Activity Project 5(Noe Lomidze)

Ordinary Differential Equations model many real-life problems

In this project we discuss SIR model of an epidemic, but in a slightly changed way

Mathematical modelling of an outbreak of Zombie Infection

At first let's take a closer view to the general case:

One of the simplest but influential models of an epidemic is the compartmental SIR model. In this model, disease propagates between the population compartments through susceptible individuals (S) becoming infected (I) after encountering other infected individuals with rate β and finally moving to a recovered population (R) with rate γ.

The following simple system of ODEs describes the dynamics of this compartmental model:

$$\frac{dS}{dt} = -\beta SI,$$
$$\frac{dI}{dt} = \beta SI - \gamma I,$$
$$\frac{dR}{dt} = \gamma I.$$

One of my favorite youtuber 3Blue1Brown has a great video about that. You can check out [it] from his series to see some cool exploration of this approach to SIR models.

Now, let's get back to our zombies..

A [zombie] is a reanimated human corpse that feeds on living human flesh(If you didn't know)

The Basic Model

For the basic model, we consider three basic classes:

- **Susceptible** (S).
- **Zombie** (Z).
- **Removed** (R).

Susceptible can become deceased through ‘natural’ causes, i.e., non-zombie-related death (parameter δ).
The removed class consists of individuals who have died, either through attack or natural causes. Humans in the removed class can resurrect and become a zombie (parameter ζ).
Susceptible can become zombies through transmission via an encounter with a zombie (transmission parameter β).
Only humans can become infected through contact with zombies, and zombies only have a craving for human flesh so we do not consider any other life forms in the model.

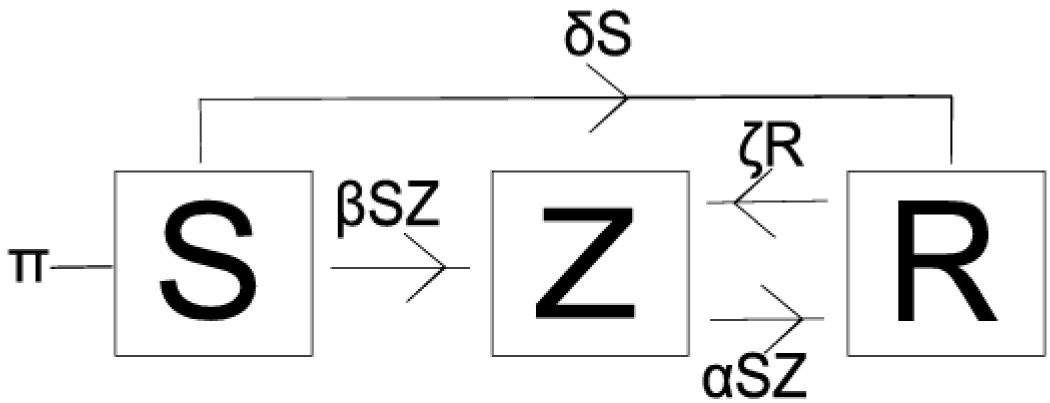
New zombies can only come from two sources:

- The resurrected from the newly deceased (removed group)
- Susceptible who have ‘lost’ an encounter with a zombie

In addition, we assume the birth rate is a constant, $\prod$.
Zombies move to the removed class upon being ‘defeated’. This can be done by removing the head or destroying the brain of the zombie (parameter α).
We also assume that zombies do not attack/defeat other zombies

Thus, the basic model is given by:

$$
\begin{aligned}
S' &= \Pi - \beta S Z - \delta S \\
Z' &= \beta S Z + \zeta R - \alpha S Z \\
R' &= \delta S + \alpha S Z - \zeta R \,.
\end{aligned}
$$



The ODEs satisfy:

$$S' + Z' + R' = \prod$$

and hence:

$$S + Z + R \rightarrow \infty$$

Let's try to try solving it using numerical methods

Euler method

$$y_{n+1} = y_n + hf(t_n, y_n).$$

```
def euler_method(f, y0, t, args):
    y = np.zeros((len(t), len(y0)))
    y[0] = y0
    for i in range(len(t) - 1):
        y[i + 1] = y[i] + dt * f(y[i], t[i], *args)
    return y
```

Backward Euler method

$$y_{n+1} = y_n + hf(t_{n+1}, y_{n+1}).$$

```
def backward_euler_method(f, y0, t, args):
    y = np.zeros((len(t), len(y0)))
    y[0] = y0
    for i in range(len(t) - 1):
```

```
        y[i + 1] = y[i] + dt * f(y[i], t[i], *args)
    return y
```

