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CH19B072

MIDSEM EXAM

My roll no : ch19b072

$m = -2.21$

$n = 3.85$

$$\frac{d^2y}{dx^2} - 2.21\frac{dy}{dx} + 3.85y = 0$$

$y(0) = 1$, $\dot{y}(1) = 0.01m = -0.0221$

1: Shooting method

Convert BVP $\longrightarrow$ IVP

Original BVP : $\ddot{y} - 2.21\dot{y} + 3.85y = 0$

$y(0) = 1$ $\dot{y}(1) = -0.0221$

Converting to a system of 1st order eqns :

$$\left.\begin{aligned} y_1 &= y \\ y_2 &= \dot{y} \end{aligned}\right\} \quad \left.\begin{aligned} \begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{bmatrix} &= \begin{bmatrix} y_2 \\ 2.21y_2 - 3.85y_1 \end{bmatrix} \begin{matrix} \to f_a \\ \to f_b \end{matrix} \\ \begin{bmatrix} y_1(0) \\ y_2(0) \end{bmatrix} &= \begin{bmatrix} 1 \\ ? \end{bmatrix} \end{aligned}\right\} \text{IVP}$$

Boundary condition to be achieved : $y_2(1) = -0.0221$

We use standard RK_2 for solving IVP

We use $h = 0.25 \rightarrow x = 0, 0.25, 0.5, 0.75, 1$

$$\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{bmatrix} = \begin{bmatrix} y_2 \\ 2.21 y_2 - 3.85 y_1 \end{bmatrix} = \begin{bmatrix} f_a (x, y_1, y_2) \\ f_b (x, y_1, y_2) \end{bmatrix}$$

(i) Given $1 \rightarrow \dot{y}(0) = 0$

$$\begin{bmatrix} y_1(0) \\ y_2(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$\longrightarrow x = 0.25$

$$k_1 a = f_a (x = x_0, y_1 = y_1(0), y_2 = y_2(0))$$
$$= 0$$

$$k_1 b = f_b (x = x_0, y_1 = y_1(0), y_2 = y_2(0))$$
$$= 2.21(0) - 3.85(1) = -3.85$$

$$k_2 a = f_a (x_0 + h, y_1(0) + h k_1 a, y_2(0) + h k_1 b)$$
$$= f_a (0.25, 1 + 0.25 \times 0, 0 + 0.25 \times -3.85)$$
$$= -0.9625$$

$$k_2 b = f_b (x_0 + h, y_1(0) + h k_1 a, y_2(0) + h k_1 b)$$
$$= f_b (0.25, 1 + 0.25 \times 0, 0 + 0.25 \times -3.85)$$
$$= -5.977125$$

$$y_1(0.25) = y_1(0) + \frac{h}{2} (k_1 a + k_2 a)$$
$$= 1 + \frac{0.25}{2} (0 - 0.9625) = 0.8796875$$

$$y_2(0.25) = y_2(0) + \frac{h}{2} (k_1 b + k_2 b)$$
$$= 0 + \frac{0.25}{2} (-3.85 - 5.977125) = -1.228390625$$

