Department of Mathematics, IIT Madras
MA 2040 : Probability, Statistics and Stochastic Processes
July - Nov 2020
Problem Set - 8
(Bernoulli and Poisson Processes)

1. Each of n packages is loaded independently onto either a red truck (with probability p) or onto a green truck (with probability $1-p$). Let R be the total number of items selected for the red truck and let G be the total number of items selected for the green truck.

   (a) Determine the PMF, expected value, and variance of the random variable R.

   (b) Evaluate the probability that the first item to be loaded ends up being the only one on its truck.

   (c) Evaluate the probability that at least one truck ends up with a total of exactly one package.

   (d) Evaluate the expected value and the variance of the difference $R-G$.

2. In the above problem, assume that $n \geq 2$. Given that both of the first two packages to be loaded go onto the red truck, find the conditional expectation, variance and PMF of the random variable R.

3. Gopal fails quizzes with probability $1/4$, independent of other quizzes. What is the probability that Gopal fails exactly two of the next six quizzes?

4. In the above problem,

   (a) What is the expected number of quizzes that Gopal will pass before he has failed three times?

   (b) What is the probability that the second and third time Gopal fails a quiz will occur when he takes his eighth and ninth quizzes, respectively?

   (c) What is the probability that Gopal fails two quizzes in a row before he passes two quizzes in a row?

5. Consider a Bernoulli process with probability of success in each trial equal to p.

   (a) Relate the number of failures before the rth success (sometimes called a negative binomial random variable) to a Pascal random variable and derive its PMF.

   (b) Find the expected value and variance of the number of failures before the rth success.

6. For a Bernoulli process (with success probability p), obtain an expression for the probability that the ith failure occurs before the rth success.

7. A train bridge is constructed across a wide river. Trains arrive at the bridge according to a Poisson process of rate $\lambda = 3$ per day. If a train arrives on day 0, find the probability that there will be no trains on days 1, 2, and 3.

8. In the above problem,

(a) Find the probability that the next train to arrive after the first train on day 0, takes more than 3 days to arrive.

(b) Find the probability that no trains arrive in the first 2 days, but 4 trains arrive on the 4th day.

(c) Find the probability that it takes more than 2 days for the 5th train to arrive at the bridge.

9. A store opens at $t = 0$ and potential customers arrive in a Poisson manner at an average arrival rate of λ potential customers per hour. As long as the store is open, and independently of all other events, each particular potential customer becomes an actual customer with probability p. The store closes as soon as ten actual customers have arrived.

   (a) What is the probability that exactly three of the first five potential customers become actual customers?

   (b) What is the probability that the fifth potential customer to arrive becomes the third actual customer?

10. In the above problem,

   (a) What is the PDF and expected value for L, the duration of the interval from store opening to store closing?

   (b) Given only that exactly three of the first five potential customers became actual customers, what is the conditional expected value of the total time the store is open?