KUTAISI INTERNATIONAL UNIVERSITY

Quiz 4(G-9)

Linear Algebra for Informatics
Fall semester 2024
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Problem

$$W = \left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \in \mathbb{R}^3 : x_1 - 4x_2 + 5x_3 = 2 \right\}$$

Is W a subspace of $\mathbb{R}^3$? Explain your answer

Solution

Since the zero vector $\mathbf{0}$ does not satisfy the defining relation $x_1^2 - 4x_2 + 5x_3 = 2$, it is not in W. Hence W is not a subspace of $\mathbb{R}^3$, thats it..

Problem

Find the kernel of the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ Given by

$$T\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 7 & 1 \\ 0 & -3 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

Solution

To find the kernel of T, we solve

$$\begin{pmatrix} 7 & 1 \\ 0 & -3 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

This gives the equations: $7x_1 + x_2 = 0, -3x_2 = 0 \implies kernel(T) = \left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right\}$

Problem

Show that given vectors are linearly dependent

$$\begin{pmatrix} 0 \\ -1 \\ 3 \end{pmatrix}, \begin{pmatrix} -2 \\ 3 \\ 0 \end{pmatrix}, \begin{pmatrix} -6 \\ 7 \\ 6 \end{pmatrix}$$

Solution

To show that those vectors are linearly dependent, we need to find coefficients a, b, c such that:

$$a\begin{pmatrix} 0 \\ -1 \\ 3 \end{pmatrix} + b\begin{pmatrix} -2 \\ 3 \\ 0 \end{pmatrix} + c\begin{pmatrix} -6 \\ 7 \\ 6 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\left(\begin{array}{ccc|c} 0 & -2 & -6 & 0 \\ -1 & 3 & 7 & 0 \\ 3 & 0 & 6 & 0 \end{array}\right) \xRightarrow{R_1 \leftrightarrow R_2} \left(\begin{array}{ccc|c} -1 & 3 & 7 & 0 \\ 0 & -2 & -6 & 0 \\ 3 & 0 & 6 & 0 \end{array}\right) \xRightarrow{R_3 = R_3 + 3R_1} \left(\begin{array}{ccc|c} -1 & 3 & 7 & 0 \\ 0 & -2 & -6 & 0 \\ 0 & 9 & 27 & 0 \end{array}\right) \xRightarrow{R_2 = R_2 * -\frac{1}{2}} \left(\begin{array}{ccc|c} -1 & 3 & 7 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 9 & 27 & 0 \end{array}\right) \xRightarrow{R_3 = R_3 - 9R_2} \left(\begin{array}{ccc|c} -1 & 3 & 7 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

We conclude that the vectors are linearly dependent as we have a free variable.

$$a = -2c, b = -3c, c = c...$$