Theory of Computation

G-4

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Homework

Idea

Prove that the language $L = \{a^n b^n : n \in \mathbb{N}\}$ is not regular.

Pumping lemma $\Rightarrow$ $|w| = n > N$ can be decomposed as $w = xyz$ such that:

$|y| > 1,$

$|xy| \le N$ and you can pump on y.

$\forall i \in \mathbb{N}_0 .\ \ xy^i z \in L$

Homework

Task 1 Regularity

Consider the string $w = a^N b^N$, for which $|w| > N$ is clear…

Next step is to find where the pumping block y is.

Consider the substring a^N and then $w = xy^i z, i \in \mathbb{N}_0$

So we have $w = a^{N-k} a^k b^N$ where $x = a^{N-k}, y = a^k$ and $z = b^N$

We attempt to pump out the pumping block y once, and obtain the

string $a^{N-k} b^N$ where $k > 0$ this string is supposed to belong to the language L according to the pumping lemma but as you can see we found a contradiction.

Homework

Task 2 $\iff$ Lemma 4 & Pumping lemma for CFLs

Lemma 4 $\iff$ Theorem 1.49, chapter 1, page 62.

Introduction to the Theory of Computation(Third edition)

Michel Sipser.

Theorem 2.34 $\iff$ Again the same book, page 125..

Homework

Task 4 $\iff$ Regularity

$L = \{1^n \mid n \text{ is prime}\}$ is not regular.
For pumping length N, since there exists arbitrary large prime numbers, there is $p > N$, $1^p = uvx$ and $uv^i x \in L$ for all i.
Say $|v| = k$, we take $i = p - k$, then $|uv^{p-k}x| = k(p-k) + (p-k) =$
$= (p-k)(k+1)$, so $uv^{p-k}x = 1^{(p-k)(k+1)} \in L$. (Contradiction)