$\rightarrow$ $x = 0.5$

$k_1 a = f_a(x = 0.25,\ y_1 = y_1(0.25),\ y_2 = y_2(0.25))$

$= -1.2284$

$k_1 b = f_b(x = 0.25,\ y_1 = y_1(0.25),\ y_2 = y_2(0.25))$

$= -6.1015$

$k_2 a = f_a(0.5,\ y_1(0.25) + h k_1 a,\ y_2(0.25) + h k_1 b)$

$= -2.7538$

$k_2 b = f_b(0.5,\ y_1(0.25) + h k_1 a,\ y_2(0.25) + h k_1 b)$

$= -8.2903$

$y_1(0.5) = y_1(0.25) + \frac{h}{2}(k_1 a + k_2 a)$

$= 0.3819$

$y_2(0.5) = y_2(0.25) + \frac{h}{2}(k_1 b + k_2 b)$

$= -3.0274$

$\rightarrow$ $x = 0.75$

$k_1 a = f_a(x\ 0.5,\ y_1(0.5),\ y_2(0.5))$

$= -3.0274$

$k_1 b = f_b(0.5,\ y_1(0.5),\ y_2(0.5))$

$= -8.1609$

$k_2 a = f_a(0.75,\ y_1(0.5) + h k_1 a,\ y_2(0.5) + h k_1 b)$

$= -5.0676$

$k_2 b = f_b(0.75,\ y_1(0.5) + h k_1 a,\ y_2(0.5) + h k_2 b)$

$= -9.7559$

$$y_1(0.75) = y_1(0.5) + \frac{h}{2}(k_1a + k_2a)$$

$$= -0.6299$$

$$y_2(0.75) = y_2(0.5) + \frac{h}{2}(k_1a + k_2a)$$

$$= -5.2669$$

→ $\underline{x = 1.0}$

$$k_1a = fa(0.75, y_1(0.75), y_2(0.75))$$

$$= -5.2669$$

$$k_1b = fb(0.75, y_1(0.75), y_2(0.75))$$

$$= -9.2147$$

$$k_2a = fa(1, y_1(0.75) + hk_1a, y_2(0.75) + hk_1b)$$

$$= -7.5706$$

$$k_2b = fb(1, y_1(0.75) + hk_1a, y_2(0.75) + hk_1b)$$

$$= -9.2363$$

$$\cancel{y_1(1) = -2.2346}$$

$$\cancel{y_2(1) = -7.57}$$

$$y_1(1) = y_1(0.75) + \frac{h}{2}(k_1a + k_2a)$$

$$= -2.2346$$

$$y_2(1) = y_2(0.75) + \frac{h}{2}(k_1a + k_2a)$$

$$= -7.5733$$

$$y_2(1) = \dot{y}(1) = -7.5733 \neq -0.0221$$

∴ NEED ANOTHER GUESS

(ii) Given 2 $\longrightarrow$ $\dot{y}(0) = 4$

$$\begin{bmatrix} y_1(0) \\ y_2(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

$\longrightarrow$ $x = 0.25$

$$k_1 a = f_a(0, 1, 4) = 4$$

$$k_1 b = f_b(0, 1, 4) = 4.99$$

$$k_2 a = f_a(0.25, y_1(\overset{0}{\cancel{0.25}}) + h k_1 a, y_2(\overset{0}{\cancel{0.25}}) + h k_1 b)$$

$$= 5.2475$$

$$k_2 b = f_b(0.25, y_1(\overset{0}{\cancel{0.25}}) + h k_1 a, y_2(\overset{0}{\cancel{0.25}}) + h k_1 b)$$

$$= 3.8970$$

$$y_1(0.25) = y_1(0) + \frac{h}{2}(k_1 a + k_2 a)$$

$$= 2.1559$$

$$y_2(0.25) = y_2(0) + \frac{h}{2}(k_1 b + k_2 b)$$

$$= 5.1109$$

$\longrightarrow$ $x = 0.5$

$$k_1 a = f_a(0.25, y_1(0.25), y_2(0.25))$$

$$= 5.1109$$

$$k_1 b = f_b(0.25, y_1(0.25), y_2(0.25))$$

$$= 2.9947$$

$$k_2 a = f_a(\overset{0.5}{\cancel{0.25}}, y_1(0.25) + h k_1 a, y_2(0.25) + h k_1 b)$$

