Quiz 4(G-1)

Linear Algebra for Informatics
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Problem

$$W = \left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \in \mathbb{R}^3 : x_1^2 + x_2 + x_3 = 0 \right\}$$

Is W a subspace of $\mathbb{R}^3$? Explain your answer

Solution

Consider **Additive Property**

$\begin{pmatrix} 1 \\ 3 \\ -4 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 6 \\ -10 \end{pmatrix}$ are both in W but their sum: $\begin{pmatrix} 3 \\ 9 \\ -14 \end{pmatrix}$ is not Since $3^2 + 9 - 14 = 4 \neq 0$

So W is not a subspace of $\mathbb{R}^3$

Problem

Find the kernel of the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ Given by

$$T\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 12 & -1 \\ 3 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

Solution

To find the kernel of T we solve:

$$\begin{pmatrix} 12 & -1 \\ 3 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

This gives the equations: $12x_1 - x_2 = 0, 3x_1 + 1 = 0 \Longrightarrow kernel(T) = \left\{\begin{pmatrix} 0 \\ 0 \end{pmatrix}\right\}$

Problem

Find a basis of the image of the matrix:

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 5 \\ 1 & 3 & 7 \end{pmatrix}$$

Solution

To find the basis of the image of matrix

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 5 \\ 1 & 3 & 7 \end{pmatrix} \text{ we perform row operations:}$$

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 5 \\ 1 & 3 & 7 \end{pmatrix} \overset{\substack{R_2=R_2-R_1 \\ R_3=R_3-R_1}}{\Longrightarrow} \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 4 \\ 0 & 2 & 6 \end{pmatrix} \overset{R_3=R_3-2R_2}{\Longrightarrow} \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 4 \\ 0 & 0 & -2 \end{pmatrix} \overset{R_3=-\frac{R_3}{2}}{\Longrightarrow} \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{pmatrix} \overset{\substack{R_1=R_1-R_3 \\ R_2=R_2-4R_3}}{\Longrightarrow} \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

The pivot columns correspond to the original matrix columns 1, 2, and 3.

Thus, a basis for the image is:

$$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 5 \\ 7 \end{pmatrix}.$$