deterministic context free languages

how to generate parse trees (derivation trees)

- what cf languages are recognized by a *deterministic* pushdown automaton (dpda)?
- hopefully C0 is an example
- if true we can constrcut derivation tree for program of length n in time $O(n)$
- construction of such trees was left out in I2OS lectures
- following Michael Sipser: Introduction to the Theory of Computation, 3rd edition, CENGAGE learning 2013

MIT textbook

1 Derivation trees, derivations and reductions

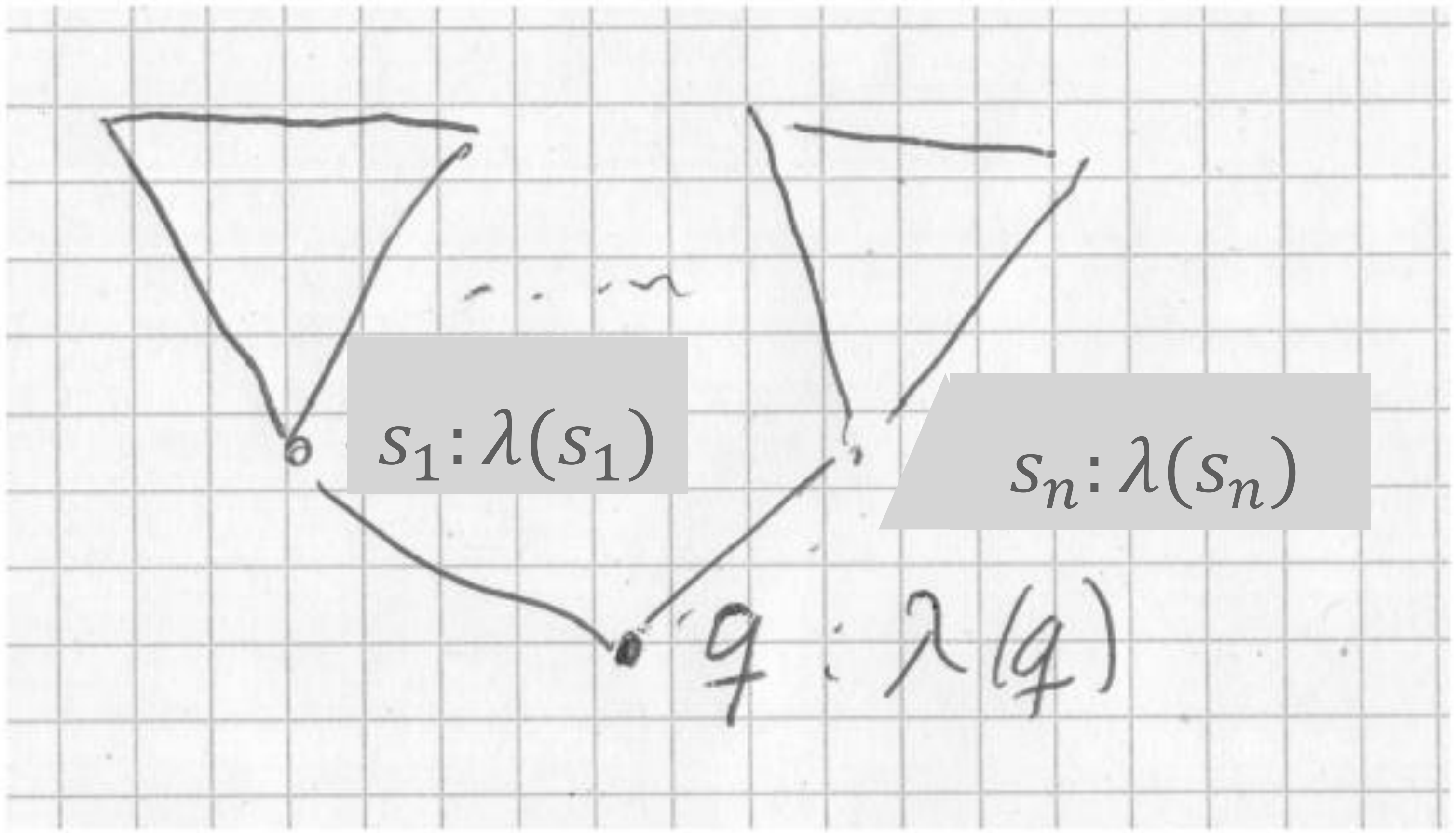

Figure 1: nodes x and their labels λ in a derivation tree.

reminder: derivation trees

- grammar

$$G = (N, T, P, S_1)$$

- tree (formally tree regions), nodes q with labels $\lambda(q) \in N \cup T$
- if node q has from left to right sons $s_1, \ldots, s_n$, then

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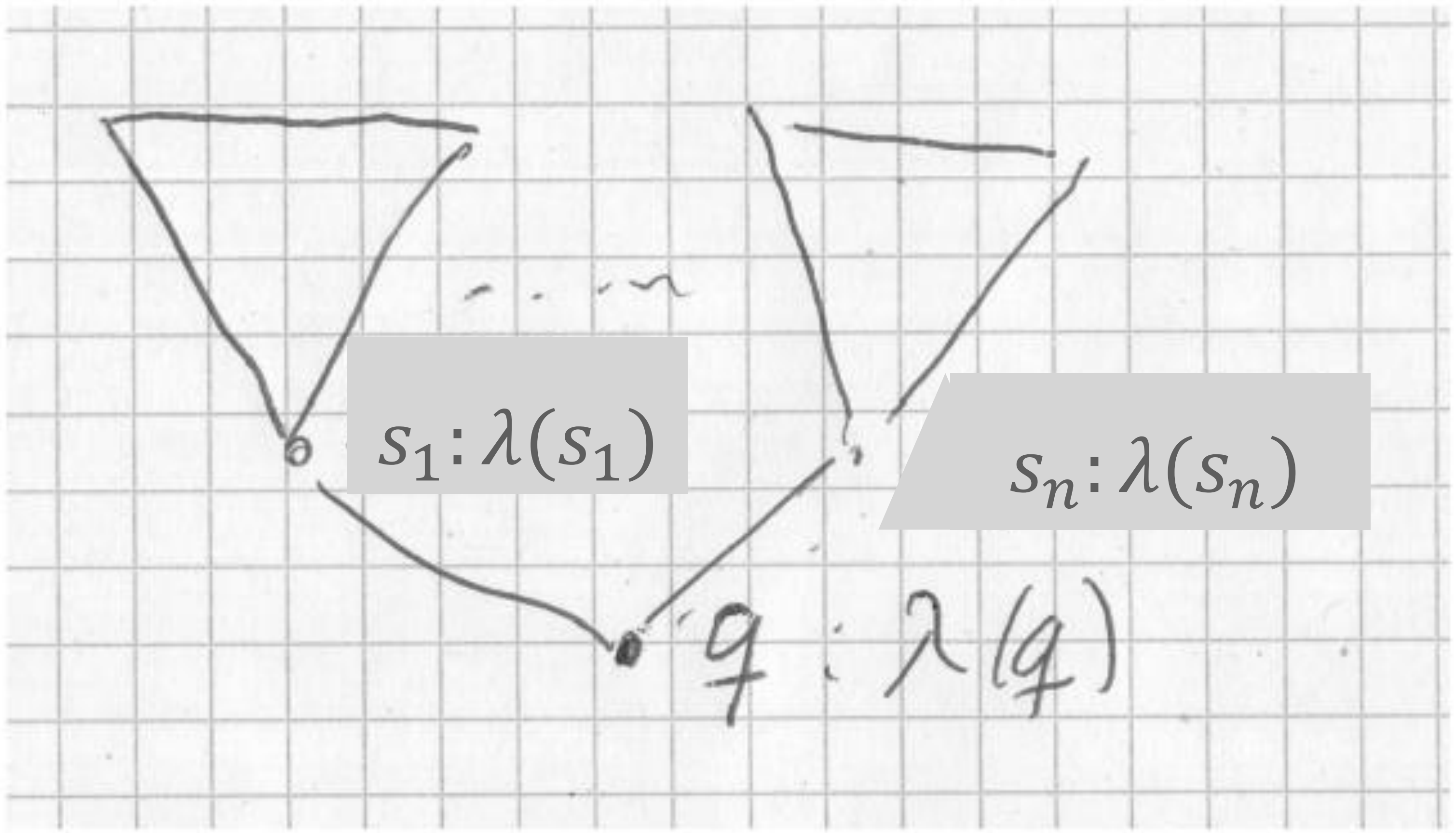