$$= 5.8595$$

$$k_2 b = f_b(0.5, y_1(0.25) + h k_1 a, y_2(0.25) + h k_1 b)$$

$$= -0.2699$$

$$y_1(0.5) = y_1(0.25) + \frac{h}{2}(k_1 a + k_2 a) = 3.5272$$

$$y_2(0.5) = y_2(0.25) + \frac{h}{2}(k_1 b + k_2 b) = 5.4514$$

→ $\underline{x = 0.75}$

$k_1 a = f_a(0.5, y_1(0.5), y_2(0.5))$

$= 5.4514$

$k_1 b = f_b(0.5, y_1(0.5), y_2(0.5))$

$= -1.5322$

$k_2 a = f_a(0.75, y_1(0.5) + h k_1 a, y_2(0.5) + h k_1 b)$

$= 5.0634$

$k_2 b = f_b(0.75, y_1(0.5) + h k_1 a, y_2(0.5) + h k_1 b)$

$= -7.6257$

$y_1(0.75) = y_1(0.5) + \frac{h}{2}(k_1 a + k_2 a) = 4.8422$

$y_2(0.75) = y_2(0.5) + \frac{h}{2}(k_1 b + k_2 b) = 4.3067$

→ $\underline{x = 1.0}$

$k_1 a = f_a(0.75, y_1(0.75), y_2(0.75))$

$= 4.3067$

$k_1 b = f_b(0.75, y_1(0.75), y_2(0.75))$

$= -9.1247$

$k_2 a = f_a(1, y_1(0.75) + h k_1 a, y_2(0.75) + h k_1 b)$

$= 2.0255$

$k_2 b = f_b(1, y_1(0.75) + h k_1 a, y_2(0.75) + h k_1 b)$

$= -18.3113$

$y_1(1) = y_1(0.75) + \frac{h}{2}(k_1 a + k_2 a)$

$= 5.6337$

$y_2(1) = y_2(0.75) + \frac{h}{2}(k_1 b + k_2 b)$

$= 0.8772$

$y_2(1) = \dot{y}(1) = 0.8772 \neq -0.0221$

∴ NEED ANOTHER GUESS

Using linear interpolation for guess - 3

Guess 1 $\longrightarrow$ $\dot{y}(0) = 0$ $\longrightarrow$ $\dot{y}(1) = -7.5733$

Guess 2 $\longrightarrow$ $\dot{y}(0) = 4$ $\longrightarrow$ $\dot{y}(1) = 0.8772$

$$\underbrace{\dot{y}(0)}_{x_g} \qquad \underbrace{\dot{y}(1)}_{y_g}$$

$$\frac{y_g - 0.8772}{x_g - 4} = \frac{-7.5733 - 0.8772}{0 - 4}$$

$$y_g - 0.8772 = 2.1126\,(x_g - 4)$$

We need x_g for which $y_g = -0.0221$

$$-0.0221 - 0.8772 = 2.1126\,(x_g - 4)$$

$$\Rightarrow \quad x_g = 3.5743$$

$\Downarrow$

New initial guess

(iii) guess 3 $\longrightarrow$ $\dot{y}(0) = 3.5743$

$$\begin{bmatrix} y_1(0) \\ y_2(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 3.5743 \end{bmatrix}$$

