KUTAISI INTERNATIONAL UNIVERSITY

Quiz (G-9)

Linear Algebra for Informatics
Fall semester 2024
Noe Lomidze

Problem

$$\text{Compute} \quad (2+2i)^{10}$$

Solution

$$(2+2i)^{10} = 2^{10}(1+i)^{10} \quad \text{so now lets just calculate that}$$

$$(1+i) \Rightarrow r = \sqrt{1+1} = 2, \ \theta = \arctan\left(\frac{1}{1}\right) = \frac{\pi}{4} \quad \text{so that complex number in polar coorinates will be:}$$

$$(1+i) \Leftrightarrow \sqrt{2}\left(\cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)\right)$$

Now we can apply the De Moivre's theorem: $z^n = r^n(\cos(n\cdot\theta) + i\sin(n\cdot\theta))$

$$\Rightarrow 2^{10}\cdot\sqrt{2}^{10}\left(\cos\left(10\cdot\frac{\pi}{4}\right) + i\sin\left(10\cdot\frac{\pi}{4}\right)\right) \Rightarrow 2^{15}\left(\cos\left(\frac{5\pi}{2}\right) + i\sin\left(\frac{5\pi}{2}\right)\right)$$

$$\Rightarrow 2^{15}\left(\cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right)\right) \Rightarrow 2^{15}\cdot i$$

Problem

$$\text{Find all } 6^{\text{th}} \text{ roots of} -64$$

Solution

$$\text{General root formula for the nth roots of a complex number } \; r(\cos(\theta) + i\sin(\theta)) :$$

$$x_i = \sqrt[n]{r} \cdot \left(\cos\left(\frac{\theta + 2\pi k_i}{n}\right) + i\sin\left(\frac{\theta + 2\pi k_i}{n}\right)\right) \quad \text{where} \quad k = 0, 1, 2, \ldots n-1$$

$$-64 \text{ in polar form } -64 = 64 \cdot (\cos(\pi) + i\sin(\pi)), r = 64, \; \theta = \pi, \; n = 6$$

$$\text{for example } x_0 = 2 \cdot \left(\cos\left(\frac{\pi + 2\pi \cdot 0}{6}\right) + i\sin\left(\frac{\pi + 2\pi \cdot 0}{6}\right)\right) \Rightarrow$$

$$x_0 = 2 \cdot \left(\cos\left(\frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{6}\right)\right) \Leftrightarrow x_0 = 2\left(\frac{\sqrt{3}}{2} + i\frac{1}{2}\right) = \sqrt{3} + i$$

$$x_1 = 2\left(\cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right)\right) \Leftrightarrow x_1 = 2(0 + i) = 2i$$

$$\text{for } \; k = 2, 3 \ldots \; \text{ you can continue like that } \ldots$$

Problem

Find all solutions of the equation $e^{ix} = -1$

Solution

We know that $e^{ix} = \cos(x) + i\sin(x)$

So in our case: $\cos(x) + i\sin(x) = -1 \Rightarrow$ Since it has no imaginary part, the $\sin(x) = 0$ and $\cos(x) = -1$

From trigonometry we know that $x = \pi + 2\pi k, \ \forall k \in Z$