CH6060

ENDSEM

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PART - B

1:

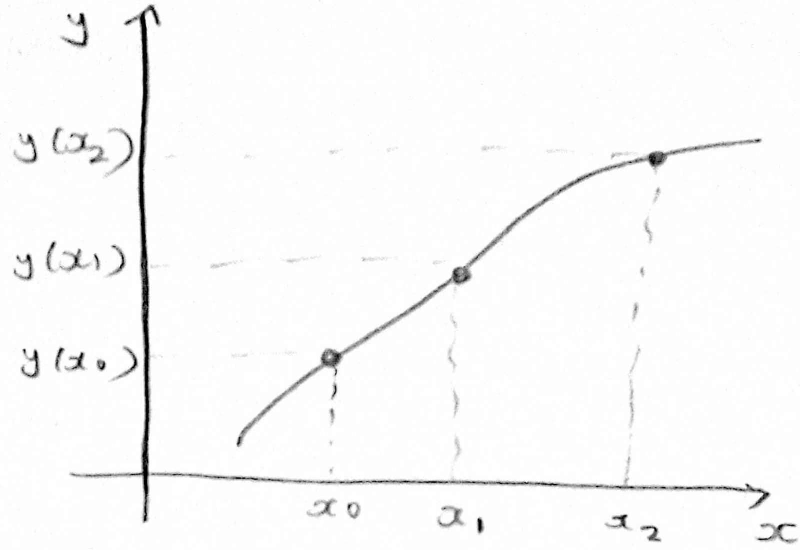
(a) Interpolation of data means producing a function that matches the given data exactly. It should be done such that the function should provide a reasonable approximation to intermediate data points.

Lagrange Interpolation

Obtain a function $P(x)$ that goes through the data points. Use $P(x)$ to estimate the values at intermediate points. $P(x)$ has a specific form in Lagrange interpolation

Suppose we have 3 data points:

$$(x_0, y(x_0))$$
$$(x_1, y(x_1))$$
$$(x_2, y(x_2))$$



We approximate $y(x)$ by $P(x)$

$$y(x) \approx P(x) \rightarrow \text{Lagrange interpolating polynomial}$$

$P(x)$ is written as :

$$P(x) = L_0(x)\,y(x_0) + L_1(x)\,y(x_1) + L_2(x)\,y(x_2)$$

where

$$L_0(x) = \frac{(x - x_1)(x - x_2)}{(x_0 - x_1)(x_0 - x_2)}$$

$$L_1(x) = \frac{(x - x_0)(x - x_2)}{(x_1 - x_0)(x_1 - x_2)}$$

$$L_2(x) = \frac{(x - x_0)(x - x_1)}{(x_2 - x_0)(x_2 - x_1)}$$

~~verification~~

$x = x_0 \longrightarrow$ $\cancel{P(x_0) =}$ $\cancel{L_0(x_0) = y(x_0)}$

verification

$x = x_0 \longrightarrow$

$$\left.\begin{aligned} L_0(x_0) &= 1 \\ L_1(x_0) &= 0 \\ L_2(x_0) &= 0 \end{aligned}\right\} \quad P(x_0) = y(x_0)$$

$x = x_1 \longrightarrow$

$$\left.\begin{aligned} L_0(x_1) &= 0 \\ L_1(x_1) &= 1 \\ L_2(x_1) &= 0 \end{aligned}\right\} \quad P(x_1) = y(x_1)$$

$x = x_2 \longrightarrow$

$$\left.\begin{aligned} L_0(x_2) &= 0 \\ L_1(x_2) &= 0 \\ L_2(x_2) &= 1 \end{aligned}\right\} \quad P(x_2) = y(x_2)$$

$\therefore$ $P(x)$ pass through all three points

Q. Lagrange interpolation (3 points)

$$\boxed{\begin{aligned} P(x) = {} & \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}\, y(x_0) \\ & + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}\, y(x_1) \\ & + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}\, y(x_2) \end{aligned}}$$

Simpson's $\frac{1}{3}$ rd rule

$$\int_{x_0}^{x_2} y(x)\,dx \approx \int_{x_0}^{x_2} P(x)\,dx$$

$$\Downarrow$$

We use Lagrange interpolation (3 points) to approximate $y(x)$ as $P(x)$ and then integrate over $P(x)$ instead of $y(x)$.

$x_1 = x_0 + h \qquad x_2 = x_0 + 2h$

$$\int_{x_0}^{x_2} P(x)\,dx = \int_{x_0}^{x_2} \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}\, y(x_0)\,dx \quad (I_1)$$

$$+ \int_{x_0}^{x_2} \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}\, y(x_1)\,dx \quad (I_2)$$

$$+ \int_{x_0}^{x_2} \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}\, y(x_2)\,dx \quad (I_3)$$

Integral - (1)

$$\int_{x_0}^{x_2} \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}\, y(x_0)\,dx$$

$$= \frac{y(x_0)}{-h \times -2h} \int_{x_0}^{x_2} \left(x^2 - (x_1+x_2)x + x_1 x_2\right) dx$$

$$= \frac{y(x_0)}{2h^2} \left[\frac{x^3}{3} - (x_1+x_2)\frac{x^2}{2} + x_1 x_2 x \right]_{x_0}^{x_2}$$

Taking $x_0 = a$,

$x_1 = a + h$

$x_2 = a + 2h = b$;

$$\int_{x_0}^{x_2} P(x)\,dx = \int_a^{a+2h} \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} y(x_0)\,dx \quad (I_1)$$

$$+ \int_a^{a+2h} \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} y(x_1)\,dx \quad (I_2)$$

$$+ \int_a^{a+2h} \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} y(x_2)\,dx \quad (I_3)$$

