Problem Sheet 11

November 6, 2020

1. A radar system uses radio waves to detect aircraft. The system receives a signal and based on the signal it needs to decide whether an aircraft is present or not. Suppose that X is the received signal, where $X = \theta + W$, where $\theta = 0$ if no aircraft is present, and $\theta = 1$ if an aircraft is present, here $W \sim Normal(0, \frac{1}{16})$. Suppose we define

$$H_0 : \text{ No aircraft is present.}$$

and

$$H_1 : \text{ An aircraft is present.}$$

   (a) Write null hypothesis and alternate hypothesis in terms of possible values of θ.

   (b) Design a level $\alpha = 0.05$ test to decide between H_0 and H_1. Find the probability of Type II error β for this test.

   (c) If we observe $X = 0.5$, is there enough evidence to reject H_0 at significance level $\alpha = 0.05$? What about at significance level $\alpha = 0.01$?

   (d) What should be the smallest possible value of α, which ensures $\beta \leq 0.05$.

2. You know batteries from company A lasts for expected time 1000 hours with standard deviation 30 hours. Now a new company B claims that their batteries lasts more than that of company A while maintaining the same standard deviation. To decide whether to buy from the of company B or not, you ask 10 customers who have purchased the product of company B to find out that their batteries last 1020 hours on average.

   (a) You decide to do hypothesis testing at significance level 5%. Based on this result will you decide to purchase from company A or B?

   (b) You decide to do hypothesis testing at significance level 1%. Based on this result will you decide to purchase from company A or B?

   (c) Suppose that the company B claims that their batteries lasts 1040 hours with standard deviation 30 hours. If you decide to do hypothesis testing at significance level 1% and Type II error at 5%, how many customers shall you ask and what should be the critical value for accepting/rejecting the hypothesis?

3. You know that robotic house cleaner from company A cleans a 1000 square-feet apartment in 30 minutes with a standard deviation of 5 minutes. You want to buy a new robotic house cleaner after your old robotic house cleaner of company A has broken down. In the store, you found out that a new company B claims that their robotic house cleaner cleans a 1000 square-feet apartment in less than 30 minutes with a standard deviation of 5 minutes. You sample 10 models of company B and found that on average their robotic house cleaner cleans a 1000 square-feet apartment in 28 minutes. You are willing to take risk and decided to do hypothesis testing at a high significance value 10% to decide. Based on this test, should you stick to company A or switch to company B ?

4. A rice manufacturer makes rice packets of expected weight 1 kg and standard deviation 10 grams. Samples are drawn hourly and checked. If the production level gets out of sync with a statistical significance more than 5%, then the process is stopped and fixed. A sample of 20 rice packets have mean 992 grams. Should the process be stopped for adjustment?