$\longrightarrow$ $x = 0.25$

$k_1a = fa(0, 1, 3.5743) = 3.5743$

$k_1b = fb(0, 1, 3.5743) = 4.0492$

$k_2a = fa(0.25,\ y_1(0) + h\,k_1a,\ y_2(0) + h\,k_1b)$

$= 4.5866$

$$k_{2b} = f_b(0.25,\ y_1(0) + hk_{1a},\ y_2(0) + hk_{1b})$$

$$= 2.8461$$

$$y_1(0.25) = y_1(0) + \frac{h}{2}(k_{1a} + k_{2a})$$

$$= 2.0201$$

$$y_2(0.25) = y_2(0) + \frac{h}{2}(k_{1b} + k_{2b})$$

$$= 4.4362$$

→ $x = 0.5$

$$k_1a = f_a(0.25,\ y_1(0.25),\ y_2(0.25))$$

$$= 4.4362$$

$$k_1b = f_b(0.25,\ y_1(0.25),\ y_2(0.25))$$

$$= 2.0266$$

$$k_2a = f_a(0.5,\ y_1(0.25) + hk_1a,\ y_2(0.25) + hk_1b)$$

$$= 4.9429$$

$$k_2b = f_b(0.5,\ y_1(0.25) + hk_1a,\ y_2(0.25) + hk_1b)$$

$$= -1.1235$$

$$y_1(0.5) = y_1(0.25) + \frac{h}{2}(k_1a + k_2a) = 3.1925$$

$$y_2(0.5) = y_2(0.25) + \frac{h}{2}(k_1a + k_2a) = 4.5491$$

→ $x = 0.75$

$$k_1a = f_a(0.5,\ y_1(0.5),\ y_2(0.5))$$

$$= 4.5491$$

$$k_1b = f_b(0.5,\ y_1(0.5),\ y_2(0.5))$$

$$= -2.2376$$

$$k_2a = f_a(0.75,\ y_1(0.4) + hk_1a,\ y_2(0.5) + hk_1b)$$

$$= 3.9897$$

$$k_{2b} = f_b(0.75,\ y_1(0.5) + hk_{1a},\ y_2(0.5) + hk_{1a})$$

$$= -7.3524$$

$$y_1(0.75) = y_1(0.5) + \frac{h}{2}(k_{1a} + k_{2a})$$

$$= 4.2598$$

$$y_2(0.75) = y_2(0.5) + \frac{h}{2}(k_{1b} + k_{2b})$$

$$= 3.2878$$

$\longrightarrow$ $\underline{x = 1.0}$

$$k_{1a} = f_a(0.75,\ y_1(0.75),\ y_2(0.75))$$

$$= 3.2878$$

$$k_{1b} = f_b(0.75,\ y_1(0.75),\ y_2(0.75))$$

$$= -9.1343$$

$$k_{2a} = f_a(1,\ y_1(0.75) + hk_{1a},\ y_2(0.75) + hk_{1b})$$

$$= 1.0043$$

$$k_{2b} = f_b(1,\ y_1(0.75) + hk_{1a},\ y_2(0.75) + hk_{1b})$$

$$= -17.3455$$

$$y_1(1) = y_1(0.75) + \frac{h}{2}(k_{1a} + k_{2a})$$

$$= \underline{\underline{4.7964}}$$

$$y_2(1) = y_2(0.75) + \frac{h}{2}(k_{1a} + k_{2b})$$

$$= \underline{\underline{-0.0221}}$$

$$y_2(1) = \dot{y}(1) = -0.0221$$

$\therefore$ OUR GUESS IS CORRECT

Final solution (based on shooting method)

x	y	$\dot{y}$
0	1	3.5743
0.25	2.0201	4.4362
0.5	3.1925	4.5491
0.75	4.2598	3.2878
1	4.7964	−0.0221

2 : Obtaining set of algebraic eqns using finite difference method

$$\frac{d^2y}{dx^2} + -2.21\frac{dy}{dx} + 3.85y = 0$$

$y(0) = 1$ $\quad\quad \dot{y}(1) = 0.01m = -0.0221$

Base points :

$x_0 = 0,$ $\quad x_1 = 0.25,$ $\quad x_2 = 0.5,$ $\quad x_3 = 0.75,$ $\quad x_4 = 1$

$y_0 = 1$ $\quad\quad y'_4 = -0.0221$

$h = 0.25$

Central difference formula :

$$\frac{dy_i}{dx} = \frac{y_{i+1} - y_{i-1}}{2h}$$

$$\frac{d^2y_i}{dx_i^2} = \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2}$$

Substituting above eqn in our ODE :

$$16(y_{i+1} - 2y_i + y_{i-1}) - 2.21 \times 2(y_{i+1} - y_{i-1}) + 3.85y_i = 0$$

$$11.58\,y_{i+1} - 28.15\,y_i + 20.42\,y_{i-1} = 0$$

$i = 1, 2, 3$

$\Downarrow$

$11.58\,y_2 - 28.15\,y_1 + 20.42\,y_0 = 0 \longrightarrow (1)$

$11.58\,y_3 - 28.15\,y_2 + 20.42\,y_1 = 0 \longrightarrow (2)$

$11.58\,y_4 - 28.15\,y_3 + 20.42\,y_2 = 0 \longrightarrow (3)$

$y_0 = 1$

$\therefore$ We have 4 variables and 3 eqns

We use $\left.\frac{dy}{dx}\right|_{x=x_4} = -0.0221$

If we use central difference, another variable (y_5) will get added.