Let $x = a + ht \Rightarrow dx = h\,dt$

$t = \frac{x-a}{h}$

I_1

$$\int_0^2 \frac{h(t-1)\,h(t-2)}{(-h)(-2h)} y(x_0)\,h\,dt$$

$$= h\,y(x_0) \int_0^2 \frac{(t-1)(t-2)\,dt}{(-1)(-2)}$$

$$= \frac{h}{2} y(x_0) \int_0^2 (t^2 - 3t + 2)\,dt$$

$$= \frac{h}{2} y(x_0) \left[\frac{t^3}{3} - \frac{3t^2}{2} + 2t \right]_0^2$$

$$= \frac{h}{2} y(x_0) \left[\frac{8}{3} - \frac{3\times 4}{2} + 2\times 2 \right]$$

$$= \frac{h}{3} y(x_0)$$

I_2

$$\int_0^2 h \frac{(t-0)(t-2)\, y(x_1)\, dt}{(1-0)(1-2)}$$

$$= \frac{h\, y(x_1)}{-1} \int_0^2 (t)(t-2)\, dt$$

$$= -h\, y(x_1) \left[\frac{t^3}{3} - t^2 \right]_0^2$$

$$= -h\, y(x_1) \left[\frac{8}{3} - 4 \right]$$

$$= \frac{4h}{3}\, y(x_1)$$

I_3

$$\int_0^2 h \frac{(t-0)(t-1)\, y(x_2)\, dt}{(2-0)(2-1)}$$

$$= \frac{h\, y(x_2)}{2} \int_0^2 (t)(t-1)\, dt$$

$$= \frac{h\, y(x_2)}{2} \left[\frac{t^3}{3} - \frac{t^2}{2} \right]_0^2$$

$$= \frac{h\, y(x_2)}{2} \left(\frac{8}{3} - 2 \right)$$

$$= \frac{h}{3}\, y(x_2)$$

$\therefore \quad I_1 = I_1 + I_2 + I_3$

$$\boxed{\int_{x_0}^{x_2} y(x)\,dx \approx \frac{h}{3}\left[y(x_0) + 4y(x_1) + y(x_2)\right]}$$

Simpson's 1/3 rule

(b) <u>Runge- Kutta 4th order</u>

RK method uses sampling of slopes through on interval and takes a weighted average slope to calculate end point

<u>Derivation</u>

$$\frac{dy}{dx} = y'(x) \Rightarrow dy = y'(x)\,dx$$

$$\int dy = \int y'(x)\,dx$$

$$y(x_{n+1}) - y(x_n) = \int_{x_n}^{x_{n+1}} y'(x)\,dx$$

~~y ta~~ $x_{n+1} = x_n + h$

$\therefore$

$$y(x_n + h) - y(x_n) = \int_{x_n}^{x_n+h} y'(x)\,dx \rightarrow (1)$$

We can approximate RHS using Lagrange interpolated polynomial

$$\int_{x_n}^{x_n+h} y'(x)\,dx \approx \int_{x_n}^{x_n+h} p(x)\,dx$$

But integrating over $p(x)$ (Lagrange) is same as doing Simpson's $\frac{1}{3}$ rd rule

$\therefore$ We use

$$z_0 = x_n$$

$$z_1 = x_n + \frac{h}{2} = x_n + h'$$

$$z_2 = x_n + h = x_n + 2h'$$

$$(h' = \frac{h}{2})$$

$$\therefore \cancel{\int_{x_n}^{x_n+h} y'(x)\,dx \approx \int_{z_0}^{z_0+2h'} P}$$

$$\therefore \int_{x_n}^{x_n+h} y'(x)\,dx \approx \frac{h'}{3}\left[y'(z_0) + 4y'(z_1) + y'(z_2)\right]$$

$$\approx \frac{h}{6}\left[y'(x_n) + 4y'\left(x_n+\frac{h}{2}\right) + y'(x_n+h)\right]$$

We split the middle term into 2:

$$\int_{x_n}^{x_n+h} y'(x)\,dx \approx \frac{h}{6}\left[y'(x_n) + 2y'\left(x_n+\frac{h}{2}\right) + 2y'\left(x_n+\frac{h}{2}\right) + y'(x_n+h)\right]$$

Each of the above 4 terms are calculated as :

$$y'(x_n) \approx k_1 = f(x_n, y_n)$$

$$y'\left(x_n + \frac{h}{2}\right) \approx k_2 = f\left(x_n + \frac{h}{2}, y_n + \frac{h}{2} k_1\right)$$

$$y'\left(x_n + \frac{h}{2}\right) \approx k_3 = f\left(x_n + \frac{h}{2}, y_n + \frac{h}{2} k_2\right)$$

$$y'(x_n + h) \approx k_4 = f(x_n + h, y_n + h k_3)$$

[where $\frac{dy}{dx} = f(x, y)$]

Substituting,

$$\int_{x_n}^{x_n + h} y'(x)\, dx = \frac{h}{6}\left[k_1 + 2k_2 + 2k_3 + k_4\right]$$

Substituting in (1)

$$\Rightarrow \boxed{y_{n+1} = y_n + \frac{h}{6}\left[k_1 + 2k_2 + 2k_3 + k_4\right]}$$

$\Downarrow$

RK 4th order

2:

(a)

STABILITY OF SOLUTIONS

3 types :

- stable → Solutions resulting from perturbations to the initial value remains close to the origin
- Asymptotically stable
  - Solutions from perturbations converge back to the original solution
- Unstable
  - Solutions from perturbations diverge from original solution without any bounds.