$$(p-k)(k+1) \text{ is definitely not a prime..} \tag{1}$$

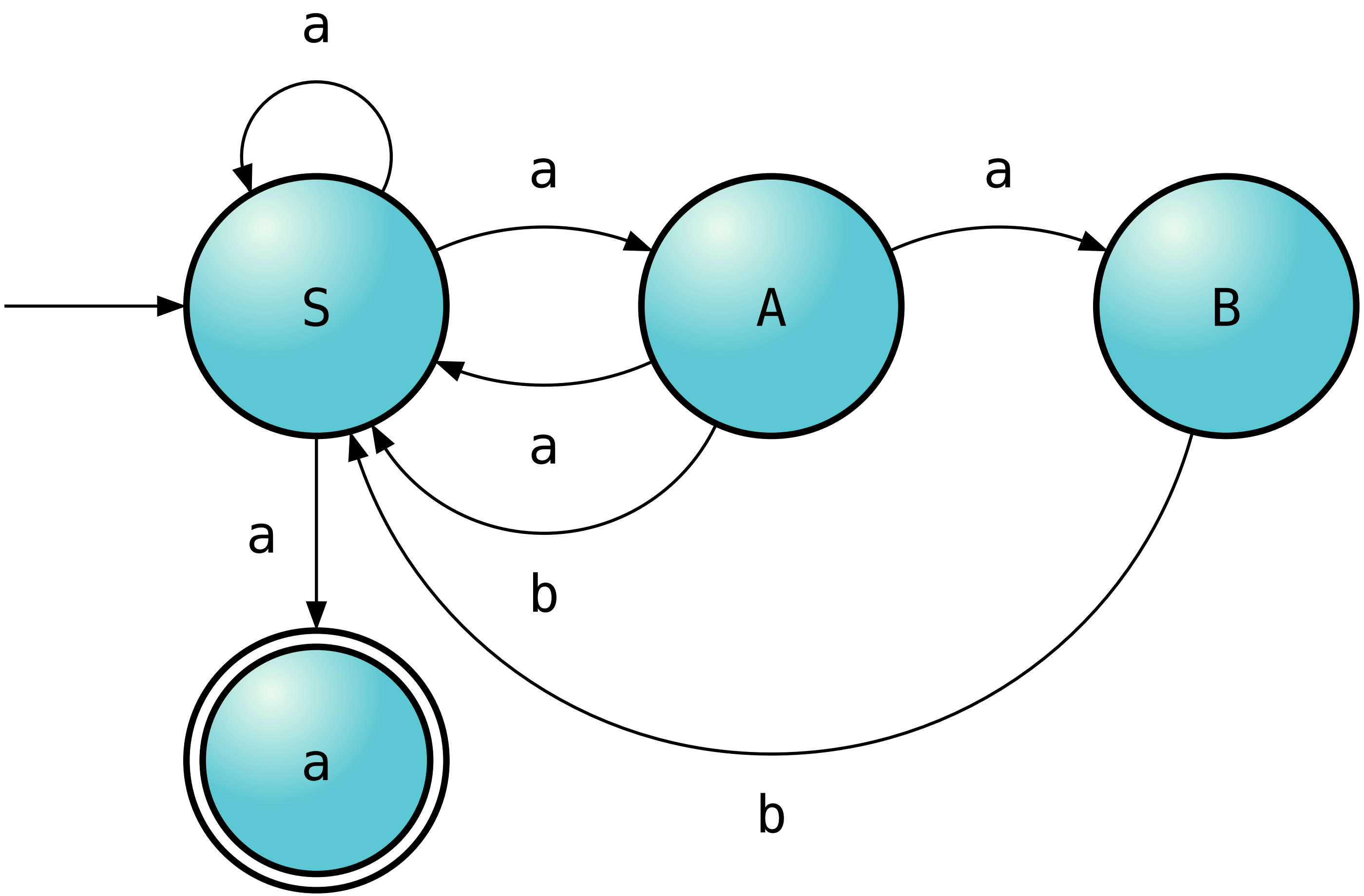
Homework Task 5

$$G = (\{S, A, B\}, \{a, b\}, P, S) \tag{2}$$

Construct NFA With productions P :

$$\begin{aligned} S &\to aS \\ S &\to aA \\ A &\to bS \\ A &\to aB \\ B &\to bS \\ S &\to a \end{aligned} \tag{3}$$

Homework Task 5



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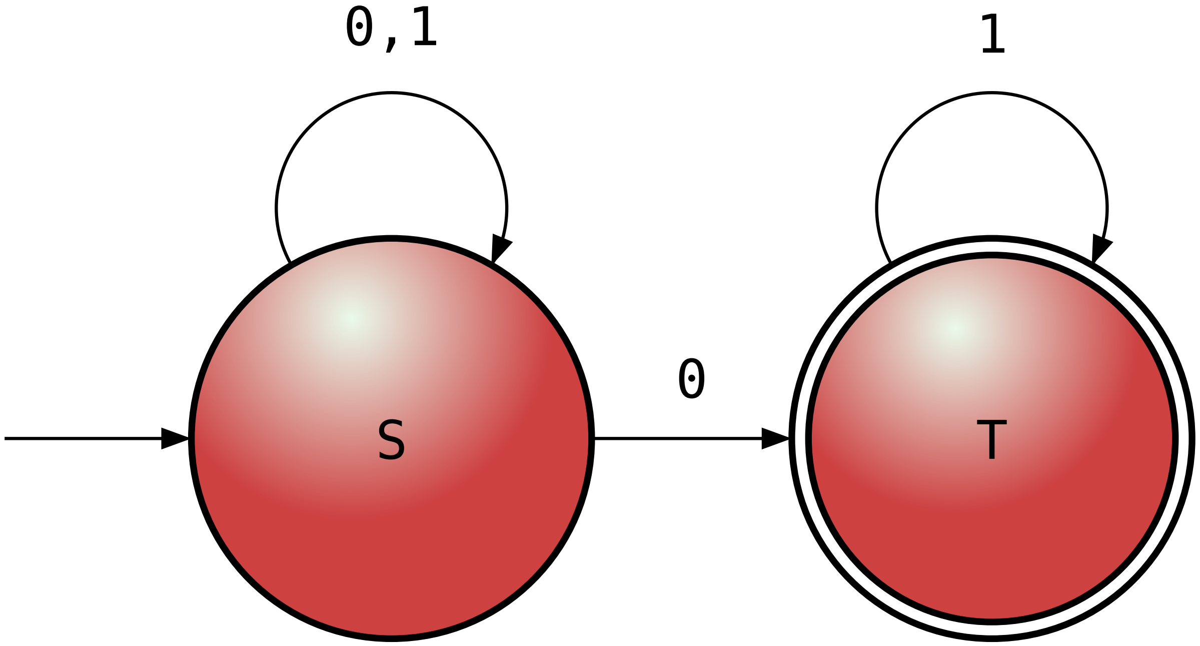
Exercise 1

Consider regular expression: $(1 \cup 0)^{*}01^{*}$

Construct 1) NFA; 2) Regular Grammar

Exercise 1



$(1 \cup 0)^{*}01^{*}$

Exercise 1

$$\begin{aligned} S &\to 0S \mid 1S \mid 0T \\ T &\to 1T \mid \varepsilon \end{aligned} \tag{4}$$

Exercise 2 (Which of the following L is regular)

$$\{a^n b^m \mid n \geq m \lor n \leq m\}$$

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$$\{a^n b^m \mid n \geq m \lor n \leq m\}$$

is the same as a^*b^* so its regular..

