Solutions to Problem Sheet - 2

1. $$P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{0.2}{0.35} = 4/7$$

$$P(C|B) = \frac{P(C\cap B)}{P(B)} = \frac{0.15}{0.35} = 3/7$$

$$P(B|A\cup C) = \frac{P(B\cap(A\cup C))}{P(A\cup C)} = \frac{0.1+0.1+0.05}{0.7} = 5/14$$

$$P(B|A\cap C) = \frac{P(B\cap A\cap C)}{P(A\cap C)} = \frac{0.1}{0.2} = 1/2.$$

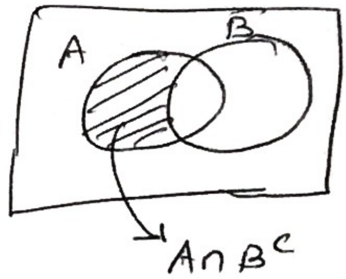
2. $G = \{$selected child is girl$\}$ &
$A = \{$all children are girls$\}$
$P(G|A) = 1$; $P(A) = 1/2^5$; $P(G) = 1/2$.

$$P(A|G) = \frac{P(G|A)\,.\,P(A)}{P(G)} = \frac{1.\,1/2^5}{1/2} = 1/2^4 = 1/16$$

3. Let B_i be the event of choosing i^{th} bag and
R be the event of choosing red marble.
$P(R|B_1) = 0.75$, $P(R|B_2) = 0.6$ & $P(R|B_3) = 0.45$

$$P(R) = P(R|B_1)\cdot P(B_1) + P(R|B_2)\cdot P(B_2) + P(R|B_3)\cdot P(B_3)$$
$$= 0.60. \quad (\because P(B_i) = 1/3 \;\forall i = 1,2,3).$$

4. $A = (A \cap B) \cup (A \cap B^c)$



$P(A) = P(A \cap B) + P(A \cap B^c)$

$= P(A) \cdot P(B) + P(A \cap B^c)$

$\therefore P(A \cap B^c) = P(A)(1 - P(B))$

$= P(A) \cdot P(B^c)$.

— x —

5. We've $P(A|C) = 1/2 = P(B|C)$

$P(A \cap B | C) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$. Thus A & B are Cond. indep. given C.

$P(A) = P(A|C) \cdot P(C) + P(A|C^c) \cdot P(C^c)$

$= \frac{1}{2} \cdot \frac{1}{2} + 1 \cdot \frac{1}{2} = 3/4$.

|||ly $P(B) = 3/4$.

$P(A \cap B) = P(A \cap B | C) \cdot P(C) + P(A \cap B | C^c) \cdot P(C^c)$

$= P(A|C) \cdot P(B|C) \cdot P(C) + P(A|C^c) \cdot P(B|C^c) \cdot P(C^c)$

$= \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} + 1 \cdot 1 \cdot \frac{1}{2} = 5/8$.

$P(A \cap B) \neq P(A) \cdot P(B)$.

$\therefore$ A & B are not independent.

6. $P(A) = 1/3$ & $P(B) = 1/2$.

$P(A \cap B) = 1/6 = P(A) \cdot P(B)$.

But $P(A|C) = 1/2 = P(B|C)$ and

$P(A \cap B | C) = 0$.

7. $P(A \cap B) = \sum_{i=1}^{n} P(A \cap B | C_i) P(C_i)$ (total prob. law)

$= \sum P(A|C_i) \cdot P(B|C_i) \cdot P(C_i)$ ($\because$ Cond. indep.)

$= \sum P(A|C_i) \cdot P(B) \cdot P(C_i)$ ($\because$ B is indep. of C_i)

$= P(B) \sum_{i=1}^{n} P(A|C_i) \cdot P(C_i)$

$= P(B) \cdot P(A)$ ($\because$ total prob. law)

8. Let $C_1 = \{$ Choosing regular coin $\}$

$C_2 = \{$ Choosing fake coin $\}$.

Given $P(H|C_1) = \frac{1}{2}$ & $P(H|C_2) = 1$.

$P(H) = P(H|C_1) \cdot P(C_1) + P(H|C_2) \cdot P(C_2)$

$= \frac{1}{2} \cdot \frac{2}{3} + 1 \cdot \frac{1}{3} = \frac{2}{3}$.

$$P(C_2|H) = \frac{P(H|C_2) \cdot P(C_2)}{P(H)} = \frac{1 \cdot \frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}.$$

9. X takes values from 2 to 10.

$f(2) = \frac{1}{25}$; $f(3) = \frac{2}{25}$; $f(4) = \frac{3}{25}$, $f(5) = \frac{4}{25}$

$f(6) = \frac{5}{25}$; $f(7) = \frac{4}{25}$, $f(8) = \frac{3}{25}$, $f(9) = \frac{2}{25}$ and

$f(10) = \frac{1}{25}$.

10. $P_Y(y)$ = prob. for $(y-1)$ failures & success at y^{th} toss

$$= \begin{cases} (1-p)^{y-1} \cdot y & y = 1, 2, \ldots \\ 0 & \text{otherwise} \end{cases}$$