Exercises below are your homework; after submission, they will also be discussed during exercise classes.

Week 13

1. Give an example of a space bound Turing machine that does not halt.

2. Show that $\{0^n1^n \mid n \in \mathbb{N}_0\} \in \mathrm{TIME}_2(n) \cap \mathrm{SPACE}_2(n)$.

3. Prove Lemma 1 and Lemma 2 from the lecture on Deterministic Complexity Classes; that is, prove

Lemma 1. *If L is decided by a $t(n)$-time bounded k-tape Turing machine, then L is decided by state by a $t(n) + O(1)$-time bounded k-tape Turing machine.*

Lemma 2. *If L is decided by state by a $t(n)$-time bounded k-tape Turing machine, then L is decided by a $O((t(n))$-time bounded k-tape Turing machine.*

4. Let $x \notin \mathbb{B}$ and

$$L = \{wx^nw \mid w \in \mathbb{B}^n, n \in \mathbb{N}_0\}.$$

Show that $L \in \mathrm{TIME}_2(n)$ and $L \in \mathrm{TIME}_1(n^2)$.

5. Sketch the proof of Lemma 12 from the lecture on Deterministic Complexity Classes; that is, sketch the argument for

Lemma 12. *There is a universal 2-tape Turing machine U which simulates t steps of 1-tape Turing machine M_u started with input v of length $|v| = n$ in time $O(|u| \cdot (n + t))$. If M_u uses space s then U uses space $O(|u| \cdot s)$.*

6. Read and understand the proof of Space Hierarchy Theorem (including the necessary Lemma).

7. Read and understand the construction of the decidable non-time constructable function $f(n) > n$.

8. Try: conclude the tighter time-hierarchy theorem then Lemma 17 from the lecture on Deterministic Complexity Classes. Recall, (1 p)

Lemma 17. *Let t be time constructible. Then*

$$\mathrm{TIME}(t(n)) \subseteq \mathrm{TIME}_2(t(n)\log(t(n))).$$