Assignment 1

Problem 1. Evaluate the function f(x,y) and the associated round-off error in terms of percent relative error.

$$f(x,y) = (x+y)/(x-y)$$

Given: x=1.5001 and y=1.4999. Use 5-digit, 4-digit, 3-digit, and 2-digit arithmetic with chopping. For example, for doing 3-digit arithmetic, use x=1.50 and y=1.49. (*10 marks*)

Problem 2. Taylor series for the exponential function.

(a) Using exp(x=0) = 1 as the base point, calculate the zeroth-, first-, and second-order approximation of the Taylor series for exp(x=0.5), such that the step size h=0.5.

(b) Report the truncation error (in the form of true fractional relative error) for each approximation (zeroth-, first-, and second-order) used in part (a). Use the true value of exp(x=0.5) as 1.649.

(c) Repeat the calculations performed in part(a) using h=0.25, 0.5, and 1. Show the variation in the truncation error due to the changing step size for different approximations (zeroth-, first-, and second-order) of the Taylor series. Explain your observations. (*30 marks*)

Problem 3. Taylor series for a general 3rd degree polynomial:

$$f(x) = ax^3 + bx^2 + cx + d,$$

(a) Using x=0 as the base point, estimate f(x=1) by 2nd order Taylor series approximation and calculate the associated truncation error using the truncation formula given by

$$R_n = f^{(n+1)}(\xi)h^{n+1}/(n+1)!$$

for n^{th} order Taylor series approximation.

(b) Show that the 3rd order Taylor series approximation is exactly equal to the given polynomial f(x) for any value of x. (*30 marks*)

Problem 4. The Stefan-Boltzmann law can be employed to estimate the rate of radiation of energy Q from a surface:

$$Q = Ae\sigma T^4$$

where A is the surface area, e is the emissivity, σ is the Stefan-Boltzmann constant (= 5.67×10^{-8}), and T is the temperature. Given A=0.15 and e=0.9. Compute the error in Q for:

(a) $T = 650\pm10$

(b) $T = 650\pm50$

For both (a) and (b) use the first-order and second-order approximation of the Taylor series to computer the error in Q. For each case, compare your results with the exact error and comment on your observation. (*30 marks*)