Exercise 2 (Which of the following L is regular)

$$\{a^n b^m \mid n > m \wedge n < m\}$$

Exercise 2 (Which of the following L is regular)

$$\{a^n b^m \mid n > m \vee n < m\}$$

is empty $\implies$ regular

Exercise 2 (Which of the following L is regular)

$$\{a^n b^m \mid n \geq m \land n \leq m\}$$

Exercise 2 (Which of the following L is regular)

$$\{a^n b^m \mid n \geq m \wedge n \leq m\}$$

is the same as $a^n b^n$ $\Rightarrow$ not regular (already seen why)

Exercise 2 (Which of the following L is regular)

$$\{a^n b^m \mid n > m \lor n < m\}$$

Exercise 2 (Which of the following L is regular)

$$\{a^n b^m \mid n > m \vee n < m\}$$

It is the same language as $\{a^n b^m \mid n \neq m\}$
which is complement to $\{a^n b^m \mid n = m\}$.

Since regularity is closed under complementation, the considered language can not be regular.
(cuz it would imply that $\{a^n b^m \mid n = m\}$ is regular)

Chomsky normal form

- $A \to BC$ with $A, B, C \in N$
- $A \to b$ with $A \in N$ and $b \in T$
- $S \to \varepsilon$

every context free grammar can be transformed into Chomsky normal form

Chomsky normal form

1. Start symbol does not appear on right (add $S' \to S$)
2. Eliminate right hand sides with more than 2 symbols $P : n \to a_1 ... a_s$ where $s \geq 2$, introduce a new non terminal x $n \to a_1 ... a_{s-2} x, \quad x \to a_{s-1} a_s$, repeat until `len(rhs) <=2`
3. Eliminate ε rules, for all non terminals 1..m (besides S)
   for i = 1 to m: if $n_i \to \varepsilon$:
   i) drop this rule
   ii) for each rule with n_i on the right side add a rule where each occurence of n_i is dropped

$$\begin{aligned} n \to x n_i \text{ or } n \to n_i x &: \text{add } n \to x \\ n_k \to n_i n_i \wedge k > i &: \qquad \text{add } n \to \varepsilon \end{aligned} \tag{5}$$

Chomsky normal form

4. for all chain rules between nonterminals $A \to B$ with $A, B \in N$ drop $A \to B$ and for all productions

$$B \to u : \text{add } A \to u \tag{6}$$

   repeat until no chain rules between nonterminals are left

5. What about $A \to bC$?
   add a nonterminal X and do the following:

$$X \to b, A \to XC \tag{7}$$

   thats it..

Convert Grammar into Chomsky normal form

$$\begin{aligned} S &\rightarrow AB \mid aB \\ A &\rightarrow aab \mid \varepsilon \\ B &\rightarrow bbA \end{aligned} \tag{8}$$

Eliminate epsilon rules

$$
\begin{aligned}
S &\to AB \mid aB \\
A &\to aab \mid \varepsilon \\
B &\to bbA
\end{aligned}
\tag{9}
$$

$$
\begin{aligned}
S &\to AB \mid B \mid aB \\
A &\to aab \\
B &\to bbA \mid bb
\end{aligned}
\tag{10}
$$

Removal of the unit-production

$$\begin{aligned} S &\rightarrow AB \mid B \mid aB \\ A &\rightarrow aab \\ B &\rightarrow bbA \mid bb \end{aligned} \tag{11}$$

$$\begin{aligned} S &\rightarrow AB \mid bbA \mid aB \mid bb \\ A &\rightarrow aab \\ B &\rightarrow bbA \mid bb \end{aligned} \tag{12}$$

Make RHS <=2

$$\begin{aligned} S &\to AB \mid bbA \mid aB \mid bb \\ A &\to aab \\ B &\to bbA \mid bb \end{aligned} \tag{13}$$

$$\begin{aligned} S &\to AB \mid V_b V_b A \mid V_a B \mid V_b V_b \\ A &\to V_a V_a V_b \\ B &\to V_b V_b A \mid V_b V_b \end{aligned} \tag{14}$$

Final result

$$
\begin{aligned}
S &\to AB \mid V_b V_b A \mid V_a B \mid V_b V_b \\
A &\to V_a V_a V_b \\
B &\to V_b V_b A \mid V_b V_b
\end{aligned} \tag{15}
$$

$$
\begin{aligned}
S &\to AB \mid V_c A \mid V_a B \mid V_b V_b \\
A &\to V_d V_b \\
B &\to V_c A \mid V_b V_b \\
V_c &\to V_b V_b \\
V_d &\to V_a V_a \\
V_a &\to a \\
V_b &\to b
\end{aligned} \tag{16}
$$

thats it..(hope u undertand how it works)

Bye