Example

Consider the ODE

$$y' = \lambda y$$

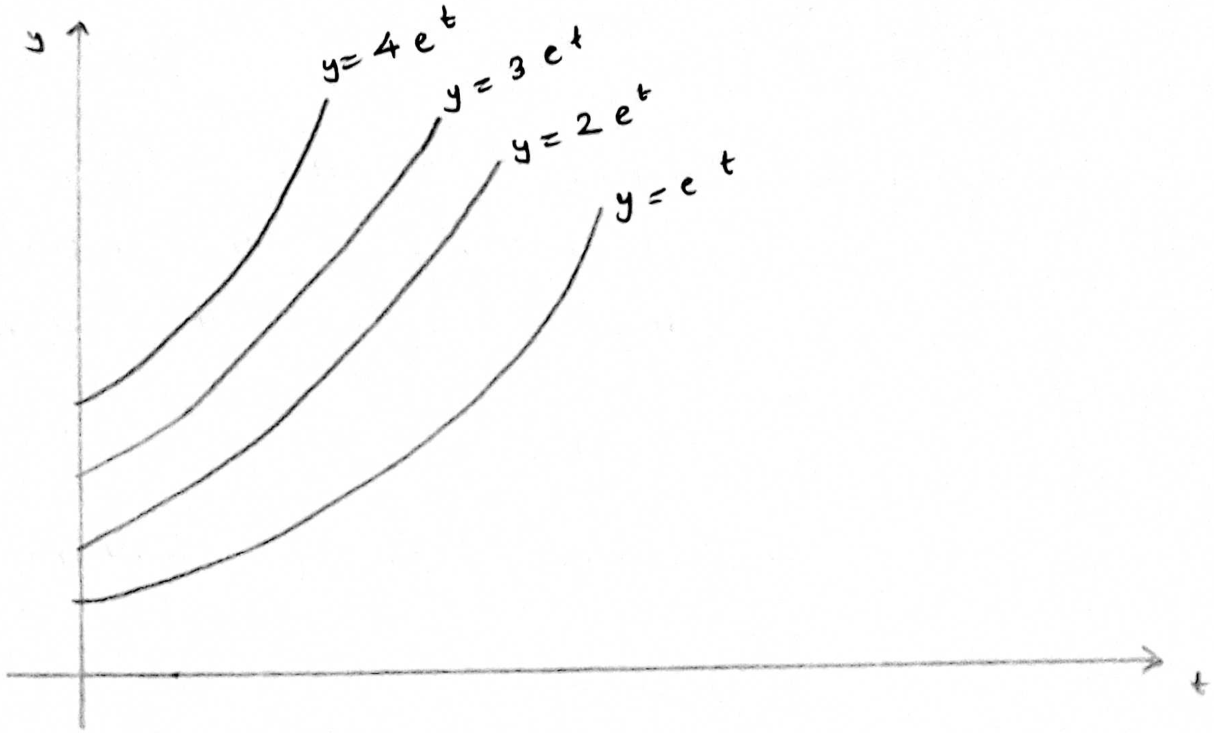
Actual solution is $y(t) = y_0 e^{\lambda t}$

$t_0 = 0$

$y(t_0) = y_0$ (initial value)

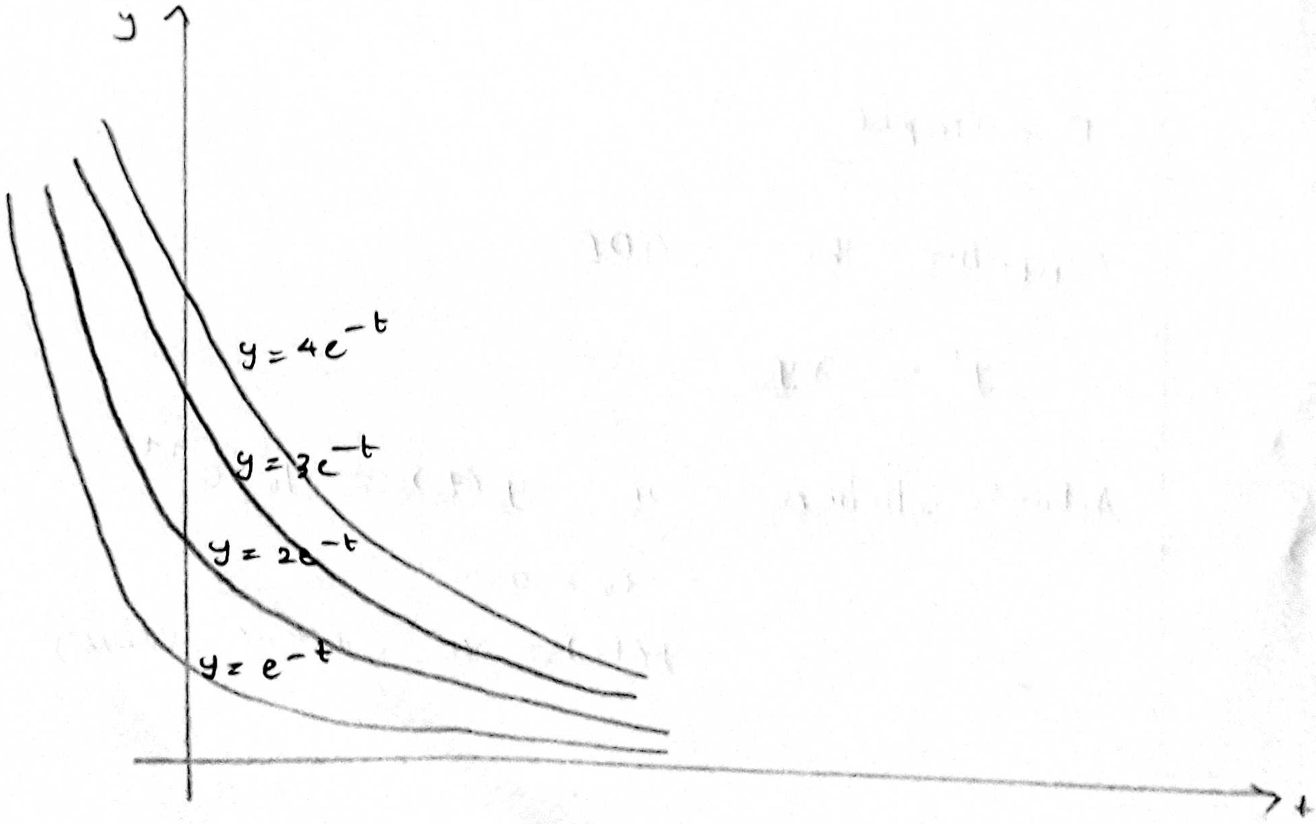
Case - 1 : $\lambda > 0 \rightarrow$ Unstable