and also

The Runge–Kutta method

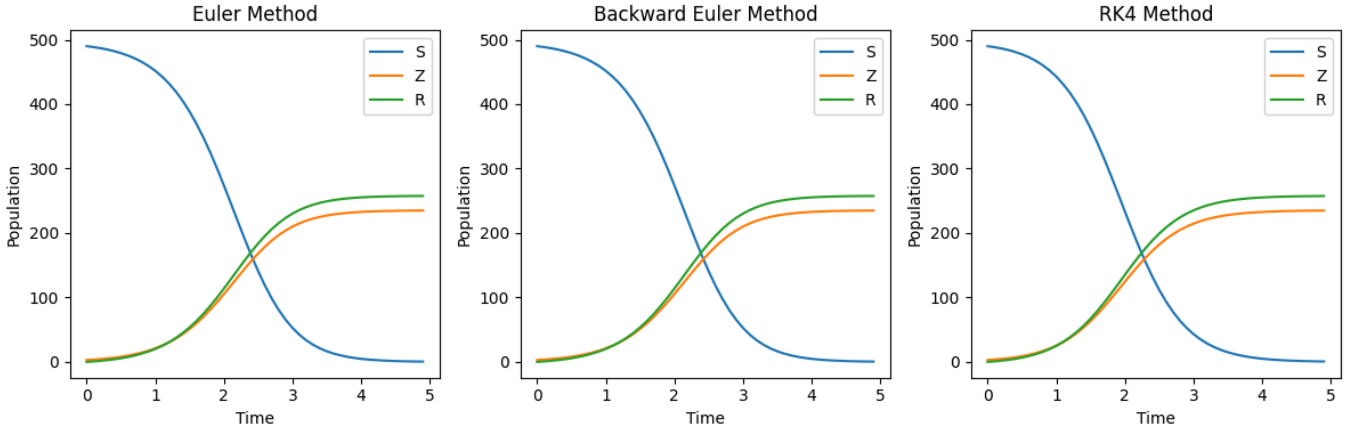
$$\begin{aligned} y_{n+1} &= y_n + \frac{h}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right), \\ t_{n+1} &= t_n + h \end{aligned}$$

for n = 0, 1, 2, 3, ..., using[3]

$$\begin{aligned} k_1 &= \ f(t_n, y_n), \\ k_2 &= \ f\left(t_n + \frac{h}{2}, y_n + h\frac{k_1}{2}\right), \\ k_3 &= \ f\left(t_n + \frac{h}{2}, y_n + h\frac{k_2}{2}\right), \\ k_4 &= \ f\left(t_n + h, y_n + hk_3\right). \end{aligned}$$

> Which as you can see is quite complex

```
def rk4_method(f, y0, t, args):
    y = np.zeros((len(t), len(y0)))
    y[0] = y0
    for i in range(len(t) - 1):
        k1 = dt * f(y[i], t[i], *args)
        k2 = dt * f(y[i] + 0.5 * k1, t[i] + 0.5 * dt, *args)
        k3 = dt * f(y[i] + 0.5 * k2, t[i] + 0.5 * dt, *args)
        k4 = dt * f(y[i] + k3, t[i] + dt, *args)
        y[i + 1] = y[i] + (1/6) * (k1 + 2 * k2 + 2 * k3 + k4)
    return y
```

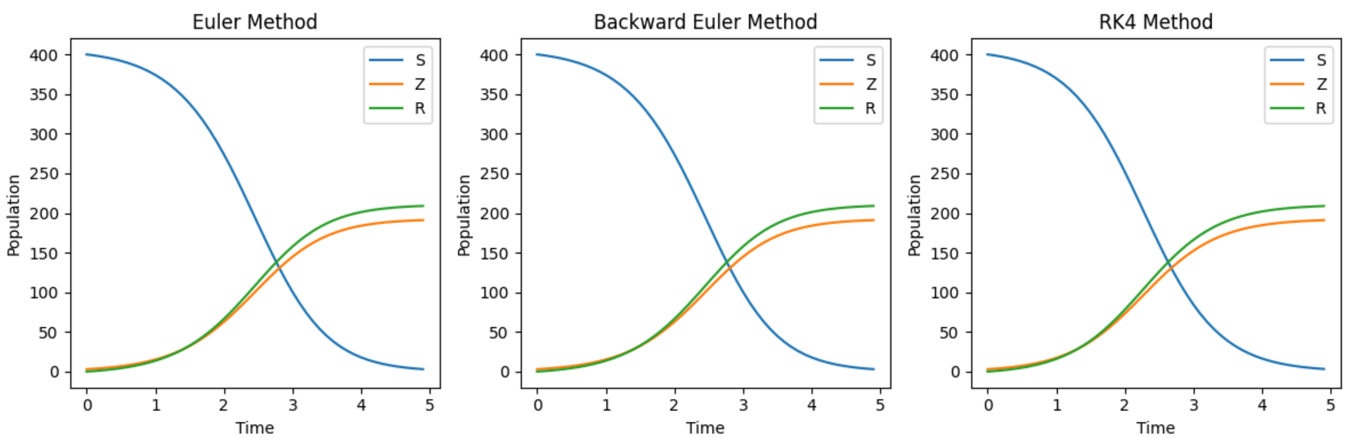


This was an easy problem so all 3 methods give almost the same result

Parameters are:

alpha = 0.005 *Zombie destruction rate*
beta = 0.0095 *New zombie rate*
zeta = 0.0003 *Zombie resurrection rate*
delta = 0.0003 *Background death rate*

