

Problem Sheet - 5

1. If $f_{x,y}(x,y) = \begin{cases} kxy^2, & 0 < x < y < 1 \\ 0 & \text{otherwise} \end{cases}$ is the joint pdf of continuous random variables x and y , find the value of k .
2. If $f_{x,y}(x,y) = \begin{cases} \frac{3}{4}, & 0 < y^2 < x < 1 \\ 0 & \text{otherwise} \end{cases}$ is the joint pdf, find the marginal pdf of x & y .
3. Let $f_{x,y}(x,y) = \begin{cases} 2x, & 0 < x < 1; 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$ be the joint pdf of x and y . Find $P(x+y \leq 1 \mid x \leq \frac{1}{2})$.
4. If $F_{x,y}(x,y) = \begin{cases} \frac{1}{5}(2x^3y + 3x^2y^2), & 0 < x < 1, 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$ is the CDF of x, y , find the joint pdf of x & y .
5. Let x & y have joint pdf which is uniform on the triangle with vertices $(0,0)$, $(0,1)$ and $(1,0)$.
 - i) Find the conditional pdf of x given y .
 - ii) $E[x \mid y = y]$
 - iii) $E[x]$.

6) Let X and Y be two Continuous Random Variables.

$$\text{let } f_X(x) = \begin{cases} 24x^2, & 0 < x < 1/2 \\ 0 & \text{otherwise.} \end{cases} \quad \text{and}$$

$$f_{Y|X}(y|x) = \begin{cases} y/2x^2, & 0 < y < 2x \\ 0 & \text{otherwise} \end{cases}$$

~~Find~~ Show that $f_{X|Y}(x|y) = \begin{cases} 2/1-y, & 0 < y < 2x < 1 \\ 0 & \text{otherwise} \end{cases}$

7. Let $X \sim \text{EXP}(1)$ and $A = \{X > 1\}$.

Show that $E[X|A] = 2$ and $\text{Var}(X|A) = 1$.

8. Let $f_{X,Y}(x,y) = \begin{cases} \frac{x^2}{4} + \frac{y^2}{4} + \frac{xy}{6}, & 0 \leq x \leq 1, 0 \leq y \leq 2 \\ 0 & \text{otherwise} \end{cases}$

Show that $E[X|Y=1] = 7/12$; ~~and~~

$\text{Var}(X|Y=1) = \frac{287}{3600}$ and $P(X < 1/2 | Y=0) = 1/8$

9. Let $D = \{(x,y) : x^2 + y^2 \leq 1\}$. A point (x,y) is

Chosen uniformly at random from D .

i) Show that $f_{X|Y}(x|y)$ follows uniform distribution.

ii) Are x and y independent?

10. Let X and Y have the joint pdf $f_{X,Y}(x,y)$

$$f_{X,Y}(x,y) = \begin{cases} x^2 + y/3, & -1 \leq x \leq 1, 0 \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Show that X and Y are NOT independent.