KUTAISI INTERNATIONAL UNIVERSITY

Exercises for TTF

Introduction to Theory of Computation
Summer semester 2025

Exercises below are your homework; after submission, they will also be discussed during exercise classes.

Week eleven

1. Show that the set of all proofs in a proof system $S = (\Sigma, L, A, R)$ is decidable.
2. Say we are given a proof system $S = (\Sigma, L, A, R)$ with $\Sigma = \{|, x, +\}$, $L = \Sigma^*$, R defined by
   - $\dfrac{uxv}{uxxv}$, for all $u, v \in \Sigma^*$;
   - $\dfrac{u + v}{u| + |v}$, for all $u, v \in \Sigma^*$;
   - $\dfrac{uxxv}{ux + xv}$, for all $u, v \in \Sigma^*$.

   (a) Describe all provable words in S, if $A = \{+\}$;
   (b) describe all provable words in S, if $A = \{x\}$.
3. *(From Exam 2024)* Let $u \in \mathbb{B}^*$ and $m \in \mathbb{N}$. Let

$$L_t = \{u\#m \mid \text{there exists an input that makes } M_u \text{ run for at least } m \text{ steps}\}$$

Prove that that L_t is decidable.

4. *(From Exam 2024)* Let $v \in \mathbb{B}^*$ and $k \in \mathbb{N}$. Let

$$L_s = \{v\#k \mid \text{there exists an input that makes } M_v \text{ visit at least } k \text{ cells}\}$$

Prove that that L_s is decidable.