Assignment problems solution

Asssignment 1

Problem 1. Evaluate the function f(x,y) and the associated round-off error in terms of percent relative error.

$$f(x,y) = (x+y)/(x-y)$$

Given: x=1.5001 and y=1.4999. Use 5-digit, 4-digit, 3-digit, and 2-digit arithmetic with chopping. For example, for doing 3-digit arithmetic, use x=1.50 and y=1.49. *(10 marks)*

Answer:

i) Function : $f(x,y) = \frac{x+y}{x-y}$

ii) Percent relative error : $\frac{|Numerical\ Soution - True\ Solution|}{Numerical\ Solution}$ x 100

for the given x,y the numerical Solution $f(1.5001,1.4999) = \frac{(1.5001+1.4999)}{(1.5001-1.4999)} = 15{,}000$

Chopping Digits	x	y	Value	PRE
2	1.5	1.4	29	99.81
3	1.50	1.49	299	98.00
4	1.500	1.499	2999	80.00
5	1.5001	1.4999	15,000	00.00

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Asssignment 1

Problem 2. Taylor series for the exponential function.

(a) Using exp(x=0) = 1 as the base point, calculate the zeroth-, first-, and second-order approximation of the Taylor series for exp(x=0.5), such that the step size h=0.5.

(b) Report the truncation error (in the form of true fractional relative error) for each approximation (zeroth-, first-, and second-order) used in part (a). Use the true value of exp(x=0.5) as 1.649.

(c) Repeat the calculations performed in part(a) using h=0.25, 0.5, and 1. Show the variation in the truncation error due to the changing step size for different approximations (zeroth-, first-, and second-order) of the Taylor series. Explain your observations. (*30 marks*)

Answers:

a) Zeroth order approximated value = 1
First order approximated value = 1.5
Second order approximated value =1.625

b) & c)

Values:

H	True Value	Zeroth order	First order	Second order
0.25	1.284	1	1.25	1.2813
0.50	1.649	1	1.5	1.625
1.00	2.718	1	2	2.5

Relative Fractional Error:

H	Zeroth order	First order	Second order
0.25	0.221	0.026	0.002
0.50	0.3936	0.0904	0.0146
1.00	0.632	0.264	0.08

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3) $\boldsymbol{f(x)} = ax^3 + bx^2 + cx + d$

a) Second order Taylor series expansion

$$\boldsymbol{f(x+h) = f(x) + f^1(x)h + \frac{f^2(x)h^2}{2!}}$$

$\boldsymbol{f^1(x)} = 3ax^2 + 2bx + c$

$\boldsymbol{f^2(x)} = 6ax + 2b$

$\boldsymbol{f^3(x)} = 6a$ (Constant function)

$\boldsymbol{f^4(x)} = 0$

Here, the base value is 0. x=0;

The function is to be evaluated at x=1. So, h=1;

$$f(0+1) = f(0) + f^1(0) * 1 + \frac{f^2(0)*1^2}{2!}$$

$\boxed{\boldsymbol{f(1) = d + c + b} \rightarrow \textbf{Value of the function evaluated with Second order Taylor series Approximation}}$

Evaluation of Truncation Error at associated with the Second-order Taylor series approximation

Reminder Term $R_n = \boldsymbol{\frac{f^{(n+1)}(\xi)h^{(n+1)}}{(n+1)!}}$

In this case, $\boxed{R_2 = \boldsymbol{\frac{f^{(3)}(\xi)h^{(3)}}{3!}} = a}$

Note that, the third derivative of this function is a constant and independent of the term ξ.

b) Proof that 3rd order Taylor series expansion is exactly equal to the polynomial f(x) for any value of x.

To prove this, it is sufficient to show that $R_3=0$;

It shows that, if we use a third order approximation, the reminder terms are zero

$\boxed{R_3 = \boldsymbol{\frac{f^{(4)}(\xi)h^{(4)}}{4!}} = 0}$

It could be also shown that, if we take a third order approximation,

$$f(x+h) = f(x) + f^1(x)h + \frac{f^2(x)h^2}{2!} + \frac{f^3(x)h^3}{3!}$$

$$f(0+1) = f(0) + f^1(0) * 1 + \frac{f^2(0)*1^2}{2!} + \frac{f^3(0)*1^3}{3!}$$

$\boxed{\boldsymbol{f(1) = d + c + b + a}}$ **which is exactly equal to the actual value of the function at x=1;**

It is also to be noted the difference between the function value using the second-order and the third order approximations is exactly equal to R_2.

④ Given: $Q = Ae\sigma T^4$

$A = 0.15$; $e = 0.9$; $\sigma = 5.67 \times 10^{-8}$.