$\therefore$ We use backward difference

$$\frac{y_4 - y_3}{h} = \left.\frac{dy}{dx}\right|_{x=x_4} = -0.0221$$

$$4y_4 - 4y_3 = -0.0221 \longrightarrow (4)$$

$\therefore$ Now we have 4 eqns and 4 unknowns :

$$-28.15y_1 + 11.58y_2 = -20.42$$

$$-28.15y_2 + 11.58y_3 + 20.42y_1 = 0$$

$$20.42y_2 - 28.15y_3 + 11.58y_4 = 0$$

$$4y_4 - 4y_3 = -0.0221$$

Matrix form

$$\begin{bmatrix} -28.15 & 11.58 & 0 & 0 \\ 20.42 & -28.15 & 11.58 & 0 \\ 0 & 20.42 & -28.15 & 11.58 \\ 0 & 0 & -4 & 4 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} -20.42 \\ 0 \\ 0 \\ -0.0221 \end{bmatrix}$$

3: Note: A matrix is not diagonally dominant (and cannot be made so), hence convergence in Gauss - Siedel is not guaranteed

From the equations:

$$y_1^{k+1} = \frac{(-20.42 - 11.58\, y_2^{k})}{-28.15}$$

$$y_2^{k+1} = \frac{(-20.42\, y_1^{k+1} - 11.58\, y_3^{k})}{-28.15}$$

$$y_3^{k+1} = \frac{(-20.42\, y_2^{k+1} - 11.58\, y_4^{k})}{-28.15}$$

$$y_4^{k+1} = \frac{(-0.0221 + 4\, y_3^{k+1})}{4}$$

Initial guess

$y_1^0 = 1$, $y_2^0 = 2$, $y_3^0 = 3$, $y_4^0 = 4$

Iteration - 1

$$y_1^1 = \frac{(-20.42 - 11.58\, y_2^0)}{-28.15}$$

$y_2^1 = (-20.42$

Iteration - 1

$$y_1^1 = \frac{(-20.42 - 11.58\, y_2^0)}{-28.15} = \frac{(-20.42 - 11.58 \times 2)}{-28.15}$$

$$= 1.5481$$

$$y_2^1 = \frac{(-20.42\, y_1^1 - 11.58\, y_3^0)}{-28.15} = \frac{(-20.42 \times 1.5481 - 11.58 \times 3)}{-28.15}$$

$$= 2.3571$$

$$y_3^1 = \frac{(-20.42\, y_2^1 - 11.58\, y_4^0)}{-28.15} = \frac{(-20.42 \times 2.3571 - 11.58 \times 4)}{-28.15}$$

$$= 3.3553$$

$$y_4^1 = \frac{(-0.0221 + 4 y_3^1)}{4} = \frac{(-0.0221 + 4 \times 3.3553)}{4}$$

