KUTAISI INTERNATIONAL UNIVERSITY

Quiz (G-9)

Linear Algebra for Informatics
Fall semester 2024
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Problem

Compute $(2+2i)^{10}$

Solution

$(2+2i)^{10} = 2^{10}(1+i)^{10}$ so now lets just calculate that

$(1+i) \Rightarrow r = \sqrt{1+1} = 2,\ \theta = \arctan\left(\frac{1}{1}\right) = \frac{\pi}{4}$ so that complex number in polar coorinates will be:

$$(1+i) \Leftrightarrow \sqrt{2}\left(\cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)\right)$$

Now we can apply the De Moivre's theorem: $z^n = r^n(\cos(n\cdot\theta) + i\sin(n\cdot\theta))$

$$\Rightarrow 2^{10}\cdot\sqrt{2}^{10}\left(\cos\left(10\cdot\frac{\pi}{4}\right) + i\sin\left(10\cdot\frac{\pi}{4}\right)\right) \Rightarrow 2^{15}\left(\cos\left(\frac{5\pi}{2}\right) + i\sin\left(\frac{5\pi}{2}\right)\right)$$

$$\Rightarrow 2^{15}\left(\cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right)\right) \Rightarrow 2^{15}\cdot i$$

Problem

Find all 6^{th} roots of -64

Solution

General root formula for the nth roots of a complex number $r(\cos(\theta)+i\sin(\theta))$:

$$x_i = \sqrt[n]{r}\cdot\left(\cos\left(\frac{\theta+2\pi k_i}{n}\right)+i\sin\left(\frac{\theta+2\pi k_i}{n}\right)\right) \quad \text{where} \quad k=0,1,2,...n-1$$

-64 in polar form $-64 = 64\cdot(\cos(\pi)+i\sin(\pi)), r=64,\ \theta=\pi,\ n=6$

$$\text{for example } x_0 = 2\cdot\left(\cos\left(\frac{\pi+2\pi\cdot 0}{6}\right)+i\sin\left(\frac{\pi+2\pi\cdot 0}{6}\right)\right)\Rightarrow$$

$$x_0 = 2\cdot\left(\cos\left(\frac{\pi}{6}\right)+i\sin\left(\frac{\pi}{6}\right)\right)\Leftrightarrow x_0 = 2\left(\frac{\sqrt{3}}{2}+i\frac{1}{2}\right)=\sqrt{3}+i$$

$$x_1 = 2\left(\cos\left(\frac{\pi}{2}\right)+i\sin\left(\frac{\pi}{2}\right)\right)\Leftrightarrow x_1 = 2(0+i)=2i$$

$$\text{for} \quad k=2,3... \quad \text{you can continue like that ...}$$

Problem

Find all solutions of the equation $e^{ix} = -1$

Solution

We know that $e^{ix} = \cos(x) + i\sin(x)$

So in our case: $\cos(x) + i\sin(x) = -1 \Rightarrow$ Since it has no imaginary part, the $\sin(x) = 0$ and $\cos(x) = -1$

From trigonometry we know that $x = \pi + 2\pi k,\ \forall k \in Z$