Problem

What is the dimension of the space of all
upper triangular $n \times n$ matrices?

Solution

You gotta count the number of nonzero entries in upper triangular matrix so:

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ 0 & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & a_{nn} \end{pmatrix} \text{ so its just } 1+2+3+\dots+n = \frac{n \times (n+1)}{2}$$

Problem

Find a basis of the space of all 2×2 matrices S such that

$$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} S = S \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}$$

Solution

$$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} S = S \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix} \Leftrightarrow$$

$$\text{By } \quad S = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \text{ we get} \rightarrow \begin{pmatrix} a+c & b+d \\ a+c & b+d \end{pmatrix} = \begin{pmatrix} 2a & 0 \\ 2c & 0 \end{pmatrix} \Rightarrow a = c, b = -d$$

$$\text{so,} \quad \begin{pmatrix} a & b \\ a & -b \end{pmatrix} \text{ can be written as : } a\begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} + b\begin{pmatrix} 0 & 1 \\ 0 & -1 \end{pmatrix}, \text{meanign that}$$

$$\begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & -1 \end{pmatrix} \text{ is a basis.}$$