Evaluating True value/Actual value:

$T = 650.$

$\therefore Q = Ae\sigma T^4$

$= 0.15 \times 0.9 \times 5.67 \times 10^{-8} \times [650]^4.$

$\boxed{Q = 1366.3760}$ (2 mark).

a) $650 \pm 10. = T \pm \Delta T.$

Taylor Series expansion:

$f(x+h) = f(x) + \frac{f'(x).h}{1!} + \frac{f''(x).h^2}{2!} + \frac{f'''(x) h^3}{3!} \ldots$

i) Ist order approximation:

Taylor Series expansion in discrete method.

$\Delta f(\tilde{x}) = |f'(x)| \Delta \tilde{x}$

5 mark.

Given: $f(x) = Ae\sigma T^4.$

Let all the constants. be grouped.

let $A.e.\sigma = S$

$f(x) = ST^4.$

differentiate $\boxed{f'(x) = 4ST^3}$

$\Rightarrow \boxed{f''(x) = 12ST^2}$

$\Delta Q_I = (4ST^3)\,\Delta T.$

$= 4 \times A \times e \times \sigma \times 650^3 \times 10.$

$= 4 \times 0.15 \times 0.9 \times 5.67 \times 10^{-8} \times 650^3 \times 10.$

$\boxed{\Delta Q_I = 84.0847}$

ii) Second Order Approximation:

Discreete Taylor Series expansion:

$$\Delta f(\tilde{x}) = |f'(x)|\,\Delta\tilde{x} + |f''(x)|\frac{(\Delta\tilde{x})^2}{2!}$$

$$\Delta Q_{II}(T) = Q'(T)\,\Delta T + Q''(T)\frac{(\Delta T)^2}{2!}$$

5 marks.

$$\Delta Q_{II}(T) = 84.0847 + 12ST^2 \times \frac{10^2}{2}.$$

$$= 84.0847 + 12 \times Ae\sigma \times (650)^2 \times \frac{100}{2}.$$

$$= 84.0847 + 1.9404.$$

$\Delta Q_{II}(T) = 84.0847 + 1.9404$

$\boxed{\Delta Q_{II}(T) = 86.0251}$

Range of deviation from True value: i) Negative Direction

$T^- = 650 - 10 = 640.$

$\therefore Q^- = Ae\sigma (T^-)^4.$

$= Ae\sigma(640)^4.$

$\boxed{Q^- = 1284.2119}$

Error in -ve Direction:

$\Delta e^- = |\text{Act value} - \text{Deviation value}|.$

$= |1366.3760 - 1284.2119|$

$\boxed{\Delta e^- = 82.1641}$

Similarly, Positive direction:

$T^+ = 650+10 = 660.$

$Q^+ = Ae\sigma(660)^4$

$\boxed{Q^+ = 1452.4211}$

Error in +ve Direction:

$\Delta e^+ = |1366.3760 - 1452.4211|.$

$\boxed{\Delta e^+ = 86.0451}$

Range of error. (1 mark)

$\boxed{82.1641 \leq \Delta e \leq 86.0451}$

b). $T = 650$; $\Delta T = 50.$

i> I^{st} Order Approximation: 5 mark

$\Delta Q_I(T) = Q'(T)\Delta T.$

$= 4ST^3 \times 50.$

$= 4Ae\sigma.(650)^3 \times 50.$

$\boxed{\Delta Q_I(T) = 420.4231}$

ii> II^{nd} Order Approx: 5 mark.

$\Delta Q_{II}(T) = Q'(T)\Delta T + Q''(T)\frac{(\Delta T)^2}{2!}$

$= \Delta Q_I(T) + 12ST^2 \times \frac{50^2}{2}.$

$= 420.4231 + 48.5112.$

$\boxed{\Delta Q_{II}(T) = 468.934}$

Range of deviation from True Value:

-ve direction:

$T^- = 650 - 50 = \cancel{640}600$

$Q^- = A e \sigma (600)^4$

$\boxed{Q^- = 992.0232}$

2 mark

+ve direction:

$T^+ = 650 + 50 = 700.$

$Q^+ = A \sigma e (700)^4$

$\boxed{Q^+ = 1837.8454}$

error in -ve direction:

$= |1366.3760 - 992.0232|$

$\boxed{\Delta e^- = 374.3528.}$

error in +ve direction:

$= |1366.3760 - 1837.8454|.$

$\boxed{\Delta e^+ = 471.4691}$

Range of error.

$\boxed{374.3528 \leq \Delta e \leq 471.4691}$

1 mark

Observation:

a). The higher Order Taylor Series is more closer to maximum error. As, the error is of Smaller Magnitude the First Order & Second Order error are closer to Max. error.

b). The higher Order Taylor Series is much more closer to the Max. error. than the first Order. (4 marks).