Problem Sheet 4

1. If a random variable X has the pdf

$$f_X(x) = \begin{cases} h\,x/2 & , 0 \le x \le 2 \\ 0 & \text{otherwise} \end{cases},$$

find the value of h. Also determine its CDF, mean and variance.

2. Let X be uniformly distributed in the interval $[0,1]$. Let $Y = g(X)$ where

$$g(x) = \begin{cases} 1 & \text{if } x \le 1/3 \\ 2 & \text{if } x > 1/3 \end{cases}.$$

Find $E[Y]$.

3. Let X have the CDF $F_X(x) = \begin{cases} 0 & \text{if } x < -1 \\ \frac{x+1}{2} & \text{if } -1 \le x < 1 \\ 1 & \text{if } y \ge 1 \end{cases}$

Show that its variance is $1/3$.

4. Let X and Y be two normal random variables with mean 0 and 1, respectively and variance 1 and 4, respectively.

a) Show that i) $P(X \le 1.5) = 0.9332$

ii) $P(X \le -1) = 0.1587$

b) Show that $\frac{Y-1}{2}$ is std. normal

c) $P(-1 \le Y \le 1) = 0.3413$

5. Let $X \sim N(0;1)$. Using normal table, find x in the following cases.

i) $P(X \leq x) = 0.6406$

ii) $P(X > x) = 0.0606$

iii) $P(0 \leq X \leq x) = 0.4783$

iv) $P(-1.5 \leq X \leq x) = 0.2313$

v) $P(-x \leq X \leq x) = 0.5467$

6. Let $X \sim N(10;2)$. Find the values of

i) $P(X \leq 10)$ and (ii) $P(8 \leq X \leq 14)$

7. The germination success rate for certain seeds is 60%. In a package of 200 seeds, use normal approx. to find the probability that atleast one half of them germinate. (Ans. 0.9975)

8. Let $X \sim EXP(2)$ and $Y = 2 + 3X$

Show that i) $P(X > 2) = e^{-4}$.

ii) $Var(Y) = 9/4$

iii) $P(X > 2 \mid Y < 11) = \dfrac{e^{-4} - e^{-6}}{1 - e^{-6}}$

9. A real number is chosen at random on the interval $[2, 6]$. Let X be the chosen number. Find $F_X(x)$. (CDF) of X and $E[X]$.