All non-zero solutions grow exponentially



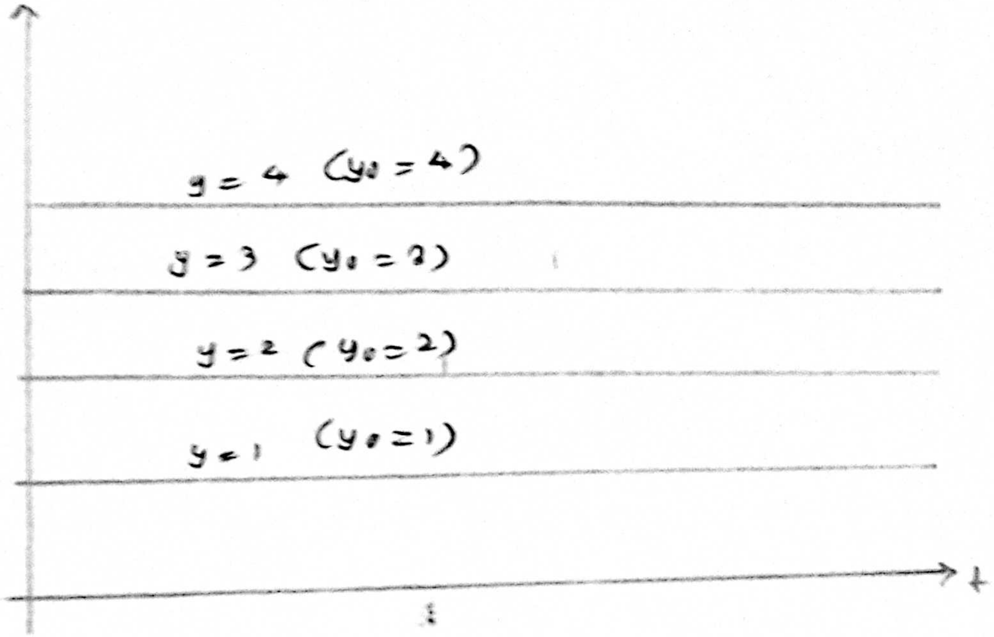
Case - 2 : $\lambda < 0 \rightarrow$ Asymptotically stable

All nonzero solutions decay asymptotically



Case 3: $\lambda = 0 \rightarrow$ Stable

Solution is $y = y_0$



(b) Heat eqn :

$$u_t - k u_{xx} = 0$$

Analytical solution:

$$u(t,x) = \sum_{-\infty}^{\infty} e^{\beta_m t} e^{i \alpha_m x}$$

$$(\beta_m + k \alpha_m^2 = 0)$$

Discrete form of the solution: (Explicit)

$$u_j^{k+1} = u_j^k + \frac{k(\Delta t)}{(\Delta x)^2}\left(u_{j+1}^k - 2u_j^k + u_{j-1}^k\right)$$

Here, we assume

$$u_j^k = G^k e^{i \alpha_m j \Delta x} \rightarrow (1)$$

where $G = e^{\beta_m \Delta t}$

Condition for stability :-

$$|G| \leq 1 \quad \text{as} \quad k \to \infty$$

$$-1 \leq G \leq 1$$

Substituting (1) in discrete form of solution

$$e^{\beta m (t+\Delta t)} e^{i \alpha m j x} = e^{\beta m t} e^{i \alpha m j x}$$

$$+ \frac{k \Delta t}{(\Delta x)^2} \Big[e^{\beta m t} e^{i \alpha m j (x + \Delta x)} - 2 e^{\beta m t} e^{i \alpha m j x} + e^{\beta m t} e^{i \alpha m j (x - \Delta x)} \Big]$$

Dividing by $e^{\beta m t} e^{i \alpha m j x}$

$$\Rightarrow \quad e^{\beta m \Delta t} = 1 + \frac{k \Delta t}{(\Delta x)^2} \Big[e^{i \alpha m \Delta x} - 2 + e^{-i \alpha m \Delta x} \Big]$$

$$\Downarrow$$

$$G = 1 + \frac{k \Delta t}{(\Delta x)^2} \Big[e^{i \alpha m \Delta x} - 2 + e^{-i \alpha m \Delta x} \Big]$$

