

♦ Problem

Solution

$$\det \begin{pmatrix} 0 & -7 & -4 \\ 2 & 4 & 6 \\ 3 & 7 & -1 \end{pmatrix} \Rightarrow 0 \cdot \det \begin{pmatrix} 4 & 6 \\ 7 & -1 \end{pmatrix} - (-7) \cdot \det \begin{pmatrix} 2 & 6 \\ 3 & -1 \end{pmatrix} + (-4) \det \begin{pmatrix} 2 & 4 \\ 3 & 7 \end{pmatrix} = 0 + 7(-2 - 18) - 4(14 - 12) = -140 - 8 = -148$$

For the second part:

$$\det(ABCD) = \det(A)\det(B)\det(C)\det(D)$$

First matrix is made by swapping the 1st and 3th rows from the above matrix, (swapping two rows in the matrix multiplies det(A) by -1) so $\det(A) = 148$

Second matrix is made by swapping the 1st and 2nd rows from the previous one so det(B) = -148

Third matrix is made by swapping the 2nd and 3rd rows from the previous matrix so det(C) = 148

4th matrix is made by multiplying the first row of the previous matrix by (-1/2) so $\det(D) = 148 \cdot \left(-\frac{1}{2}\right) \Rightarrow -74$

so finally $\det(ABCD) = \det(A)\det(B)\det(C)\det(D) = 148 \cdot (-148) \cdot 148 \cdot (-74)$

♠ Problem

Compute
$$\det \begin{pmatrix} \pi & e & 11 \\ 3\pi & 3e & 33 \\ 12 & -7 & 2 \end{pmatrix}$$

Solution

You can easily see that $R_2=3\cdot R_1$, second row is dependant on the first one so.. $\det\begin{pmatrix} \pi & e & 11\\ 3\pi & 3e & 33\\ 12 & -7 & 2 \end{pmatrix}=0$

Problem

Bonus

Find the area of the interior E of the ellipse defined by the equation :

$$\left(\frac{2x-y}{2}\right)^2 + \left(\frac{y+3x}{3}\right)^2 = 1$$

Hint: The ellipse is obtained from the unit circle $X^2 + Y^2 = 1$ by linear change of coordinates..

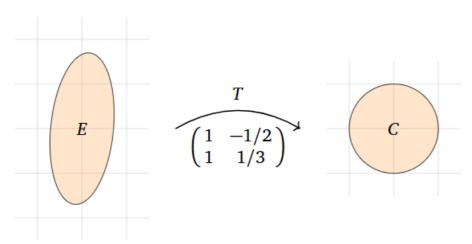
Solution

$$X = \frac{2x - y}{2}, \ Y = \frac{y + 3x}{3}$$

We can define the transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$ such that:

$$T \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{2x-y}{2} \\ \frac{y+3x}{3} \end{pmatrix}$$

$$T \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \ T \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} \\ \frac{1}{3} \end{pmatrix} \Longrightarrow A = \begin{pmatrix} 1 & -\frac{1}{2} \\ 1 & \frac{1}{3} \end{pmatrix}$$



Therefore, T scales areas by a factor of $|\det(A)| = \frac{5}{6}$. The area of the unit circle is π , so

$$\pi = \operatorname{vol}(C) = \operatorname{vol}(T(E)) = |\det(A)| \cdot \operatorname{vol}(E) = \frac{5}{6} \operatorname{vol}(E) \Rightarrow \operatorname{vol}(E) \Longrightarrow \pi \cdot \frac{6}{5}$$