Problem Sheet 6:

1. Let $X \sim EXP(2)$ and $Y = \sqrt{X}$. Find the PDF of Y.
2. Let $X \sim UNIF(0,1)$ and $Y = \frac{1}{X}$. Find the PDF of Y.
3. If X and $Y \sim UNIF(0,1)$ and are independent, find the PDF of $Z = X+Y$.
4. If X and Y are independent and uniformly distributed on $[0,1]$, find the PDF of $Z = |X-Y|$.
5. Let $X, Y \sim EXP(1)$ and independent. Let $Z = X+Y$. Find the PDF of Z.
6. Let $X \sim EXP(3)$ and $Y \sim EXP(4)$. If X and Y are independent, Show that $Z = \min\{X,Y\} \sim EXP(7)$.
7. If $E[X] = 0$, $E[X^2] = 1$, $E[X^3] = 0$ and $E[X^4] = 3$, find $\rho(X,Y)$ where $Y = 1 + 2X + 3X^2$.
8. Let X and Y be two random variables with $Var(X) = 4$ and $Var(Y) = 9$. If $2X-Y$ and $X+Y$ are independent, find $Cov(X,Y)$ and $\rho(X,Y)$
9. Let X and $Y \sim N(0;1)$. Find $\rho(7+X+Y, 1+Y)$, if X and Y are independent.
10. Let $f_{X,Y}(x,y) = \begin{cases} 2 & \text{if } y+x \leq 1, x>0, y>0 \\ 0 & \text{otherwise} \end{cases}$ be the joint PDF of X & Y. Show that $\rho(X,Y) = -1/2$.