Quiz 4

Linear Algebra for Informatics
Fall semester 2024
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Problem

Find all matrices X that satisfy the given matrix equation $\begin{pmatrix} 2 & 3 \\ 1 & -1 \end{pmatrix} X = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

Solution

$$\begin{pmatrix} 2 & 3 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 2a+3c & 2b+3d \\ a-c & b-d \end{pmatrix} \Rightarrow 2a+3c=0, a-c=0, 2b+3d=0, b-d=0$$

$$a = -\frac{3}{2}c \wedge a = c \Longrightarrow c = a = 0$$

$$b = -\frac{3}{2}d \wedge b = d \Longrightarrow d = b = 0$$

$$\text{Thus} \quad X = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

Problem

$$\text{Find the inverse of the matrix } \begin{pmatrix} 1 & 1 & 2 \\ 2 & -1 & 0 \\ 2 & 2 & 1 \end{pmatrix}$$

Solution

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 2 & -1 & 0 & 0 & 1 & 0 \\ 2 & 2 & 1 & 0 & 0 & 1 \end{array}\right) \xRightarrow{R_2=R_2-2R_1} \left(\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 0 & -3 & -4 & -2 & 1 & 0 \\ 2 & 2 & 1 & 0 & 0 & 1 \end{array}\right) \xRightarrow{R_3=R_3-2R_1} \left(\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 0 & -3 & -4 & -2 & 1 & 0 \\ 0 & 0 & -3 & -2 & 0 & 1 \end{array}\right)$$

$$\xRightarrow{R_3=-\frac{R_3}{3}} \left(\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 0 & -3 & -4 & -2 & 1 & 0 \\ 0 & 0 & 1 & \frac{2}{3} & 0 & -\frac{1}{3} \end{array}\right) \xRightarrow{R_2=R_2+4R_3} \left(\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 0 & -3 & 0 & \frac{2}{3} & 1 & -\frac{4}{3} \\ 0 & 0 & 1 & \frac{2}{3} & 0 & -\frac{1}{3} \end{array}\right) \xRightarrow{R_2=-\frac{R_2}{3}} \left(\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & -\frac{2}{9} & -\frac{1}{3} & \frac{4}{9} \\ 0 & 0 & 1 & \frac{2}{3} & 0 & -\frac{1}{3} \end{array}\right) \xRightarrow[R_1=R_1-R_2]{R_1=R_1-2R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{1}{9} & \frac{1}{3} & \frac{2}{9} \\ 0 & 1 & 0 & -\frac{2}{9} & -\frac{1}{3} & \frac{4}{9} \\ 0 & 0 & 1 & \frac{2}{3} & 0 & -\frac{1}{3} \end{array}\right)$$

$$\text{Thus, the inverse is } \begin{pmatrix} 1 & 1 & 2 \\ 2 & -1 & 0 \\ 2 & 2 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} -\frac{1}{9} & \frac{1}{3} & \frac{2}{9} \\ -\frac{2}{9} & -\frac{1}{3} & \frac{4}{9} \\ \frac{2}{3} & 0 & -\frac{1}{3} \end{pmatrix}$$

Problem

Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the orthogonal projection onto the vector $u = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$. Is T invertible?

If so, what is T^{-1}?

Solution

An orthogonal projection onto a vector $u = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ in $\mathbb{R}^2$ is not invertible

When you project a vector onto u, you essentiall map $\mathbb{R}^2$ onto a line in the direction of u.

The projection matrix T can be expressed as:

$T = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ Which you can easily check that isn't invertible