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def: derivations

- For $w, w' \in (N \cup T)^*$ we define $w \to w'$ (w' is directly derived from w) if there are u, v, C, r with

$$w = uCv \ , \ w' = urv \ , \ C \to r \text{ in } P$$

- sequence of words

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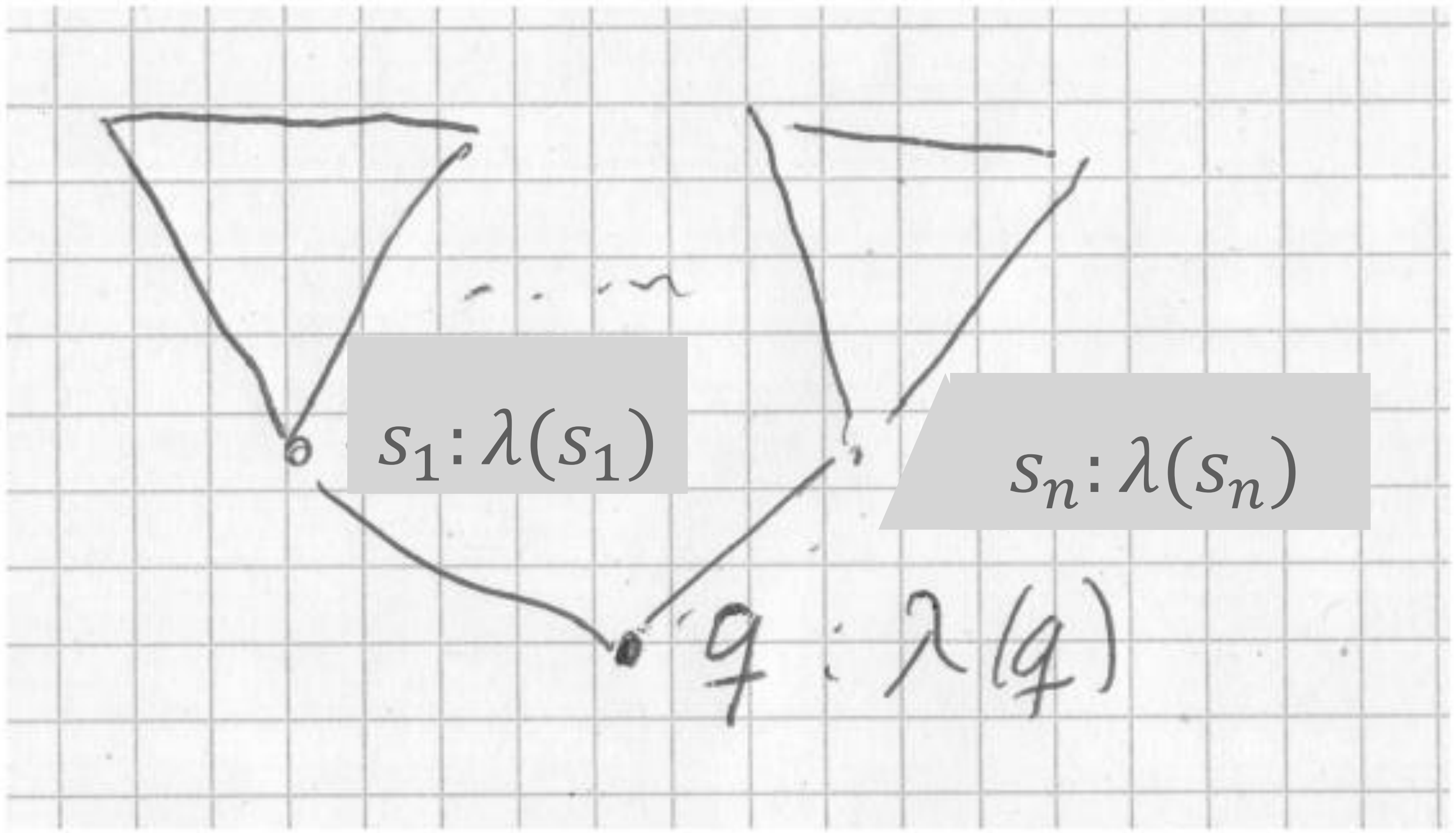

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$$w_i \to w_{i+1} \quad \text{for all } i < n$$
- it is a *rightmost derivation* if for all i in step $w_i \to w_{i+1}$ the rightmost non-terminal in w_i is replaced.

- generating a rightmost derivation from a derivation tree by preorder traversal of trees, subtrees from right to left.

preorder traversal: recursively treat

- sons of v, then v

postorder traversal: recursively treat

- v, then sons of v

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- example grammar for derivation tree in figure 2

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have you seen this grammar?

where?

- generating a rightmost derivation from a derivation tree by preorder traversal of trees, subtrees from right to left.

