Exercises below are your homework; they will be discussed during exercise classes. Problems marked with a (*) are more challenging.

Week Seven

1. What follows are the exercises from the lecture on Turing Machine. See the mentioned lecture for precise definitions.

   Construct a Turing machine for

   (a) "decrementing binary numbers." That is, given an input of a binary number $\text{bin}(n)$, it gives an output $\text{bin}(n-1)$;

   (b) "concatenating tape inscriptions." That is two machines, one giving $\text{tape1} = \text{tape1}\#\text{tape2}$ and the other $\text{tape1} = \text{tape2}\#\text{tape1}$.

   (c) "head and tail of tapes." That machines giving $\text{tape2} = \text{head}(\text{tape1})$ and $\text{tape2} = \text{tail}(\text{tape1})$.

2. Construct a Turing machine which shifts inscription $w \in \{0, 1, \#\}^*$ of tape one cell to the left. Is you machine regular?

3. Describe Turing machine "next state" relation $\vdash$ for $k = 2$ tapes. You can also try for $k > 2$.

4. Give a precise mathematical description of the Turing machine T with input symbols $\Sigma = \{0, 1\}$ informally given by the following:

   If T is given a tape with a finite run of consecutive 1's and is started on the leftmost cell of the run, it will erase all the 1's and then stop. If T is started on a tape consisting entirely of 1's, it will never stop.

5. Construct a two-tape Turing machine that simulates a deterministic push down automaton.

6. Allow Turing machine M to have accepting states Z_A and rejecting states Z_R. M is acceptor for L if for all w,

$$w \in L \Leftrightarrow M \text{ started with } w \text{ halts in an accepting state.}$$

   So rejection is now possible by halting in a rejecting state or by not halting.

   Show: a language has an acceptor in the sense of the new definition if and only if it has an acceptor in the sense of the definition of the lecture on non computable functions.

7. In this years midterm, there was the following problem to solve:

   Let $G = (\{R, X, S, T\}, \{a, b\}, P, R)$ where P is given by

$$\begin{aligned} R &\to XRX \mid S, \\ S &\to aTb \mid bTa, \\ T &\to XTX \mid X \mid \varepsilon \\ X &\to a \mid b \end{aligned}$$

   Give an informal description of NPDA that accepts $L(G)$.

   Some of the students attempted to construct the DFA instead of the NPDA. Prove that there does not exist a DFA that accepts $L(G)$.