Using the relation

$$\sin^2\left(\frac{\alpha_m \Delta x}{2}\right) = -\frac{\left(e^{i\alpha_m \Delta x} + e^{-i\alpha_m (\Delta x)} - 2\right)}{4}$$

$$\Rightarrow \quad G = 1 + \frac{K\Delta t}{(\Delta x)^2}\left(-4\sin^2\left(\frac{\alpha_m \Delta x}{2}\right)\right)$$

$$-1 \leq G \leq 1$$

$$-1 \leq 1 - \left(\frac{k\Delta t}{(\Delta x)^2}\right)\left(4\sin^2\left(\frac{\alpha_m \Delta x}{2}\right)\right) \leq 1$$

$$-2 \leq -4\frac{k\Delta t}{(\Delta x)^2}\sin^2\left(\frac{\alpha_m \Delta x}{2}\right) \leq 0$$

$$0 \leq 4\frac{k\Delta t}{(\Delta x)^2}\sin^2\left(\frac{\alpha_m \Delta x}{2}\right) \leq 2$$

$$0 \leq \sin^2\left(\frac{\alpha_m \Delta x}{2}\right)\frac{k\Delta t}{(\Delta x)^2} \leq \frac{1}{2}$$

$\sin^2$ is $[0, 1]$

$$\therefore \quad 0 \leq \frac{k\Delta t}{(\Delta x)^2} \leq \frac{1}{2}$$

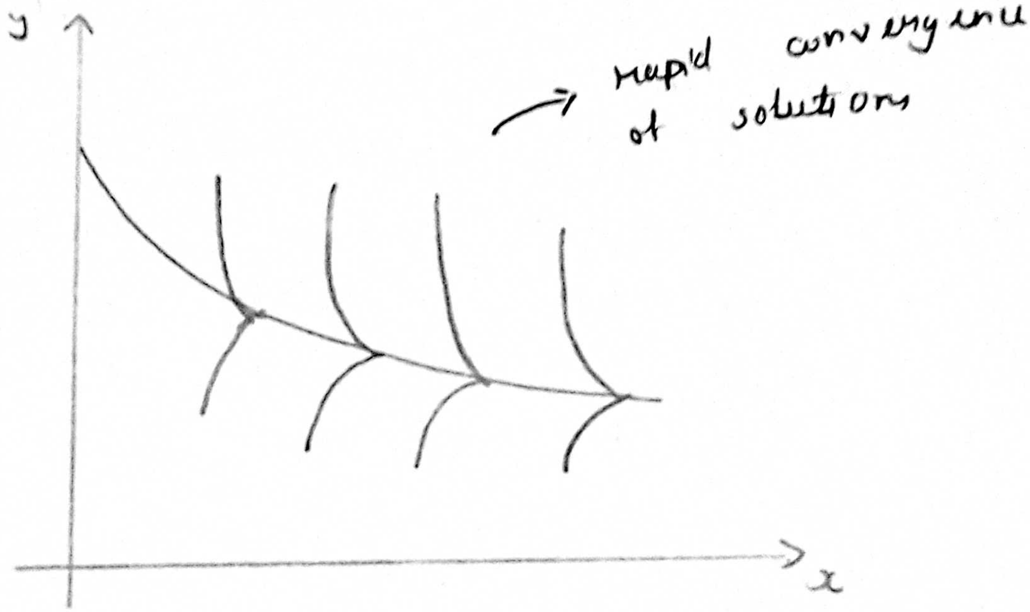
$$\boxed{\Delta t \leq \frac{(\Delta x)^2}{2k}}$$

$\Downarrow$

From Von-Neumann stability criterion

3: STIFF ODES

While solving an ODE using numerical techniques, in some cases, the convergence of solutions is too rapid. This leads to many issues. Such ODEs are called stiff ODEs



Condition for stiffness :

A system of ODEs $y' = f(x, y)$ is stiff if eigenvalues of its Jacobian matrix J_f differ greatly (in magnitude).

- large $-ve$ real parts of eigenvalues
  $\Rightarrow$ strongly damped components of soln
- large Imginary parts ~~real parts~~ of eigenvalues
  $\Rightarrow$ rapidly oscillating components of soln

Problems with stiff ODEs

→ Rapidly varying component of solution forces very small step size to ensure stability in stiff ODEs

~~→ Slowly varying component of solution forces accuracy restriction~~

→ For eg, Euler's method is extremely inefficient for solving stiff ODEs because of stability restriction in step size. But Backward Euler method is unconditionally stable, and hence can be used for stiff ODEs.

Example

Consider ODE : $y' = -100y + 100t + 101$

$y(0) = 1$

gen soln : $y(t) = 1 + t + ce^{-100t}$

Particular soln : $y(t) = 1 + t$ (satisfies $y(0) = 1$)

Theoretically, Euler's method is exact since solution is linear

But suppose we perturb the initial value slightly (to 0.99 and 1.01):

$y(0) = 0.99$

t	0	0.1	0.2	0.3	0.4
Euler soln	0.99	1.19	0.39	8.59	-14.2

$y(0) = 1.01$

t	0	0.1	0.2	0.3	0.4
Euler soln	1.01	1.01	2.01	-5.99	67.0

whereas the exact soln for $y(0) = 1$ is:

t	0	0.1	0.2	0.3	0.4
Exact	1	1.10	1.20	1.30	1.40

$\Downarrow$

Computed solution is extremely sensitive to initial value. Each small perturbation results in very different solution.

