Assignment 3

Problem 1. **Gauss elimination and LU decomposition** (*50 marks*)

Consider a system of three linear equations

$$2x + y + z = 4$$
$$x + 3y + z = 0$$
$$x - y + 3z = 6$$

a) Write the above equation in the form of $Ap = b$.

b) Find the solution p of the above equations using Gauss Elimination.

c) Decompose the coefficient matrix A into a lower triangular matrix L and upper triangular U matrix using Gauss Elimination. **Hint**: You already have the elements of L and U after solving step (b).

d) Find the solution p of the above equations using forward and back substitution with L and U, respectively, i.e., by solving

$$Lm = b$$
$$Up = m$$

e) Determinant of a triangular matrix (e.g., L and U) can be easily calculated by multiplying the main diagonal elements. For example,

$$det(L) = L_{11}\, L_{22} \ldots L_{nn} = \prod_{i=1}^{n} L_{ii}$$
$$det(U) = U_{11}\, U_{22} \ldots U_{nn} = \prod_{i=1}^{n} U_{ii}$$

where n is the size of matrices L and U. Therefore, Gauss elimination provides a convenient way to calculate the determinant of a matrix. Check if the determinant of A is same as the determinant of U.

f) Calculate the inverse of matrix A using the triangular matrices L and U.

Problem 2. Thomas Algorithm/Tridiagonal Matrix Algorithm (*20 marks*)

$$0.8x - 0.4y = 41$$
$$-0.4x + 0.8y - 0.4z = 25$$
$$-0.4y + 0.8z = 105$$

a) Write the above equations in the form of $Ap = b$.

b) Find the solution p of the above equations using the Thomas algorithm.

c) Comment on the number of calculations performed in part (b), compared to Problem 1, where you used Gauss Elimination.

Problem 3. Gauss Seidel and relaxation (*30 marks*)

Consider a system of two linear equations

$$x + 2y = 1 \qquad (1)$$

$$x - y = 4 \qquad (2)$$

a) Use **four iterations** of the Gauss Seidel method to solve for x and y. Use Eq. (1) to solve for x and Eq. (2) to solve for y. Report the values of x and y and the approximate percent relative error in x and y at the end of each iteration. What do you observe? Is the method converging?

b) Use underrelaxation with $\lambda = 0.6$ and repeat part (a). Does underrelaxation help?

c) Make the system of equations diagonally dominant. Repeat step (a). What do you observe? Is the method converging?