Question 1

Examples of custom commands: $X_1, ..., X_n \overset{\text{iid}}{\sim} \mathcal{N}\left(\mu, \sigma^2\right)$, $\text{trace}(\mathbf{H}) = p$. Code example:

Code Listing 1: Smoothed estimation of mean trajectory

```
# Compute the optimally-smoothed mean
lambdaOpt <- 10^logLambda[which(estimationError == min(estimationError))]
smoothOperatorOpt <- fdPar(bSplineBasis, int2Lfd(2), lambdaOpt)
smoothSpectrumOpt <- smooth.basis(logFreq, rawData, smoothOperatorOpt)
smoothMeanOpt <- PhiMatrix %*% mean.fd(smoothSpectrumOpt$fd)$coefs

# Plot data along unsmoothed mean and optimally-smoothed mean
matplot(logFreq, rawData, type="l", col=brewer.pal(8, "Set2"), xlab="Log frequency", ylab="Log power spectrum", main="Raw data with optimally-smoothed and\n unsmoothed means")
lines(logFreq, unsmoothedMean, lwd=2.5, col="firebrick3")
lines(logFreq, smoothMeanOpt, lwd=2.5, col="blue3")
```

Theorem (Leibniz integral rule)**.** Let $f(x,t)$ and its partial derivative $\frac{\partial}{\partial x} f(x,t)$ be continuous in x and t in some region of the (x,t) plane that includes $a(x) \leq t \leq b(x)$ and $x_0 \leq x \leq x_1$. Suppose also that the functions $a(x)$ and $b(x)$ are continuous and have continuous derivatives for $x_0 \leq x \leq x_1$. Then, for $x_0 \leq x \leq x_1$, we have:

$$\frac{\mathrm{d}}{\mathrm{d}x} \int_{a(x)}^{b(x)} f(x,t)\,\mathrm{d}t = f(x, b(x)) \cdot \frac{\mathrm{d}}{\mathrm{d}x} b(x) - f(x, a(x)) \cdot \frac{\mathrm{d}}{\mathrm{d}x} a(x) + \int_{a(x)}^{b(x)} \frac{\partial}{\partial x} f(x,t)\,\mathrm{d}t$$

If $a(x) = a$ and $b(x) = b$, where a and b are constants, then the above reduces to:

$$\frac{\mathrm{d}}{\mathrm{d}x} \int_a^b f(x,t)\,\mathrm{d}t = \int_a^b \frac{\partial}{\partial x} f(x,t)\,\mathrm{d}t$$

References

[1] Last name, first name. *Book Title*. Publisher, Year. Print.

[2] Last name, first name. "Webpage Title". Website name, Organization name. Online; accessed Month Date, Year. `www.URLhere.com`