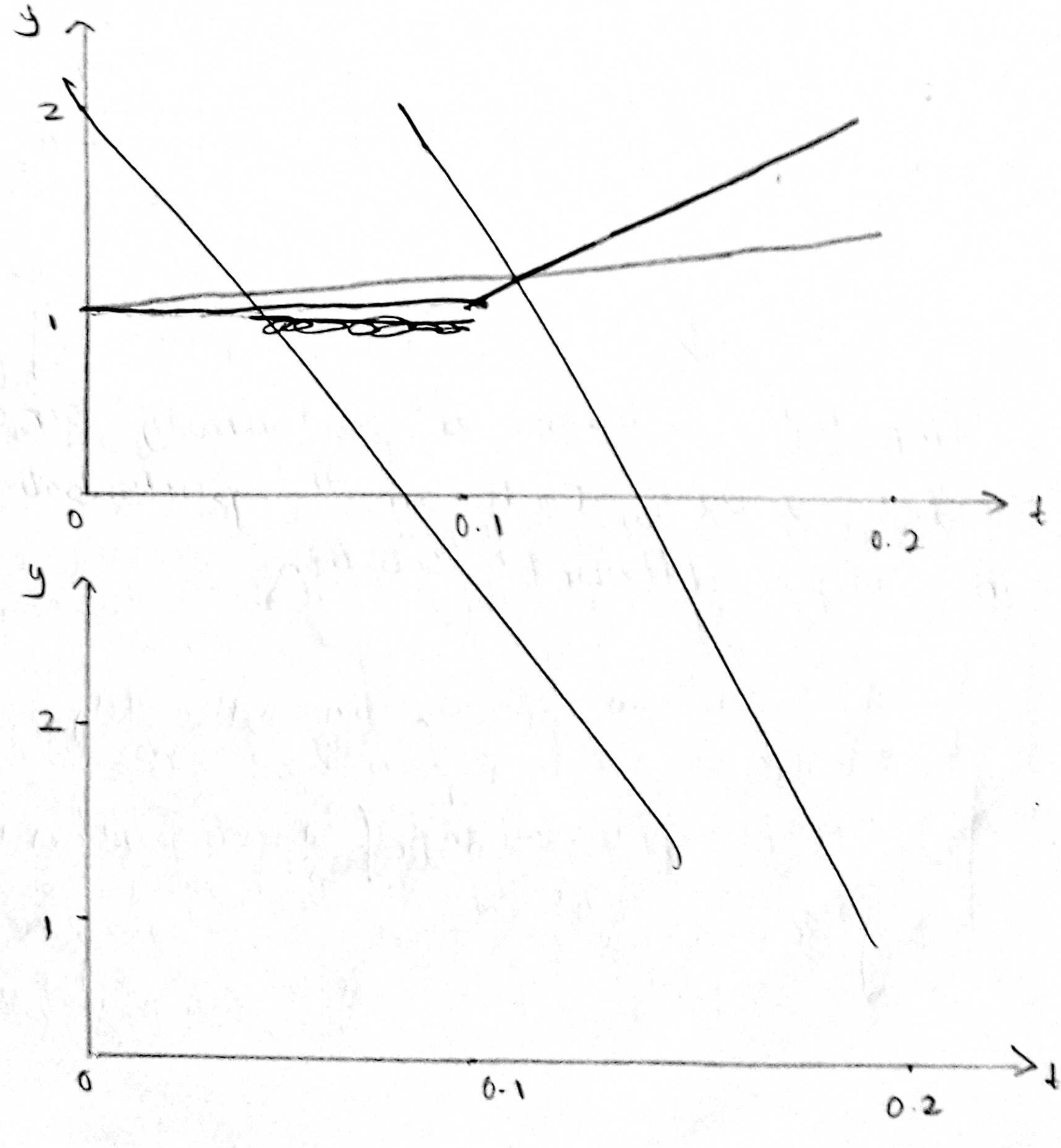
Reason :

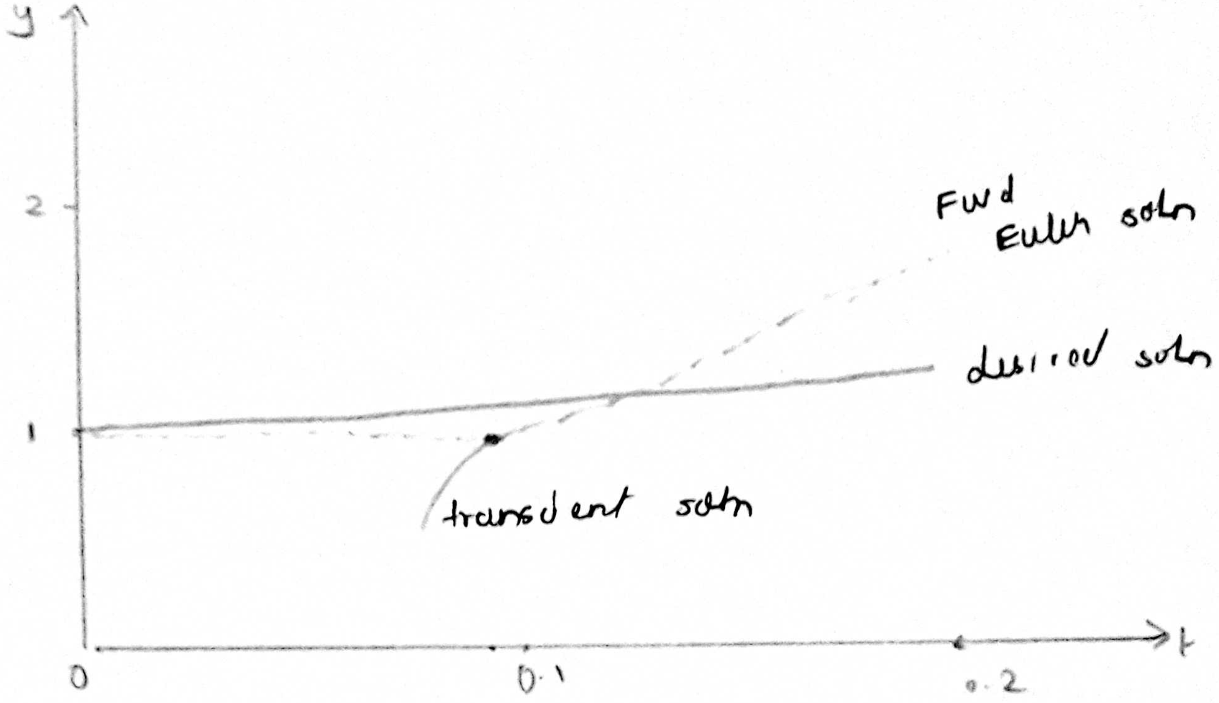
→ Euler's method bases its projection on derivative at current point, and resulting large value caused numerical soln to diverge from desired solution.

