Problem Sheet 10

November 1, 2020

1. Let $X_1, \ldots, X_n$ be a random sample from a Bernoulli distribution with parameter θ. Find MLE of θ.

2. Let $X_1, \ldots, X_n$ be a random sample from a Geometric distribution with parameter θ. Find MLE of θ.

3. Let $X_1, \ldots, X_n$ be a random sample from a Poisson distribution with parameter θ. Find MLE of θ.

4. Let $X_1, \ldots, X_n$ be a random sample from a Normal distribution with parameter $\mu = 1$ and σ^2. Find MLE of σ^2. Is the estimator unbiased?

5. Let $X_1, \ldots, X_n$ be a random sample from a Normal distribution with parameter μ and σ^2 (both μ and σ^2 unknown).

   (a) Find MLE of μ. Is it unbiased? Is it consistent? What the MSE of this estimator?

   (b) Find MLE $\widehat{\sigma^2_{ML}}$ of σ^2. Show that $\widehat{\sigma^2_{ML}}$ is a biased estimator of σ^2. Show that $s^2 = \frac{n}{n-1}\widehat{\sigma^2_{ML}}$ is an unbiased estimator of σ^2. Show that $s = \sqrt{\frac{n}{n-1}\widehat{\sigma^2_{ML}}}$ is a biased estimator of σ. We call s^2 (resp. s) the sample variance (resp. sample standard deviation).

6. Let $X_1, \ldots, X_n$ be a random sample from the interval $[0, \theta]$ with $\theta > 0$, that is,

$$f_X(x;\theta) = \begin{cases} \frac{1}{\theta} & \text{if } 0 \le x \le \theta, \\ 0 & \text{otherwise.} \end{cases}$$

   (a) Show that $\widehat{\Theta}_n = \max\{X_1, \ldots, X_n\}$ is MLE of θ.

   (b) Is $\widehat{\Theta}_n$ an unbiased estimator of θ?

   (c) Is $\widehat{\Theta}_n$ consistent?

   (d) Compute MSE of $\widehat{\Theta}_n$.

7. Let $X_1, \ldots, X_n$ be a random sample from the interval $(0, \theta)$ with $\theta > 0$, that is,

$$f_X(x;\theta) = \begin{cases} \frac{1}{\theta} & \text{if } 0 < x < \theta, \\ 0 & \text{otherwise.} \end{cases}$$

   Show that MLE of θ does not exist.

8. Let $X_1, \ldots, X_n$ be a random sample from the interval $[\theta, \theta + 1]$, that is,

$$f_X(x;\theta) = \begin{cases} 1 & \text{if } \theta \leq x \leq \theta + 1, \\ 0 & \text{otherwise.} \end{cases}$$

   Show that MLE of θ is not unique.

9. Let $X_1, \ldots, X_{1600}$ be random sample from a Normal distribution with mean μ and variance 25. Find the 95% confidence interval of μ.

10. Let $X_1, \ldots, X_n$ be random sample from a Normal distribution with mean μ and variance 25. How large must the sample size be so that the 95% confidence interval of μ has length 0.98?

11. Let $X_1, \ldots, X_{1600}$ be random sample from a distribution with mean μ and variance 25. Find approximate 95% confidence interval of μ.

12. Let $X_1, \ldots, X_{1600}$ be random sample from a Bernoulli distribution with parameter θ. Find approximate 95% confidence interval of θ.

13. Let $X_1, \ldots, X_{1600}$ be random sample from a distribution with mean μ and unknown variance $\sigma^2 < \infty$. If the sample mean is 76 and sample variance is 12, find approximate 95% confidence interval of μ.