Assignment 2

Problem 1. **Bisection method** (*10 marks*)

Water is flowing in a trapezoidal channel at a rate of Q = 40 m^3/s. The critical depth y for such a channel must satisfy the equation

$$0 = 1 - (Q^2B / gA_c^3)$$

where g = 9.81 m/s^2, A_c = the cross-sectional area (m^2), and B = the width of the channel at the surface (m). For this case, the width and the cross-sectional area can be related to y by

$$B = 3 + y \quad \text{and} \quad A_c = 3y + y^2/2$$

Solve for the critical depth using bisection method. Use initial guesses of x_l = 1.0 and x_u = 3.0 and iterate until the approximate percent relative error falls below 1% or the number of iterations exceed 10.

Problem 2. Regula fasli/False position method (*10 marks*)

The velocity v of a falling parachutist is given by

$$v = \frac{gm}{c}(1 - \exp(-c/m)t)$$

where g=9.81 m/s^2. For a parachutist with a drag coefficient c = 15 kg/s, compute the mass m so that the velocity is v = 35 m/s at t = 9 s. Use the false position/regula fasli method to determine m until the approximate error falls below 1%. You can choose the initial guesses to cover a wide range of mass for an adult human.

Problem 3. Newton-Raphson method (*20 marks*)

a) We have a defective calculator that could only add, subtract, and multiply but not divide. Using this calculator and the Newton-Raphson method, find the value of $\frac{1}{2.834}$ up to 5 decimal places. Use the initial guess as 0.4. Take assumptions wherever necessary and state them clearly. With this, write down a general equation to compute the reciprocal of any number x.

b) The same calculator has a defective square-root button too. With the help of this calculator and the Newton-Raphson method, compute the square root of 12.425 up to 5 decimal places. Use the initial guess as 3.5. With this, develop a general model to calculate the r^{th} root of any number N using the Newton-Raphson method.

Trivia: Google and find out why such calculations are/were necessary in the present/past (not for grading).

Tip: Don't just solve it. Try to graphically interpret how the solutions are being approached with increasing iterations (you don't have to submit any graphs).

Problem 4. Secant method (*15 marks*)

Using the Secant method, find the **positive root** of the following circle (i.e., the point where the circle intersects the x-axis)

$$(x+1)^2 + (y-2)^2 = 16$$

Comment on the following

- How can the negative root of the same circle be found?
- You have the freedom to choose the initial conditions. What initial conditions will you choose and what are the limits for choosing the same?

Problem 5. **Fixed-point iteration method** (*15 marks*)
We will use a modified fixed-point iteration procedure to find the roots of equation:

$$G(x) = 2 - x + ln(x) = 0$$

For fixed-point iteration of the form $x_{i+1} = g(x_i)$, let us define $g(x) = x + \beta G(x)$. Find the sufficient condition that fixed-point iteration with the above choice of $g(x)$ converges. Choose the initial guess for x that satisfies the sufficient condition and use fixed-point iteration until the approximate percent relative error is less than 1%.

Problem 6. **Multiple roots** (*10 marks*)
The function $x^3 - 2x^2 - 4x + 8$ has a double root at $x = 2$. Use **(a)** the standard Newton-Raphson, **(b)** the modified Newton-Raphson (method 1 from class notes) and **(c)** the modified Newton- Raphson (method 2 from class notes) to solve for the root at $x = 2$. Compare and discuss the rate of convergence using an initial guess of x_0 =1.2.

Problem 7. **System of non-linear equations** (*10 marks*)
Determine the roots of the following simultaneous nonlinear equations using **(a)** fixed-point iteration and **(b)** the Newton-Raphson method:

$$y = -x_2 + x + 0.75$$

$$y + 5xy = x_2$$

Employ initial guesses of $x = y = 1.2$ and discuss the results.

Problem 8. **Mueller's method** (*10 marks*)
A two-dimensional circular cylinder is placed in a high-speed uniform flow. Vortices shed from the cylinder at a constant frequency, and pressure sensors on the rear surface of the cylinder detect this frequency by calculating how often the pressure oscillates. Given three data points, use Müller's method to find the time where the pressure was zero. **Hint:** You can fit a polynomial through the given data set.

Time	0.6	0.62	0.64
Pressure	20	50	60