

Problem Sheet 3.

1. Let X be a discrete r.v with PMF

$$p_x(x) = \begin{cases} 0.1 & \text{if } x = 0.2 \\ 0.2 & \text{if } x = 0.4 \\ 0.2 & \text{if } x = 0.5 \\ 0.3 & \text{if } x = 0.8 \\ 0.2 & \text{if } x = 1 \\ 0 & \text{otherwise.} \end{cases}$$

i) Show that $P(\cancel{x \leq 0.5}) P(0.25 \leq x \leq 0.75) = 0.4$

ii) Show that $P(X = 0.2 | X < 0.6) = 0.2$

2. A fair coin is tossed twice. Let X be the no. of heads observed. Find the CDF of X .

3. If X has the PMF $p_x(x) = \frac{1}{2^x}$, $x = 1, 2, 3, \dots$

Show that $P(X > 4) = 1/16$.

4. If X follows geometric distribution with parameter p , show that i) $E[X] = 1/p$ and ii) $E\left[\frac{1}{2^X}\right] = \frac{p}{1+p}$

5. Let X have the PMF $p_x(x) = \begin{cases} 0.2 & \text{if } x = 0 \\ 0.2 & \text{if } x = 1 \\ 0.3 & \text{if } x = 2 \\ 0.3 & \text{if } x = 3 \\ 0 & \text{otherwise} \end{cases}$

Find the PMF of $Y = X(X-1)(X-2)$.

6. let $Y = -2X + 3$ with $E[Y] = 1$ and $E[Y^2] = 9$. Find the Variance of X .

7. let $p_{x,y}(x,y) = kxy$, $x=1,2,3$ & $y=1,2,3$.
be the joint pmf of the r.v's X and Y .

Find k .

8. let $p_{x,y}(x,y) = \begin{cases} \frac{1}{21}(x+y) & , \quad x=1,2 \\ & y=1,2,3 \\ 0 & \text{otherwise} \end{cases}$

Find the marginal pmf's of X and Y .

9. let X denote the no. observed when a fair die is rolled. let $A = \{X < 5\}$. Find the Conditional PMF of X given A .

10. A point (X,Y) is picked randomly from the set $\{(x,y) : x,y \in \mathbb{Z} \text{ and } |x|+|y| \leq 2\}$.

i) Find the joint and marginal PMF's of X and Y .

ii) Find the Conditional PMF of X given $Y=1$

iii) Are X and Y independent?