$J_A = -100$ for this ODE

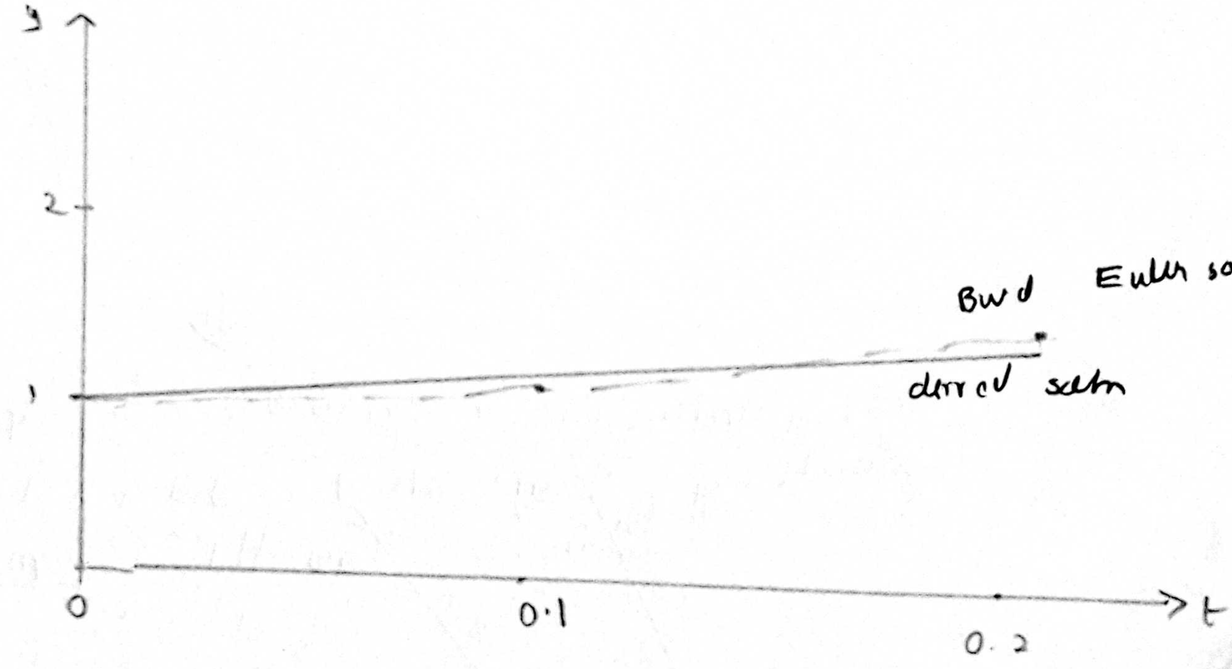
⇒ stability condition for Euler's method requires $h < 0.02$, which is violated.

→ Backward Euler solution has ~~not~~ no stability limitation and hence, solves the ODE correctly.





(Using Forward Euler)



(Using Backward Euler)

PART – A

1: (a) $A = \begin{bmatrix} 0 & 10 & 1 \\ 1 & 3 & -1 \\ 2 & 4 & 1 \end{bmatrix}$

Applying Gauss Elimination method :

(1) $R_1 \longleftrightarrow R_2$

$$u = \begin{bmatrix} 1 & 3 & -1 \\ 0 & 10 & 1 \\ 2 & 4 & 1 \end{bmatrix} \qquad L = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad P = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(2)

$R_3 \leftarrow R_3 - (2) \times R_1 \qquad (L_{3,1} = 2)$

$$u = \begin{bmatrix} 1 & 3 & -1 \\ 0 & 10 & 1 \\ 0 & -2 & 3 \end{bmatrix} \qquad L = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} \qquad P = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(3) $R_3 \leftarrow R_3 - (0.2) \times R_2$ $[L_{3,2} = -0.2]$

$$u \Rightarrow \begin{bmatrix} 1 & 3 & -1 \\ 0 & 10 & 1 \\ 0 & 0 & 3.2 \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & -0.2 & 1 \end{bmatrix}$$

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\therefore$ $PA = LU$

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad u = \begin{bmatrix} 1 & 3 & -1 \\ 0 & 10 & 1 \\ 0 & 0 & 3.2 \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & -0.2 & 1 \end{bmatrix}$$

(b) In (a), we obtained P, L, U ~~exist~~ s.t

(i) $PA = LU$

$b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$

Sys of eqn : $Ax = b$

$$PAx = Pb$$

$$L\underbrace{Ux}_{b'} = Pb$$

$$Pb = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} b_2 \\ b_1 \\ b_3 \end{bmatrix}$$

$$Lb' = Pb$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & -0.2 & 1 \end{bmatrix} \begin{bmatrix} b_1' \\ b_2' \\ b_3' \end{bmatrix} = \begin{bmatrix} b_2 \\ b_1 \\ b_3 \end{bmatrix}$$

$$b_1' = b_2$$

$$b_2' = b_1$$

$$2b_1' - 0.2b_2' + b_3' = b_3$$

$$\Rightarrow b_3' = b_3 - 2b_1' + 0.2b_2'$$

$$= b_3 - 2b_2 + 0.2b_1$$

$$\therefore b' = \begin{bmatrix} b_2 \\ b_1 \\ b_3 - 2b_2 + 0.2b_1 \end{bmatrix}$$