- there is 1-1 relationship between rightmost derivations and derivation trees

exercise: make this statement precise and prove it

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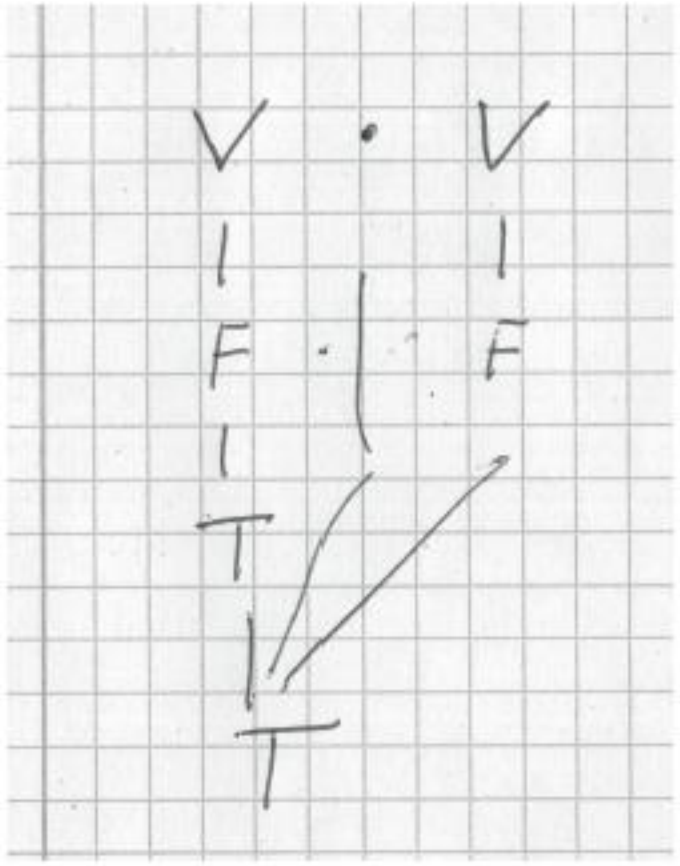

$$T \;,\; T \cdot F \;,\; T \cdot V \;,\; F \cdot V \;,\; V \cdot V$$

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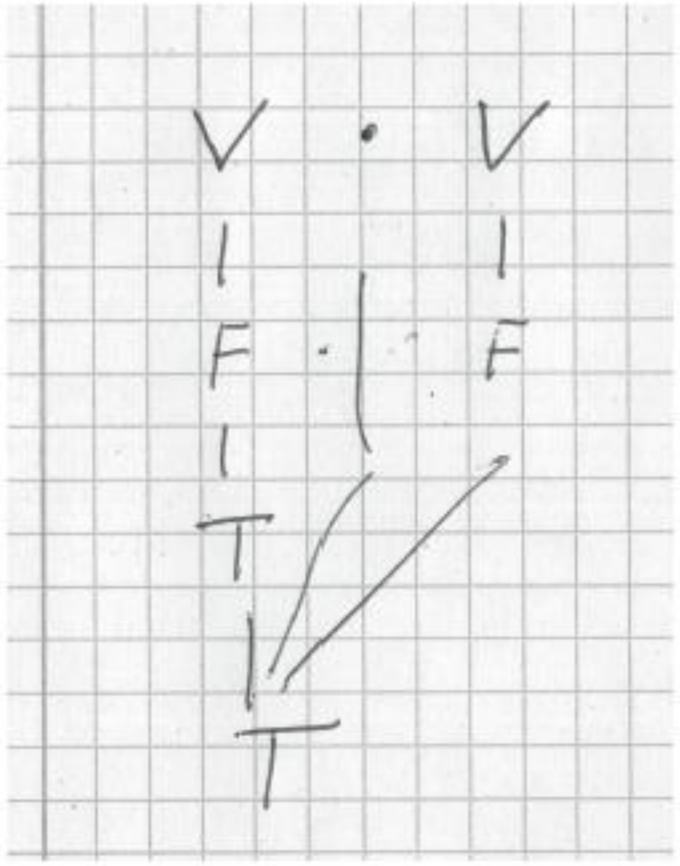

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$$w_i \leftarrow w_{i+1} \quad \text{for all } i < n$$

it's just a derivation run backwards

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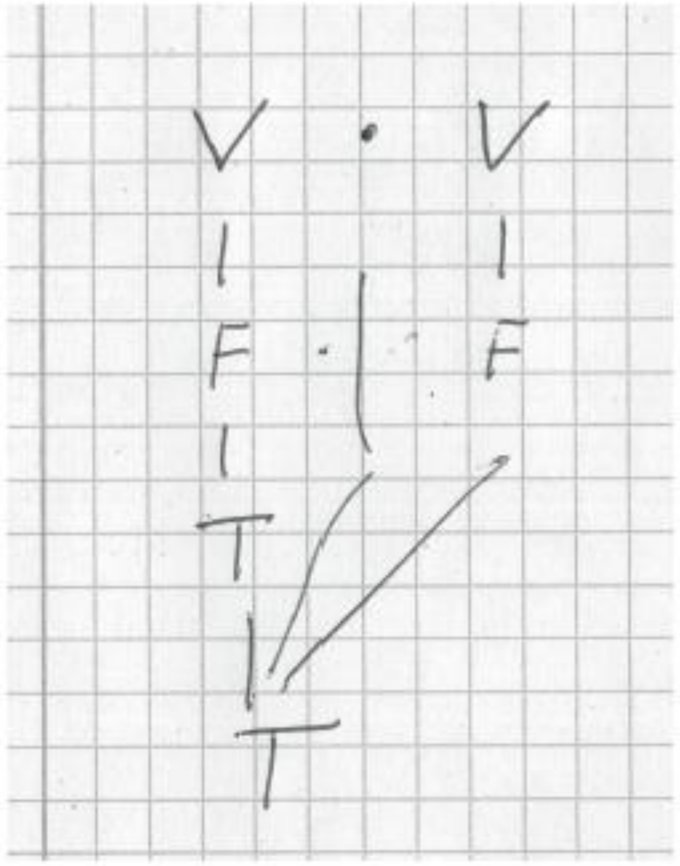

