Problem Sheet - 7

1. Let X have mean $5/4$ and variance $5/48$.
   Estimate $P(X \geq 2.5)$ using Chebyshev's inequality.
   Find a so that $P(X \geq a) \leq \frac{15}{100}$

2. Let $X \sim UNIF(0, 10)$. Find the bound for $P(2 \leq X \leq 8)$ using Chebyshev inequality. Compare it with the actual probability.

3. Let $X \sim UNIF(0, 4)$. Using Markov inequality, compute the bounds for i) $P(X \geq 2)$ (ii) $P(X \geq 4)$. Compare them with the actual probabities.

4. 100 luggages are loaded in a flight. Assume that weights are indep. random variables that are uniformly distributed between 5 and 50 Kg's. Estimate the probability that total weight will exceed 3000 Kgs.

5. Let $X \sim BIN(20; 1/2)$. Use CLT, to estimate $P(8 \leq X \leq 10)$.

6. The service time X for a customer has mean 2 and variance 1. Let Y be the total service time spent serving 50 customers. Use CLT, to estimate $P(90 < Y \leq 100)$.

7. Let $X_1, X_2, \ldots$ be iid r.v's with mean $\mu = 71.43$ and variance 56.25. Find $P(68.91 \leq X_1 + \cdots X_{25} \leq 71.97)$ using CLT.

8. Light bulbs are installed into a socket and allowed to burn continuously. Let X, the life time of bulb have mean $\mu = 2$ months and std. deviation $\sigma = 0.25$ month. How many bulbs n are needed so that one can be 95% sure that the supply of n bulbs will last 5 years.

9. Let X be any r.v. and $X_n = X + Y_n$ where $E[Y_n] = 1/n$ and $Var(Y_n) = \sigma^2/n$ where $\sigma > 0$ is a constant. Show that $X_n \xrightarrow{P} X$