(ii)

$\cancel{PA = LU}$

$\cancel{LUx = Pb}$

$$Ax = b$$

$$PAx = Pb$$

$$\boxed{\begin{aligned} LUx &= Pb \\ Lb' &= Pb \end{aligned}} \Rightarrow Ux = b'$$

We know b' in terms of b_1, b_2, b_3 (from (i))

$$Ux = b'$$

$$\begin{bmatrix} 1 & 3 & -1 \\ 0 & 10 & 1 \\ 0 & 0 & 3.2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_2 \\ b_1 \\ b_3 - 2b_2 + 0.2b_1 \end{bmatrix}$$

Backward substitution

(1) $3.2x_3 = b_3 - 2b_2 + 0.2b_1$

$$\Rightarrow x_3 = \frac{b_3 - 2b_2 + 0.2b_1}{3.2}$$

(2) $10x_2 + x_3 = b_1$

$$\Rightarrow x_2 = \frac{b_1 - x_3}{10}$$

$$= \frac{b_1 - \left[\dfrac{b_3 - 2b_2 + 0.2b_1}{3.2}\right]}{10}$$

$$= \frac{3b_1 + 2b_2 - b_3}{32}$$

(3) $x_1 + 3x_2 - x_3 = b_2$

$$x_1 = b_2 - 3x_2 + x_3$$

$$= b_2 - \frac{3(3b_1 + 2b_2 - b_3)}{32} + \frac{(b_3 - 2b_2 + 0.2b_1)}{3.2}$$

$$= b_2 + \frac{-9b_1 - 6b_2 + 3b_3}{32} + \frac{10b_3 - 20b_2 + 2b_1}{32}$$

$$= b_2 + \frac{-7b_1 - 26b_2 + 13b_3}{32}$$

$$= \frac{-7b_1 + 6b_2 + 13b_3}{32}$$

$\therefore$ Solution

$$x_1 = \frac{-7b_1 + 6b_2 + 13b_3}{32}$$

$$x_2 = \frac{3b_1 + 2b_2 - b_3}{32}$$

$$x_3 = \frac{b_3 - 2b_2 + 0.2b_1}{3.2}$$

2. (a) $x^3 - 6x^2 + 8x + 0.8 = 0 = f(x)$

Secant method : $x_{i+1} = x_i - \dfrac{f(x_i)}{m_{i,i-1}}$

$m_{i,i+1}$ is slope of line joining $(x_i, f(x_i))$ and $(x_{i-1}, f(x_{i-1}))$

$x_0 = 2.5$, $x_1 = 2$

$\rightarrow$ $x_2 = x_1 - \dfrac{f(x_1)}{m_{1,0}}$

$m_{1,0} = \dfrac{f(2.5) - f(2)}{2.5 - 2}$

$f(2.5) = 2.5^3 - 6 \times 2.5^2 + 8 \times 2.5 + 0.8$

$= \underline{-1.075}$

$f(2) = 2^3 - 6 \times 2^2 + 8 \times 2 + 0.8$

$= \underline{\underline{0.8}}$

$\therefore m_{1,0} = 2(-1.075 - 0.8) = \underline{\underline{-3.75}}$

$\rightarrow$ $\cancel{x_3 = x_2 - \dfrac{f(x_2)}{m_{2,1}}}$

$\cancel{m_{2,1} = \dfrac{f(x_2) - f(2)}{x_2 - 2}}$

$\cancel{x_2}$ x_1 -

$$x_2 = x_1 - \frac{f(x_1)}{m_{1,0}}$$

$$= 2 - \frac{f(2)}{-3.75}$$

$$= 2 + \frac{0.8}{3.75} = \underline{\underline{2.2134}}$$

$$\rightarrow x_3 = x_2 - \frac{f(x_2)}{m_{2,1}}$$

$$m_{2,1} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$f(x_2) = f(2.2134) = -0.043624$$

$$x_2 = 2.2134$$

$$\therefore m_{2,1} = \frac{-0.043624 - 0.8}{2.2134 - 2}$$

$$m_{21} = \underline{\underline{-3.9532}}$$

$$x_3 = x_2 - \frac{f(x_2)}{m_{2,1}}$$

$$= 2.2134 - \frac{f(2.2134)}{-3.9532}$$

$$= 2.2134 + \frac{-0.043624}{3.9532}$$

$\therefore x_3 = \underline{\underline{2.2024}}$

Final answers

$m_{10} = -3.75$

$x_2 = 2.2134$

$m_{21} = -3.9532$

$x_3 = 2.2024$

(b) Using Newton Raphson method:

$x_0 = 2.5$

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$f(x) = x^3 - 6x^2 + 8x + 0.8$

$f'(x) = 3x^2 - 12x + 8$

$$\rightarrow \quad x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$x_0 = 2.5$

$f(2.5) = -1.075$

$f'(2.5) = 3(2.5)^2 - 12(2.5) + 8$

$= -3.25$

$$\therefore \quad x_1 = 2.5 - \frac{(-1.075)}{-3.25}$$

$$\underline{\underline{x_1 = 2.1692}}$$

$$\rightarrow \quad x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$x_1 = 2.1692$

$f(x_1) = (2.1692)^3 - 6(2.1692)^2 + 8(2.1692) + 0.8$

$= 0.12804$

$f'(x_1) = 3(2.1692)^2 - 12(2.1692) + 8$

$= -3.914114$

$$\therefore x_2 = 2.1692 - \frac{(0.12804)}{-3.914114}$$

$$= \underline{2.2019}$$

$$\therefore \boxed{\begin{aligned} x_1 &= 2.1692 \\ x_2 &= 2.2019 \end{aligned}}$$