KUTAISI INTERNATIONAL UNIVERSITY

Quiz (G-9)

Linear Algebra for Informatics
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Definition

if V is a subspace of $\mathbb{R}^n$ with an orthonormal basis $\vec{u}_1, \ldots, \vec{u}_m$, then

$$\operatorname{proj}_V \vec{x} = (\vec{u}_1 \cdot \vec{x})\vec{u}_1 + \cdots + (\vec{u}_m \cdot \vec{x})\vec{u}_m \quad \forall \vec{x} \in \mathbb{R}^n$$

Problem

Find the orthogonal projection of $\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ onto the subspace of $\mathbb{R}^4$ spanned by $\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ -1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix}$

Solution

So since the 3 given vectors in the subspace are orthogonal, we have the orthonormal basis:

$$\vec{u}_1 = \frac{1}{2}\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \vec{u}_2 = \frac{1}{2}\begin{pmatrix} 1 \\ 1 \\ -1 \\ -1 \end{pmatrix}, \vec{u}_3 = \frac{1}{2}\begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix}$$

Now using the theorem above with $x = e_1$: $\operatorname{proj}_V x = (\vec{u}_1 \cdot \vec{x})\vec{u}_1 + (\vec{u}_2 \cdot \vec{x})\vec{u}_2 + (\vec{u}_3 \cdot \vec{x})\vec{u}_3 = \frac{1}{4}\begin{pmatrix} 3 \\ 1 \\ -1 \\ 1 \end{pmatrix}$

Problem

Find the angle between the vectors

$$\vec{x} = (1, 0, 0, 0)^T \text{ and } \vec{y} = (1, 1, 1, 1)^T.$$

Solution

Dot product of two vectors can be written as $\vec{x} \cdot \vec{y}$ as well as $\|\vec{x}\|\|\vec{y}\| \cos(\theta)$ where θ is the angle between those vectors so from that we get $\theta = \arccos \frac{\vec{x} \cdot \vec{y}}{\|\vec{x}\|\|\vec{y}\|} = \arccos\left(\frac{1}{2}\right) = \frac{\pi}{3}$

Problem

Find all vectors orthogonal to both $v = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$ and $w = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

Solution

We have to solve the system of two homogeneous equations

$$0 = x \cdot v = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} = x_1 + x_2 - x_3$$

$$0 = x \cdot v = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = x_1 + x_2 + x_3 \text{ (solve it however u want..)}$$

$$\begin{pmatrix} 1 & 1 & -1 \\ 1 & 1 & 1 \end{pmatrix} \overset{\text{RREF}}{\Longrightarrow} \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

The parametric vector form of the solution set is

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = x_2 \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$$

$$\text{Therefore the answer is line Span}\left\{ \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \right\}$$