$$= 3.3497$$

Iteration - 2

$$y_1^2 = \frac{(-20.42 - 11.58 \times 2.3571)}{-28.15} = 1.6950$$

$$y_2^2 = \frac{(-20.42 \times 1.6950 - 11.58 \times 3.3553)}{-28.15} = 2.6098$$

$$y_3^2 = \frac{(-20.42 \times 2.6098 - 11.58 \times 3.3497)}{-28.15} = 3.2712$$

$$y_4^2 = \frac{(-0.0221 + 4 \times 3.2712)}{4} = 3.2657$$

Iteration - 3

$$y_1^3 = \frac{(-20.42 - 11.58 \times 2.6098)}{-28.15} = 1.7990$$

$$y_2^3 = \frac{(-20.42 \times 1.7990 - 11.58 \times 3.2712)}{-28.15} = 2.6507$$

$$y_3^3 = \frac{(-20.42 \times 2.6507 - 11.58 \times 3.2657)}{-28.15} = 3.2662$$

$$y_4^3 = \frac{(-0.0221 + 4 \times 3.2662)}{4} = 3.2606$$

Iteration - 4

$$y_1^4 = \frac{(-20.42 - 11.58 \times 2.6507)}{-28.15} = 1.8158$$

$$y_2^4 = \frac{(-20.42 \times 1.8158 - 11.58 \times 3.2662)}{-28.15} = 2.6608$$

$$y_3^4 = \frac{(-20.42 \times 2.6608 - 11.58 \times 3.2606)}{-28.15} = 3.2714$$

$$y_4^4 = \frac{(-0.0221 + 4 \times 3.2714)}{4} = 3.2659$$

Iteration - 5

$$y_1^5 = \frac{(-20.42 - 11.58 \times 2.6608)}{-28.15} = 1.8199$$

$$y_2^5 = \frac{(-20.42 \times 1.8199 - 11.58 \times 3.2714)}{-28.15} = 2.6660$$

$$y_3^5 = \frac{(-20.42 \times 2.6660 - 11.58 \times 3.2659)}{-28.15} = 3.2774$$

$$y_4^5 = \frac{(-0.0221 + 4 \times 3.2774)}{4} = 3.2719$$

Iteration - 6

$$y_1^6 = \frac{(-20.42 - 11.58 \times 2.6600)}{-28.15} = 1.8221$$

$$y_2^6 = \frac{(-20.42 \times 1.8221 - 11.58 \times 3.2774)}{-28.15} = 2.6699$$

$$y_3^6 = \frac{(-20.42 \times 2.6699 - 11.58 \times 3.2719)}{-28.15} = 3.2827$$

$$y_4^6 = \frac{(-0.0221 + 4 \times 3.2827)}{4} = 3.2772$$

Iteration - 7

$$y_1^7 = \frac{(-20.42 - 11.58 \times 2.6699)}{-28.15} = 1.8237$$

$$y_2^7 = \frac{(-20.42 \times 1.8237 - 11.58 \times 3.2827)}{-28.15} = 2.6733$$

$$y_3^7 = \frac{(-20.42 \times 2.6733 - 11.58 \times 3.2772)}{-28.15} = 3.2874$$

$$y_4^7 = \frac{(-0.0221 + 4 \times 3.2874)}{4} = 3.2818$$

Iteration - 8

$$y_1^8 = \frac{(-20.42 - 11.58 \times 2.6733)}{-28.15} = 1.8251$$

$$y_2^8 = \frac{(-20.42 \times 1.8251 - 11.58 \times 3.2874)}{-28.15} = 2.6763$$

$$y_3^8 = \frac{(-20.42 \times 2.6763 - 11.58 \times 3.2818)}{-28.15} = 3.2914$$

$$y_4^8 = \frac{(-0.0221 + 4 \times 3.2914)}{4} = 3.2859$$

Iteration - 9

$$y_1^{9} = \frac{(-20.42 - 11.58 \times 2.6763)}{-28.15} = 1.8263$$

$$y_2^{9} = \frac{(-20.42 \times 1.8263 - 11.58 \times 3.2914)}{-28.15} = 2.6788$$

$$y_3^{9} = \frac{(-20.42 \times 2.6788 - 11.58 \times 3.2859)}{-28.15} = 3.2949$$

$$y_4^{9} = \frac{(-0.0221 + 4 \times 3.2949)}{4} = 3.2894$$

Iteration - 10

$$y_1^{10} = \frac{(-20.42 - 11.58 \times 2.6788)}{-28.15} = 1.8274$$

$$y_2^{10} = \frac{(-20.42 \times 1.8274 - 11.58 \times 3.2949)}{-28.15} = 2.6810$$

$$y_3^{10} = \frac{(-20.42 \times 2.6810 - 11.58 \times 3.2894)}{-28.15} = 3.2979$$

$$y_4^{10} = \frac{(-0.0221 + 4 \times 3.2979)}{4} = 3.2924$$

Iteration - 11

$$y_1^{11} = \frac{(-20.42 - 11.58 \times 2.6810)}{(-28.15)} = 1.8283$$

$$y_2^{11} = \frac{(-20.42 \times 1.8283 - 11.58 \times 3.2979)}{-28.15} = 2.6829$$

$$y_3^{11} = \frac{(-20.42 \times 2.6829 - 11.58 \times 3.2924)}{-28.15} = 3.3006$$

$$y_4^{11} = \frac{(-0.0221 + 4 \times 3.3006)}{4} = 3.2950$$

Iteration - 12

$$y_1^{12} = \frac{(-20.42 - 11.58 \times 2.6829)}{-28.15} = 1.8290$$

$$y_2^{12} = \frac{(-20.42 \times 1.8290 - 11.58 \times 3.3006)}{-28.15} = 2.6845$$

