Task 1

Find the values of k for which the system:

$$\begin{cases} x + ky = 1 \\ kx + y = 1 \end{cases}$$

1. has no solution 2. has exactly one solution 3. has infinitely many solutions
4. When there is exactly one solution, find x and y

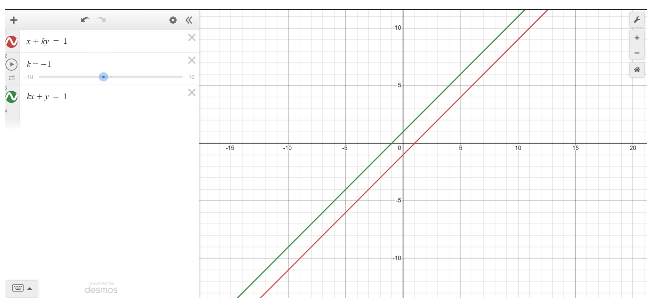
Solution for Task 1

We represent the system as an augmented matrix:

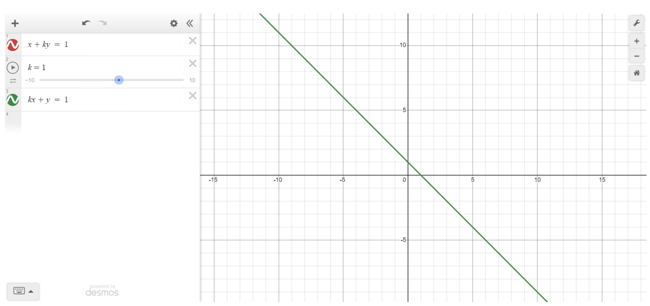
$$\begin{pmatrix} 1 & k & | & 1 \\ k & 1 & | & 1 \end{pmatrix}$$

To help visualize how the lines change as k varies, consider the following graphs:

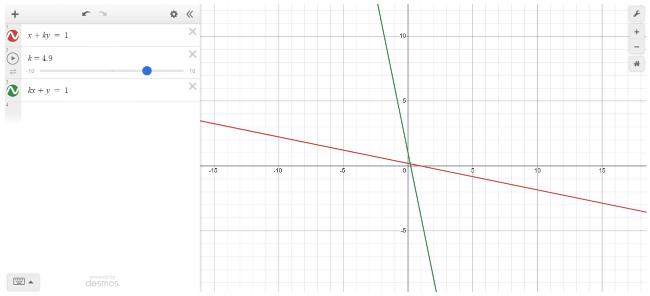
Case 1: $k = -1$ (No Solution)



Case 2: $k = 1$ (Infinitely Many Solutions)



Case 3: $k \neq \pm 1$ **(Exactly One Solution)**



Next, we perform row operations to reduce the system. Subtract $k\times$ (Row 1) from (Row 2):

$$R_2 \to R_2 - kR_1$$

This gives:

$$\begin{pmatrix} 1 & k & | & 1 \\ 0 & 1-k^2 & | & 1-k \end{pmatrix}$$

1. **No solution**: If $k = -1$, the second row becomes $0 = 2$, which is inconsistent:

$$\begin{pmatrix} 1 & -1 & | & 1 \\ 0 & 0 & | & 2 \end{pmatrix}$$

This is a contradiction, so the system has no solution for $k = -1$.

2. **Exactly one solution**: If $k \neq 1$ and $k \neq -1$, the system reduces to:

$$y = \frac{1-k}{1-k^2}, \quad x = 1 - ky = \frac{1-k}{1-k^2}$$

Therefore, the system has exactly one solution for x and y.

3. **Infinitely many solutions**: If $k = 1$, the system becomes:

$$\begin{pmatrix} 1 & 1 & | & 1 \\ 0 & 0 & | & 0 \end{pmatrix}$$

This corresponds to the equation $x + y = 1$, which has infinitely many solutions.

Task 2

Suppose matrix A is transformed into matrix B by means of an elementary row operation. Is there an elementary row operation that transforms B into A? Explain.

Solution for Task 2

Yes, each elementary row operation is reversible, that is, it can be "undone." For example, the operation of row swapping can be undone by swapping the same rows again. The operation of dividing a row by a scalar can be reversed by multiplying the same row by the same scalar.