KUTAISI INTERNATIONAL UNIVERSITY

Quiz 4

Linear Algebra for Informatics
Fall semester 2024
Noe Lomidze

Problem

Find all matrices X that satisfy the given matrix equation $\begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix} X = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

Solution

$$\begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a+2c & b+2d \\ 2a+4c & 2b+4d \end{pmatrix} \Rightarrow a+2c=0,\ b+2d=0,\ 2a+4c=0,\ 2b+4d=0$$

$$\Rightarrow a=-2c \quad \text{and} \quad b=-2d$$

$$\text{Thus} \quad X = \begin{pmatrix} -2c & -2d \\ c & d \end{pmatrix} \text{ where } \ c,d \text{ are arbitrary constants}$$

Problem

$$\text{Find the inverse of the matrix } \begin{pmatrix} 1 & 0 & 4 \\ 0 & 1 & 2 \\ 0 & -3 & -4 \end{pmatrix}$$

Solution

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 4 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & -3 & -4 & 0 & 0 & 1 \end{array}\right) \xrightarrow{R_3=R_3+3R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & 4 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 & 3 & 1 \end{array}\right) \xrightarrow[R_2=R_2-R_3]{R_1=R_1-2R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -6 & -2 \\ 0 & 1 & 0 & 0 & -2 & -1 \\ 0 & 0 & 2 & 0 & 3 & 1 \end{array}\right)$$

$$\text{And finally} \xrightarrow{R_3=\frac{R_3}{2}} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -6 & -2 \\ 0 & 1 & 0 & 0 & -2 & -1 \\ 0 & 0 & 1 & 0 & \frac{3}{2} & \frac{1}{2} \end{array}\right) \text{ So we get that } \begin{pmatrix} 1 & 0 & 4 \\ 0 & 1 & 2 \\ 0 & -3 & -4 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & -6 & -2 \\ 0 & -2 & -1 \\ 0 & \frac{3}{2} & \frac{1}{2} \end{pmatrix}$$

Problem

Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the reflection over the y-axis. Is T invertible?

If so, what is T^{-1}?

Solution

We've seen that matrix for T is

$A = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ So it's easy to see that $A^{-1} = A$

$\left(\begin{array}{cc|cc} -1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{array}\right) \overset{R_1=-R_1}{\Longrightarrow} \left(\begin{array}{cc|cc} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 \end{array}\right)$ thats it. $T^{-1} = T$

This is another way of saying that a reflection “undoes” itself