Exercises below are your homework; they will be discussed during exercise classes. Problems marked with a (*) are more challenging.

Week 4

1. Show that the language $L = \{a^n b^n c^n : n \geq 1\}$ is not context free.

2. Do the argument in Lemma 6 from the lecture on Chomsky hierarchy so that $vx \neq \epsilon$ and $|vwx| \leq n_0$ follow.

3. Consider the grammar $(\{S, A, B\}, \{0, 1\}, P, S)$ with productions

$$\begin{array}{ll} S \to 1A, & S \to 0B, \\ A \to 0S, & B \to 1S, \\ A \to 0, & B \to 1, \\ A \to 1AA, & B \to 0BB. \end{array}$$

Find an equivalent grammar in Chomsky normal form.

4. Show
   (1) if L and L' are context free, then $L \cup L'$ is context free.
   (2) If L is context free, then it is possible that $\overline{L}$ is not context free. *Hint:* try to write a non context free language as the intersection of context free languages.

5. We have defined that an npda accepts if both the pushdown store is empty and the state is accepting. There are obvious variants of this definition:
   - accept by (reaching an) accepting state.
   - Accept by (reaching an) empty pushdown store.

Show that all these are equivalent, that is
   (1) if an npda M accepts L by accepting state, then there is an npda M' which accepts L by accepting state and empty pushdown store.
   (2) If an npda M accepts L by empty pushdown store, then there is an npda M' which accepts L by accepting state and empty pushdown store.

6. (*From Midterm 2024*) Construct (in detail) a pushdown automaton that accepts

$$L = \{w_1 \# w_2 \mid w_i \in \mathbb{B}^*, w_1 \text{ contains the same exact number of 0's as } w_2\}.$$