$$y_3^{12} = \frac{(-20.42 \times 2.6845 - 11.58 \times 3.2950)}{-28.15} = 3.3028$$

$$y_4^{12} = \frac{(-0.0221 + 4 \times 3.3028)}{4} = 3.2973$$

Iteration - 13

$$y_1^{13} = \frac{(-20.42 \cancel{x} - 11.58 \times 2.6845)}{-28.15} = 1.8297$$

$$y_2^{13} = \frac{(-20.42 \times 1.8297 - 11.58 \times 3.3028)}{-28.15} = 2.6859$$

$$y_3^{13} = \frac{(-20.42 \times 2.6859 - 11.58 \times 3.2973)}{-28.15} = 3.3048$$

$$y_4^{13} = \frac{(-0.0221 + 4 \times 3.3048)}{4} = 3.2992$$

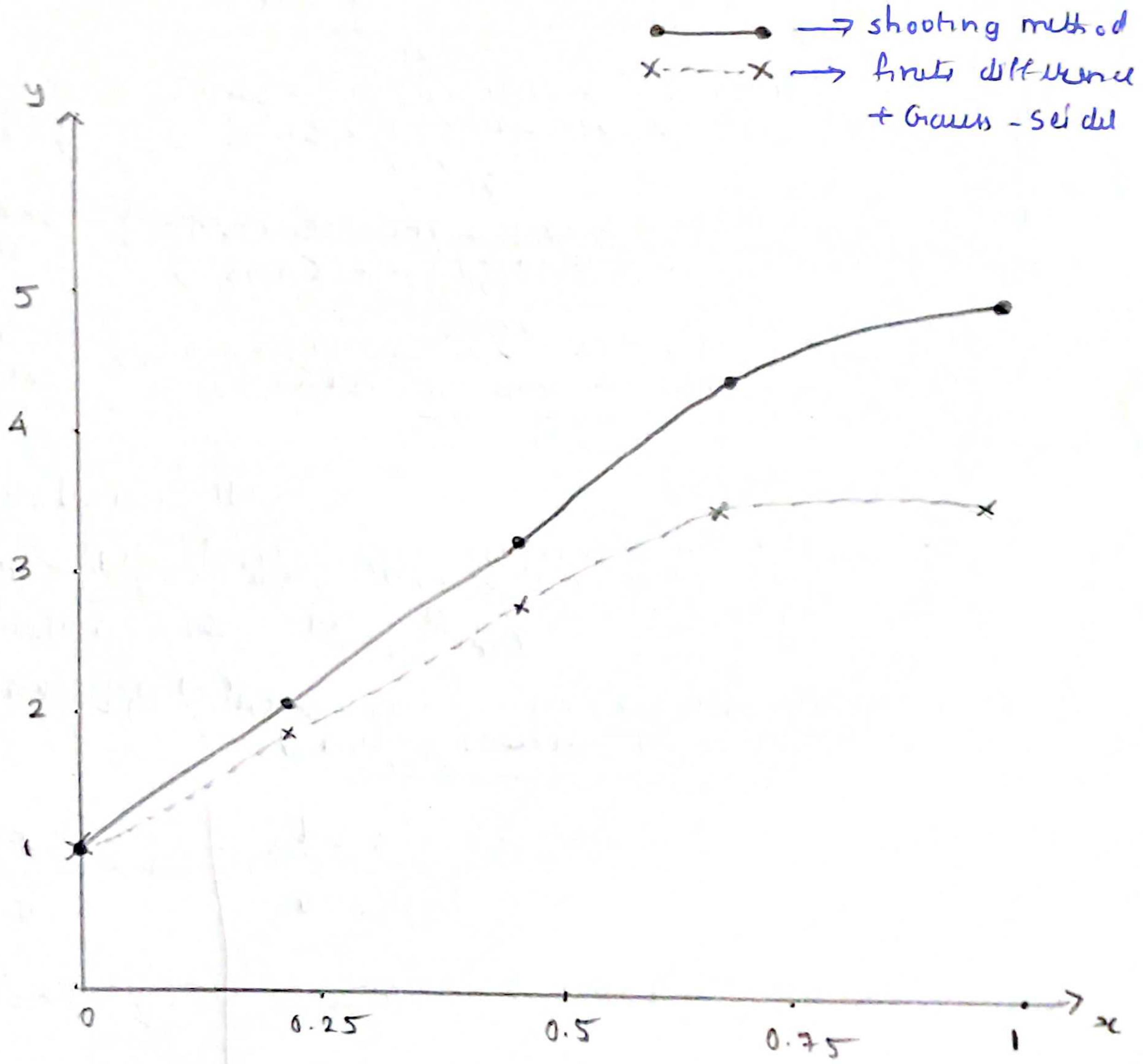
Since the first two decimal places are equal for iteration 12 and 13, we can stop iteration. (convergence)