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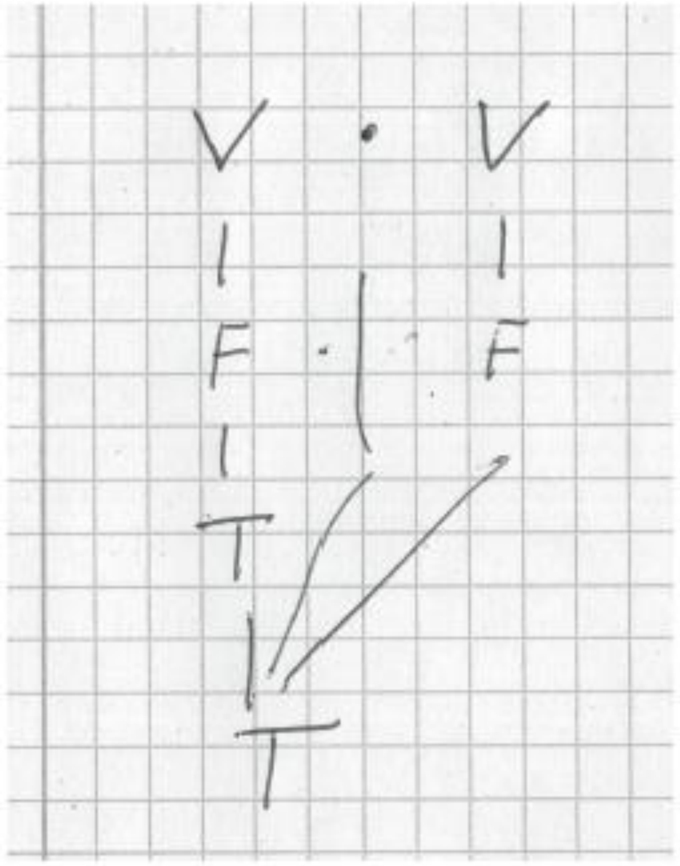

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- a string $v \in (N \cup T)^*$ is *valid* if it can occur in a leftmost reduction of a terminal string.

2 Handles and deterministic cfg's

def: handles A *handle* of a valid string v is a pair $(n, B \to h)$ specifies the first step in a leftmost reduction of v

- h occurs in v behind position n, i.e. there are x, y such that

$$v = xhy \quad \text{and} \quad n = |x|$$

- the production

$$B \to h \quad \text{in} \quad P$$

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definitions in literature amazingly imprecise

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- note (proof exercise)

$$y \in T^*$$

- if the production is clear we only specify h
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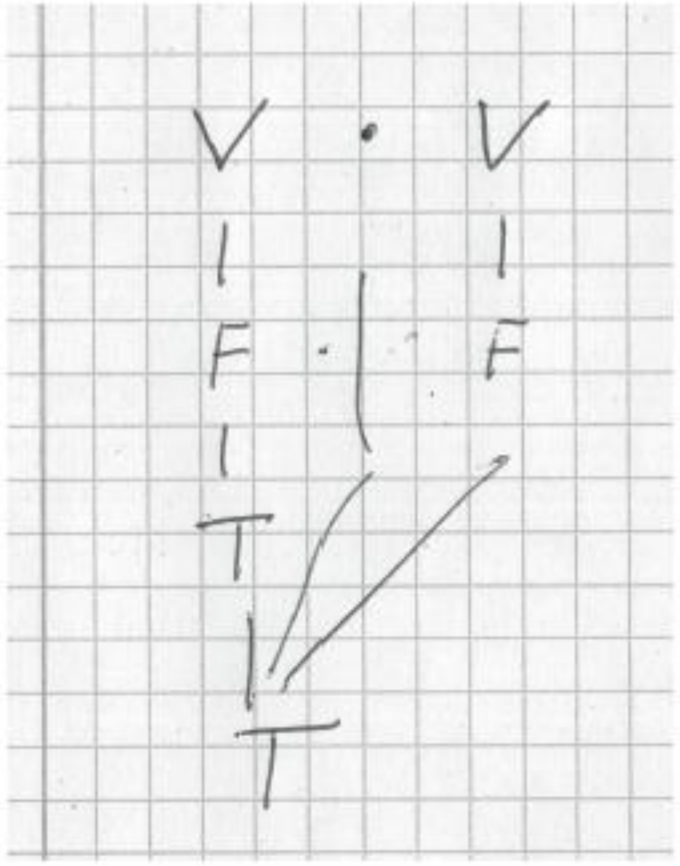

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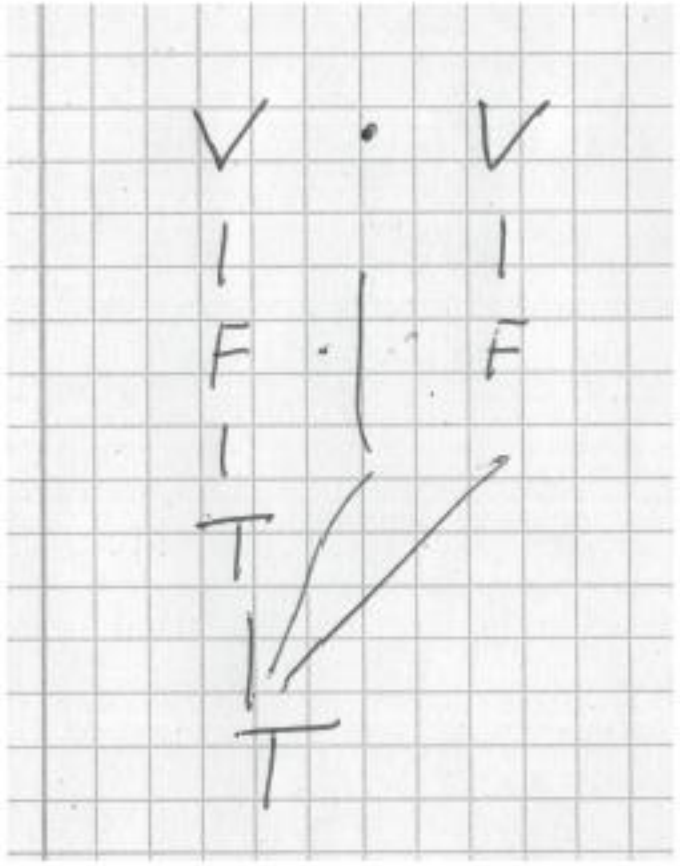

if we can identify handles, we can construct reductions and hence derivation trees

Lemma 1. *In unambiguous grammars handles for valid strings v are unique*

Proof. derivation trees are unique □

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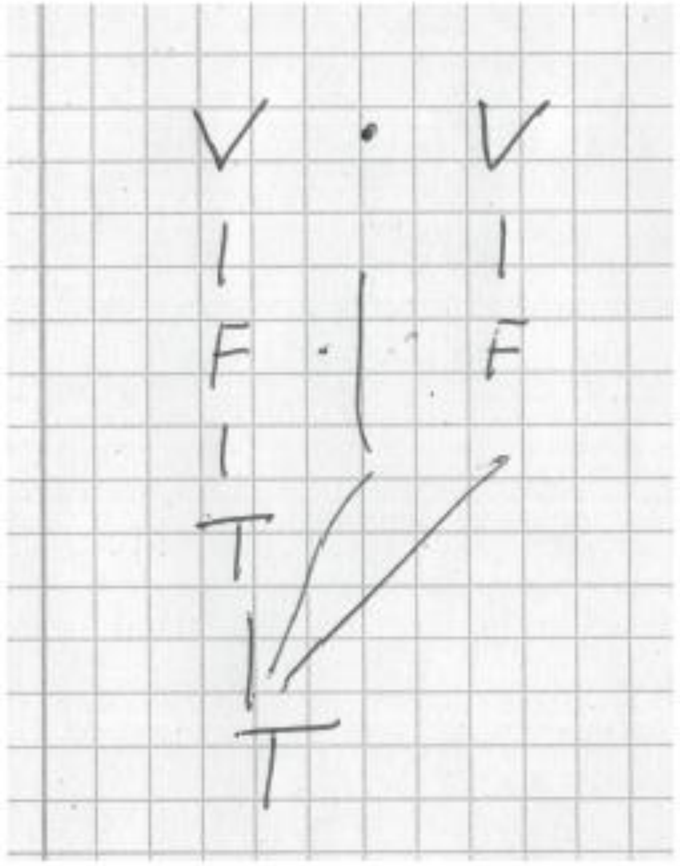

