KUTAISI INTERNATIONAL UNIVERSITY

Quiz (G-9)

Linear Algebra for Informatics
Fall semester 2024
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Problem

$$\text{Compute} \quad (4+4i)^{10}$$

Solution

$$(4+4i)^{10} = 4^{10}(1+i)^{10} \quad \text{so now lets just calculate that}$$

$$(1+i) \Rightarrow r = \sqrt{1+1} = 2, \; \theta = \arctan\left(\frac{1}{1}\right) = \frac{\pi}{4} \quad \text{so that complex number in polar coorinates will be:}$$

$$(1+i) \Leftrightarrow \sqrt{2}\left(\cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)\right)$$

$$\text{Now we can apply the De Moivre's theorem: } z^n = r^n(\cos(n\cdot\theta) + i\sin(n\cdot\theta))$$

$$\Rightarrow 4^{10}\cdot\sqrt{2}^{10}\left(\cos\left(10\cdot\frac{\pi}{4}\right) + i\sin\left(10\cdot\frac{\pi}{4}\right)\right) \Rightarrow 2^{25}\left(\cos\left(\frac{5\pi}{2}\right) + i\sin\left(\frac{5\pi}{2}\right)\right)$$

$$\Rightarrow 2^{25}\left(\cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right)\right) \Rightarrow 2^{25}\cdot i$$

Problem

Find all 5^{th} roots of -32

Solution

General root formula for the nth roots of a complex number $r(\cos(\theta)+i\sin(\theta))$:

$$x_i=\sqrt[n]{r}\cdot\left(\cos\left(\frac{\theta+2\pi k_i}{n}\right)+i\sin\left(\frac{\theta+2\pi k_i}{n}\right)\right)\quad\text{where}\quad k=0,1,2,\ldots n-1$$

$$-32\text{ in polar form }-32=32\cdot(\cos(\pi)+i\sin(\pi)),r=32,\ \theta=\pi,\ n=5$$

$$\text{for example } x_0=2\cdot\left(\cos\left(\frac{\pi+2\pi\cdot 0}{5}\right)+i\sin\left(\frac{\pi+2\pi\cdot 0}{5}\right)\right)\Rightarrow 2\cdot\left(\cos\left(\frac{\pi}{5}\right)+i\sin\left(\frac{\pi}{5}\right)\right)\text{ for }\ k=1,2,3\ldots\ \text{ you can continue like that }\ldots$$

$$x_0=2\left(\cos\left(\frac{\pi}{5}\right)+i\sin\left(\frac{\pi}{5}\right)\right)$$

$$x_1=2\left(\cos\left(\frac{3\pi}{5}\right)+i\sin\left(\frac{3\pi}{5}\right)\right)\ldots$$

Problem

Find all solutions of the equation $e^{ix} = i$

Solution

We know that $e^{ix} = \cos(x) + i\sin(x)$

So in our case: $\cos(x) + i\sin(x) = i \Rightarrow$ Since it has no real part, the $\cos(x) = 0$ and $\sin(x) = 1$

From trigonometry we know that $x = \dfrac{\pi}{2} + 2\pi k, \ \forall k \in Z$