KUTAISI INTERNATIONAL UNIVERSITY

Exercises for TTF

Introduction to Theory of Computation
Summer semester 2025

Exercises below are your homework; they will be discussed during exercise classes. Problems marked with a (*) are more challenging.

Week 10

1. Do the "trivial exercise" from the lecture on non-computable functions, that is, show that if language L is decidable, then there exists an acceptor of L.

2. We call a language *Turing-acceptable* if it has an acceptor.
   Show that the collection of Turing-acceptable languages is closed under the operation of
   (a) union;
   (b) concatenation;
   (c) star;
   (d) intersection.

3. Give descriptions of Turing machine that decides the language
$$\{w \mid w \in \{0,1\}^* \text{ contains an equal number of 0s as 1s}\}.$$

4.
   (a) Show that the relation $\equiv$ is indeed an equivalence relation on set of all languages over some alphabet A.
   (b) Prove that reducibility of languages is a partial order relation on the equivalence classes of $\equiv$.

5. Prove Lemma 2 from the lecture on non-computable functions. That is, show

Lemma 2. *Let $L \leq L'$. Then*
- *if L' is decidable then L is decidable;*
- *if L is undecidable then L' is undecidable.*

6. Let $w \in \mathbb{B}^*$. Are the following languages decidable? Sketch your arguments.
   (a) $L_{\text{dfa}} = \{(M, w) : M \text{ is a deterministic finite automaton accepting } w\}$;
   (b) $L_{\text{cfg}} = \{(G, w) : G \text{ is a context free grammar deriving string } w\}$;
   (c) a context free language.