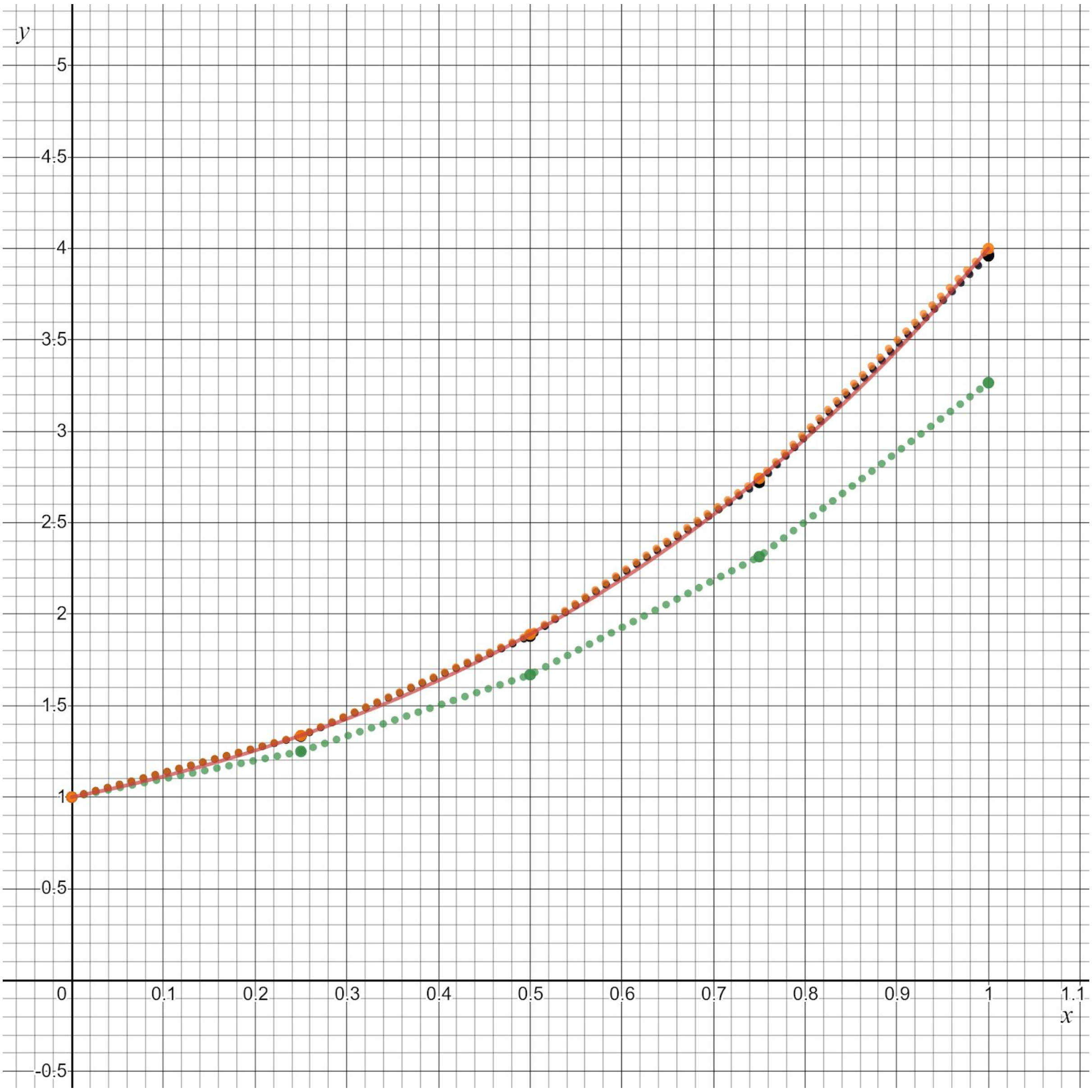
$y_5 = y_4 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) h = \underline{3.99984}$

x	y
0	1
0.25	1.336898
0.5	1.89058
0.75	2.74307
1	3.99984

ALL -METHODS COMPARISON

	x	(a) Analytical (True soln) y	(b) Euler's method y	(c) Heun's method y	(d) Ralston method y	(e) 4th order RK y
(Given)	0	1	1	1	1	1
	0.25	1.33691	1.25	1.335	1.33306	1.336898
	0.5	1.89062	1.669	1.8841	1.87975	1.89058
	0.75	2.74316	2.315	2.7283	2.72070	2.74307
	1	4	3.266	3.9716	3.95999	3.99984

PLOT



_____: analytical solution graph (plot of the function)

----O----: Euler's method

----O----: Heun's method

----O-----: Ralston method

----O-----: 4th order RK method

Note: Only Euler's method is far from the analytical solution. Solutions obtained by other methods are very close to each other and therefore, we cannot distinguish them properly