Final solution (Finite diff + Gauss - Siedel)

x	y
0	1
0.25	1.8297
0.5	2.6859
0.75	3.3048
1	3.2992

4) Comparing both solutions

x	y (shooting method)	y (finite difference + Gauss - Seidel)
0	1	1
0.25	2.0201	1.8297
0.5	3.1925	2.6859
0.75	4.2598	3.3048
1	4.7964	3.2992



Intuition and results

Given ODE : $\ddot{y} + m\dot{y} + ny = 0$

$m^2 - 4n = (-2.21)^2 - 4 \times 3.85 < 0$

$\therefore$ General solution is of the form:

$$\boxed{y(x) = e^{\alpha x}(c_1 \cos \beta x + c_2 \sin \beta x)} \rightarrow (1)$$

$\alpha = \frac{-m}{2}$ $\quad \beta = \frac{\sqrt{4n - m^2}}{2}$

For my roll number, $m < 0$

$\therefore \alpha > 0, \; \beta > 0$

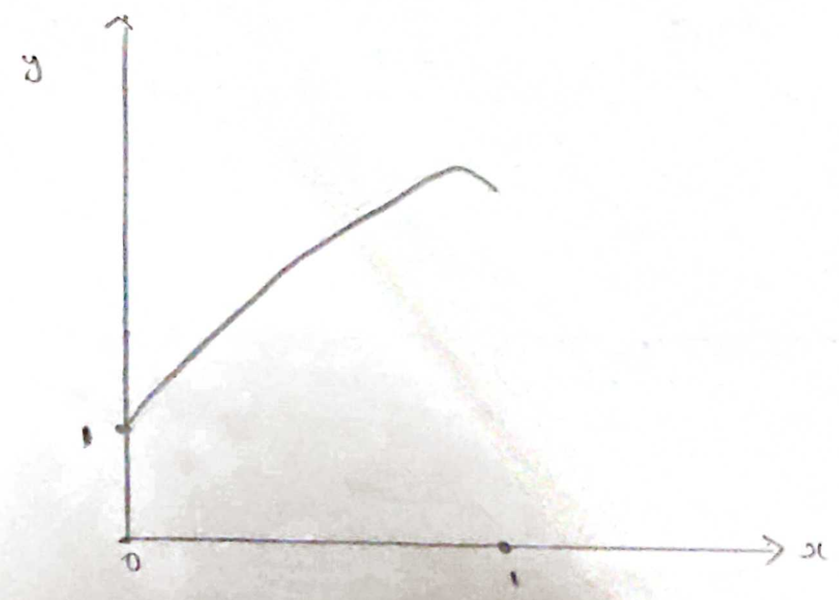
c_1 and c_2 depends on initial & boundary condition

When $\alpha, \beta > 0$, the form given in (1) is increasing for values just above zero

From boundary conditions and initial conditions, we know $y(0) = 1$ and $y'(1) = -0.0221$

Since $y'(1) < 0$, a peak occurred just before $x = 1$ & y is increasing from $x = 0$

$\therefore$ Graph by intuition (rough)



graph from intuition almost matches with our solution from shooting method & Finite difference + Gauss Seidel method.

∴ Our results are almost correct

Note:

Both numerical methods are approximations, hence their solutions are not exactly same, but trend is repeated.