

# Quiz 3

October 18, 2024

## Task 1

Let  $T$  be a linear transformation. It is intuitively known that  $T(0) = 0$ , where  $0$  is the zero vector. Your task is to formally prove that  $T(0) = 0$ .

## Task 2

Determine whether the following transformations are linear or not. Justify your answer in each case.

1.  $T : \mathbb{R} \rightarrow \mathbb{R}$  defined by  $T(x) = x + 1$
2.  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  defined by  $T(x) = 1.5x$
3.  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^3$  defined by

$$T(x, y) = \begin{pmatrix} 3x - y \\ y \\ x \end{pmatrix}$$

4.  $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$  defined by

$$T(x) = x + \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

Hints:

Since  $0 = -0$ ,  $T(0) = T(-0) = \dots$  do the rest (Task 1)

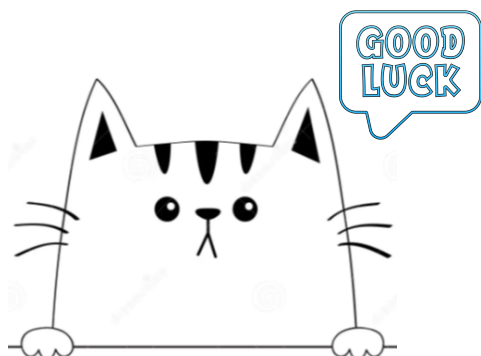
$$T(\vec{v} + \vec{w}) = T(\vec{v}) + T(\vec{w}), \quad \forall \vec{v}, \vec{w} \in \mathbb{R}^n \text{ (Task 2)}$$

$$T(k\vec{v}) = kT(\vec{v}), \quad \forall \vec{v} \in \mathbb{R}^n \text{ and } \forall k \in \mathbb{R}$$

$$\text{proj}_L(\vec{x}) = \left( \frac{\vec{x} \cdot \vec{u}}{\vec{u} \cdot \vec{u}} \right) \vec{u} \text{ (Task 3)}$$

## Task 3

Compute the orthogonal projection of  $\mathbf{x} = \begin{pmatrix} -6 \\ 4 \end{pmatrix}$  onto the line  $L$  spanned by  $\mathbf{u} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ . Also, find the distance from  $\mathbf{x}$  to  $L$ .



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