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is this grammar unambiguous

why or why not?

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2 Handles and deterministic cfg's

recognizing unique handles might require large lookahead

$$\begin{array}{rcl} R & \rightarrow & S \mid T \\ S & \rightarrow & aSb \mid ab \\ T & \rightarrow & aTbb \mid abb \end{array}$$

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recognising a handle once one sees it (dpda-like) A handle $(|x|, h)$ of a valid string $v = xhy$ is called a *forced handle* if is the unique handle of each string

$$x h \hat{y} \quad \text{with} \quad \hat{y} \in T^*$$

def: deterministic cfg A cfg is *deterministic* if every valid string has a forced handle.

3 Testing if a cfg is deterministic

The (amazing) DK-test: for every cfg there is a fa DK which identifies handles. In particular DK accepts input z if

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Construction of dfa DK from an nfa K by applying the power set construction.

running example

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states: using items of a cfg :

- constructed from productions in P. For each

$$B \rightarrow u_1 \dots u_k$$

include *dotted rules*

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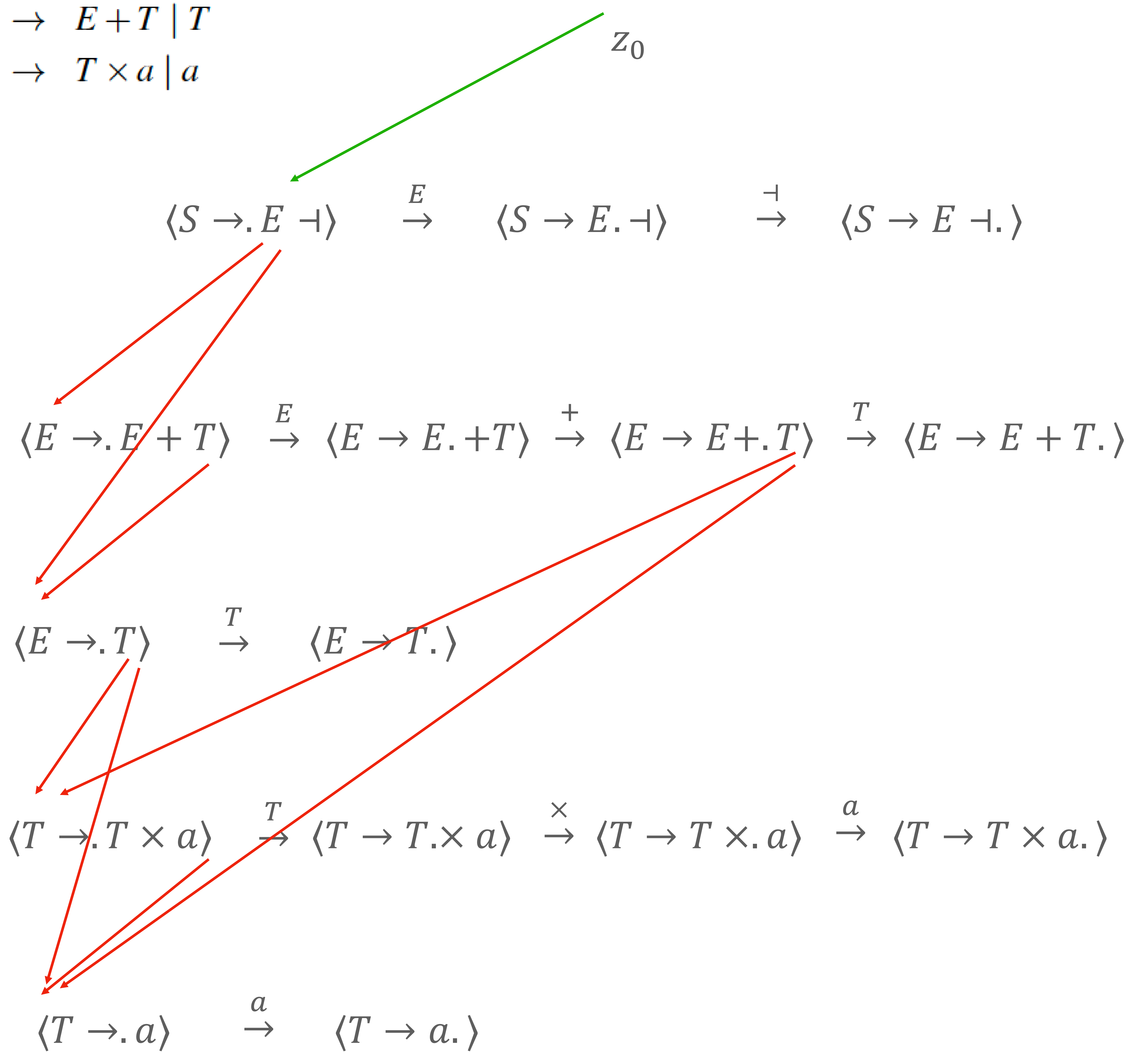

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$$z_0 \xrightarrow{\varepsilon} \langle S_\perp \to .u \rangle$$

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accepting end states: states

$$\langle B \to u. \rangle$$

corresponding to a completed production

states: using items of a cfg :

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$$B \to u_1 \dots u_k$$

include *dotted rules*

$$\begin{array}{lcll} \langle B & \to & .u_1 \dots u_k \rangle & \text{(before processing)} \\ & \dots & & \\ \langle B & \to & u_1 \dots u_i . u_{i+1} \dots u_k \rangle & \text{(after processig until } u_i) \\ & \dots & & \\ \langle B & \to & u_1 \dots u_k . \rangle & \text{(after processing)} \end{array}$$

