Exercises below are your homework; they will be discussed during exercise classes. Problems marked with a (*) are more challenging.

Week 5

1. In the lecture on context free languages and pushdown automata, prove the part of Lemma 9 for construction 3 (slide 49), that is, show the induction step for the proof of $x \in L(\langle q, A, p\rangle) \rightarrow (q, x, A) \vdash^* (p, \varepsilon, \varepsilon)$ in case $(r, \text{push}\, a) \in \delta(q, u, \varepsilon)$ with $u \in \Sigma_\varepsilon$ and $a \in \Gamma_\epsilon$.

2. Call a push down automaton *extended* if it can push more than one symobl on the stack simultaneously. Prove that notions of extended and (regular) push down automatons are equivalent.

3. Prove that (1 p)
   a) if sets A and B are countable then $A \times B$ is countable;
   b) $\mathbb{N}_0^k$ is countable for any $k \in \mathbb{N}$;
   c) the set of all subsets of natural numbers in not countable;
   d) the set of all real numbers is not countable.

4. Recall that informally, *procedure* is a finite sequence of instructions that can be mechanically carried out, such as a computer program.

   Say we are given a procedure P for testing a string to see if the string is in a language L. That is, P halts with the answer "yes" for strings in the language and either does not terminate or else halts with the answer "no" for strings not in the language. Assume that P can be broken down into discrete steps so that it makes sense to talk about the i-th step of P for any given string.

   Give a procedure for generating all strings in L.

   Hint: Enumerate all strings in A^* for the alphabet A of L. Then use the bijection $f\colon \mathbb{N} \rightarrow \mathbb{N} \times \mathbb{N}$.