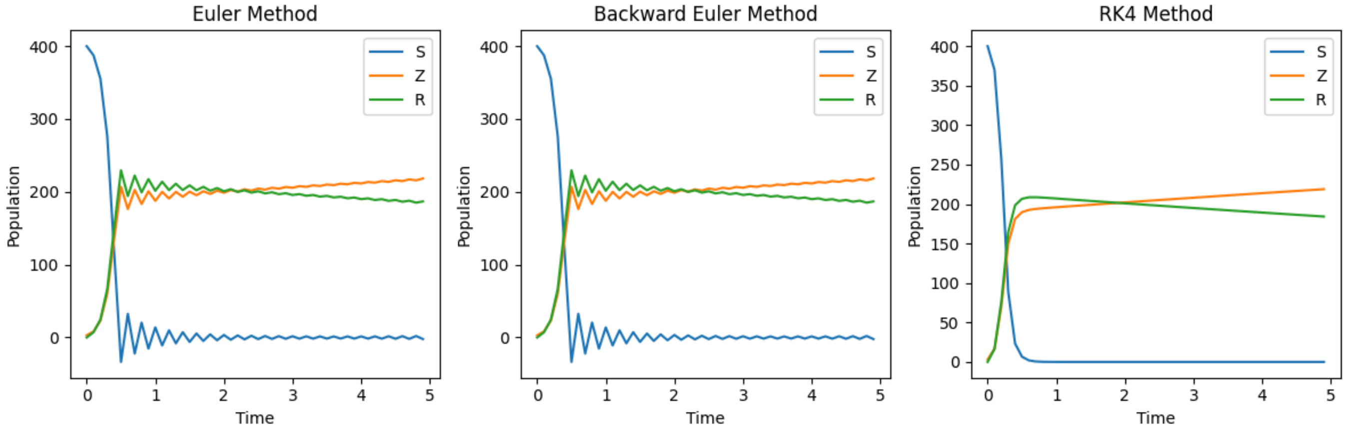
N = 500 *initial population size*
S0 = N - 10 *initial susceptible population*
Z0 = 3 *initial zombie population*
R0 = 0 *initial removed population*



Changing Initial susceptible population from N - 10 to N - 50

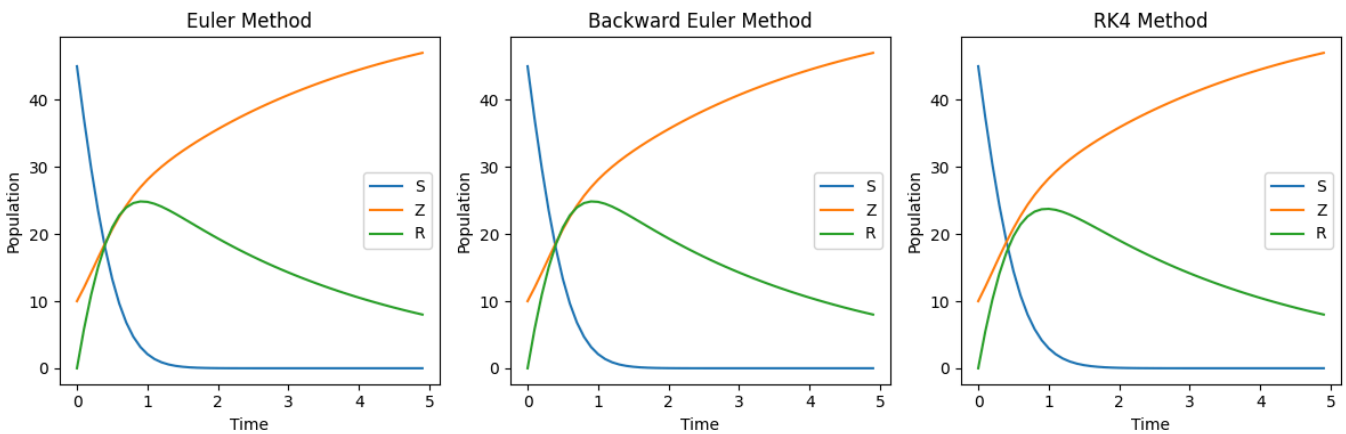
Again little change to the parameters:

alpha = 0.05 # Zombie destruction rate
beta = 0.095 # New zombie rate
zeta = 0.03 # Zombie resurrection rate
delta = 0.03 # Background death rate



The difference now is clear

δ = 0.8 # Background death rate



> see how Z yellow line increases

To sum up everything

The flexibility of mathematical modeling allows for a wide variety of challenges in biology.

In summary, a zombie outbreak is likely to lead to the collapse of civilization, unless it is dealt with quickly. While aggressive quarantine may contain the epidemic, or a cure may lead to coexistence of humans and zombies, the most effective way to contain the rise of the undead is to hit hard and hit often. As seen in the movies, it is imperative that zombies are dealt with quickly, or else we are all in a great deal of trouble...