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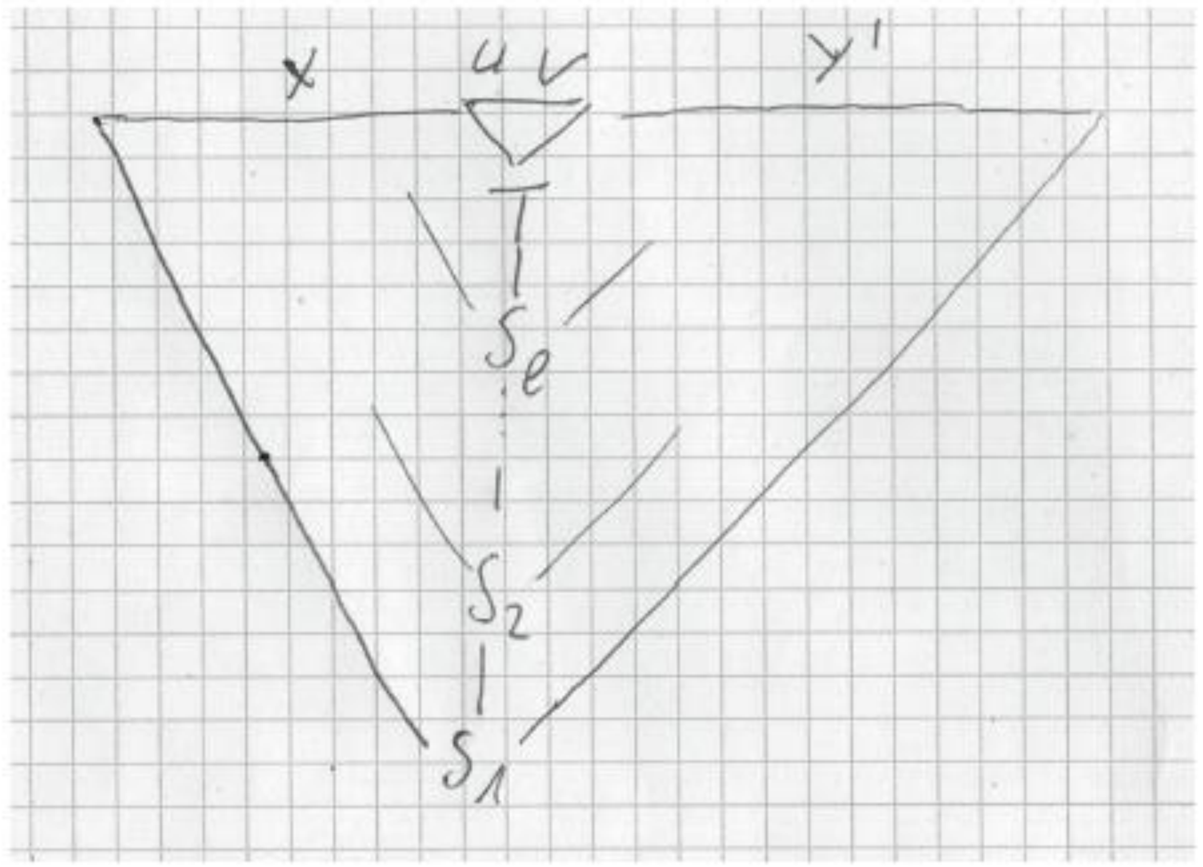

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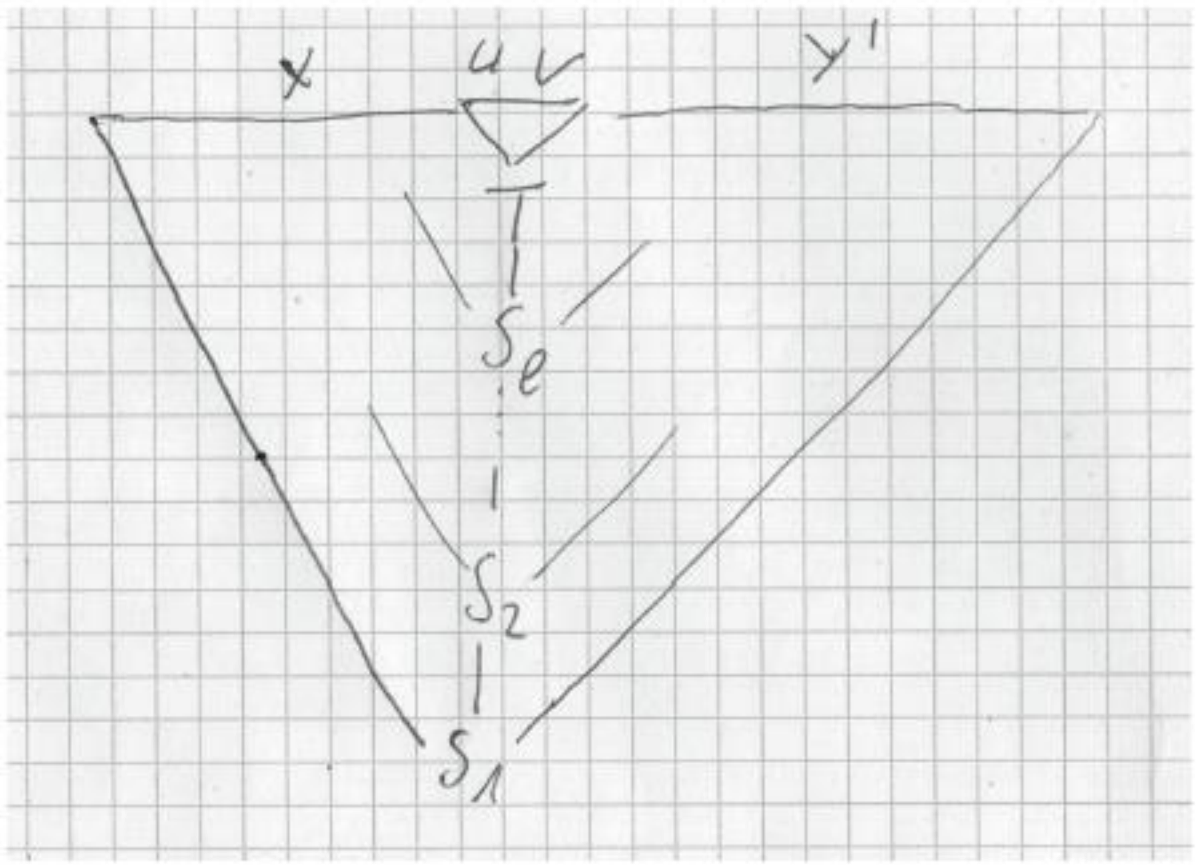

