Quiz 3

October 18, 2024

Task 1

Let T be a linear transformation. It is intuitively known that $T(0) = 0$, where 0 is the zero vector. Your task is to formally prove that $T(0) = 0$.

Task 2

Determine whether the following transformations are linear or not. Justify your answer in each case.

1. $T : \mathbb{R} \to \mathbb{R}$ defined by $T(x) = x + 1$
2. $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x) = 1.5x$
3. $T : \mathbb{R}^2 \to \mathbb{R}^3$ defined by
$$T(x, y) = \begin{pmatrix} 3x - y \\ y \\ x \end{pmatrix}$$
4. $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by
$$T(x) = x + \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

Hints:

Since $0 = -0$, $\quad T(\mathbf{0}) = T(-\mathbf{0}) = \ldots$ do the rest (Task 1)

$$T(\vec{v} + \vec{w}) = T(\vec{v}) + T(\vec{w}), \quad \forall \vec{v}, \vec{w} \in \mathbb{R}^n \text{ (Task 2)}$$

$$T(k\vec{v}) = kT(\vec{v}), \quad \forall \vec{v} \in \mathbb{R}^n \text{ and } \forall k \in \mathbb{R}$$

$$\mathrm{proj}_L(\vec{x}) = \left(\frac{\vec{x} \cdot \vec{u}}{\vec{u} \cdot \vec{u}}\right)\vec{u} \text{ (Task 3)}$$

Task 3

Compute the orthogonal projection of $\mathbf{x} = \begin{pmatrix} -6 \\ 4 \end{pmatrix}$ onto the line L spanned by $\mathbf{u} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$. Also, find the distance from $\mathbf{x}$ to L.