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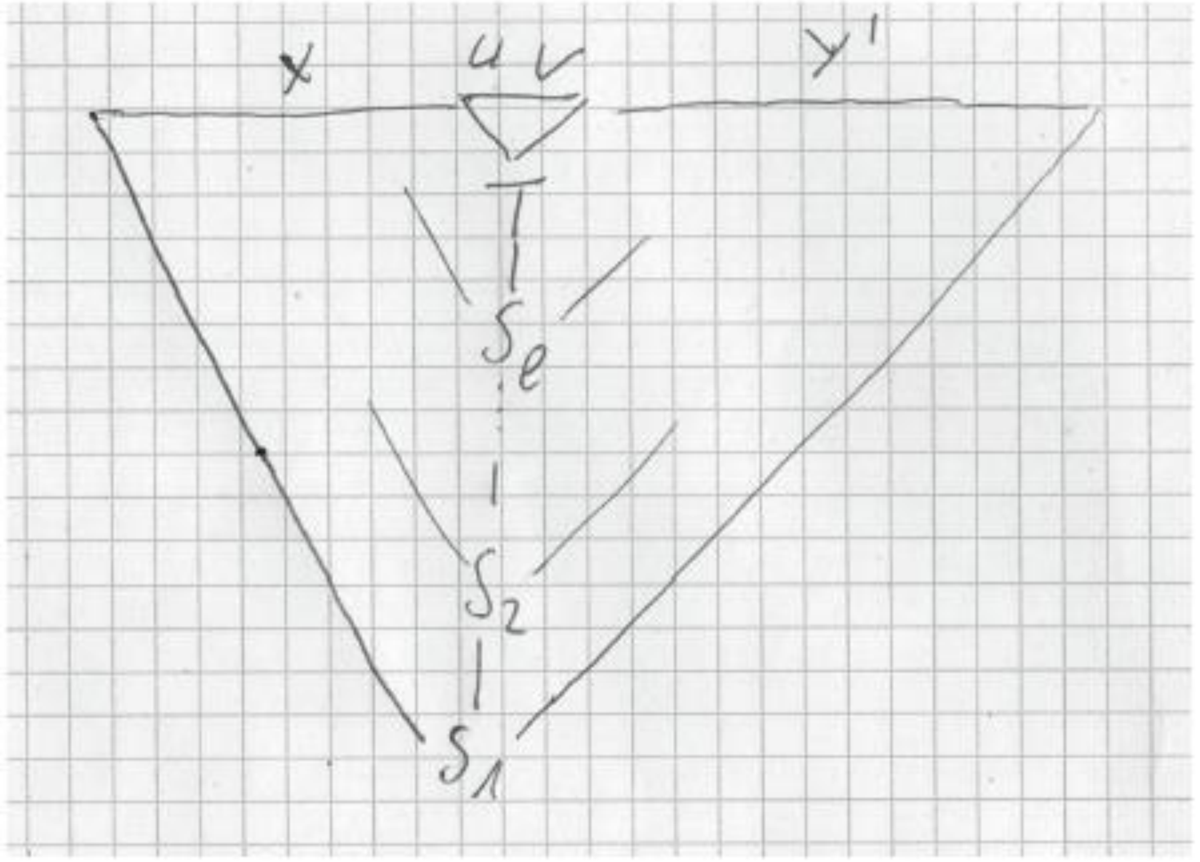

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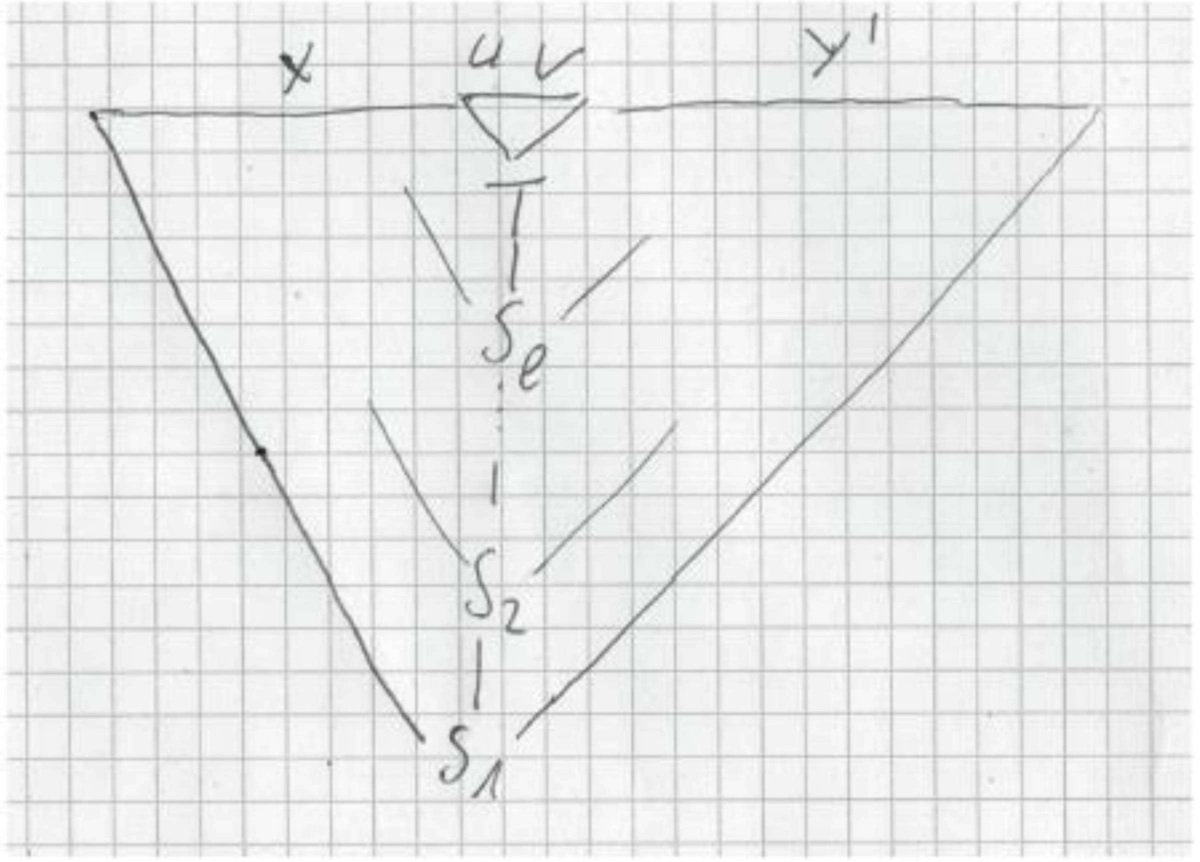
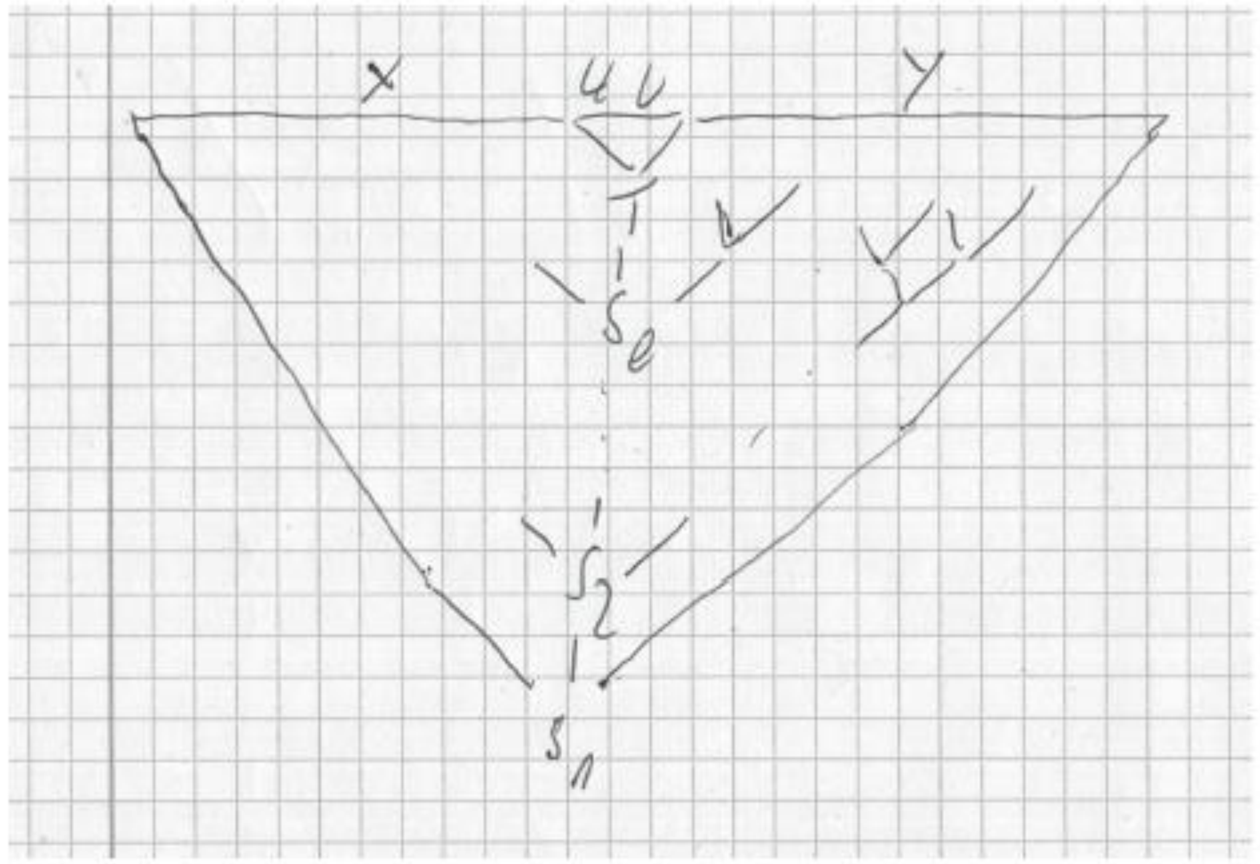


- then $S_1 \to^* xuvy'$, i.e. $xuvy'$ derivable in G by derivation tree of figure 3 and construction of automaton.

- obtain y by expanding all nonterminals in y' to terminal strings with derivation tree of figure 4 (w.l.o.g there are no redundant nonterminals, which cannot be expanded to terminal string).



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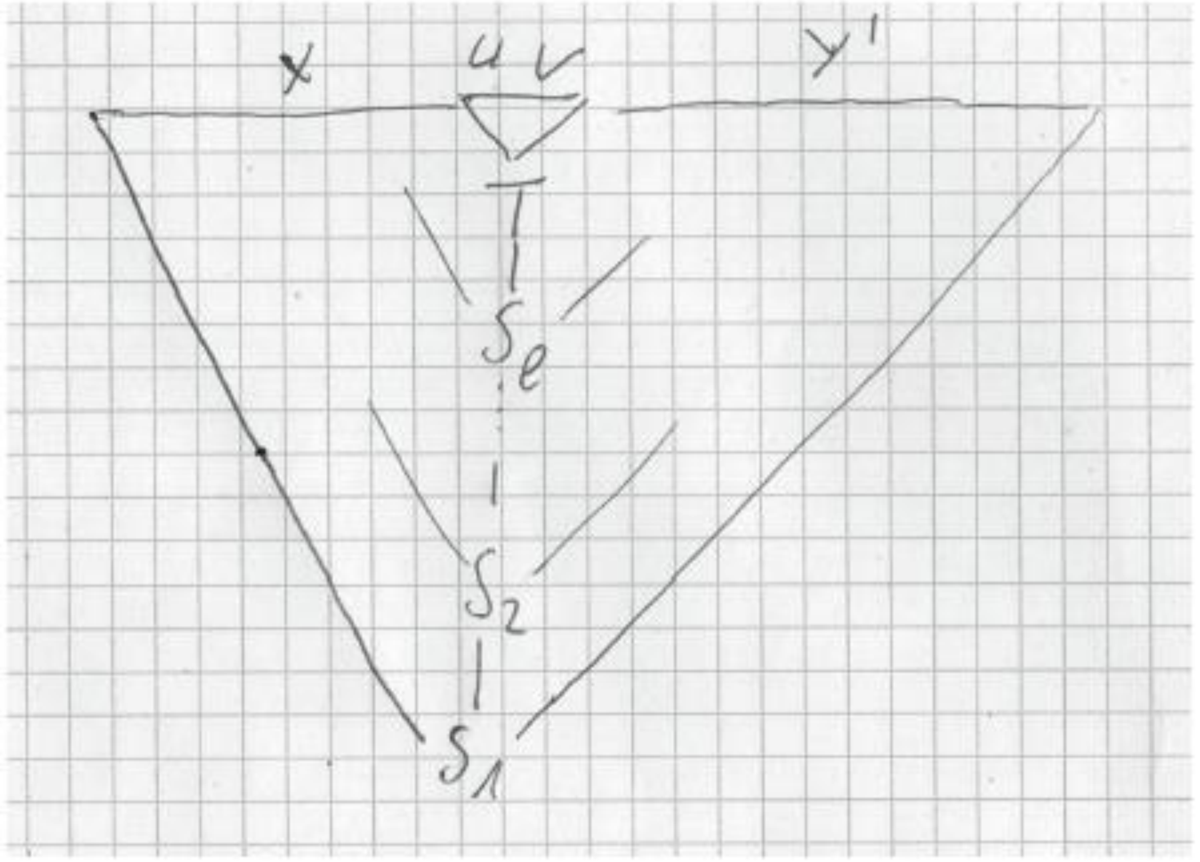

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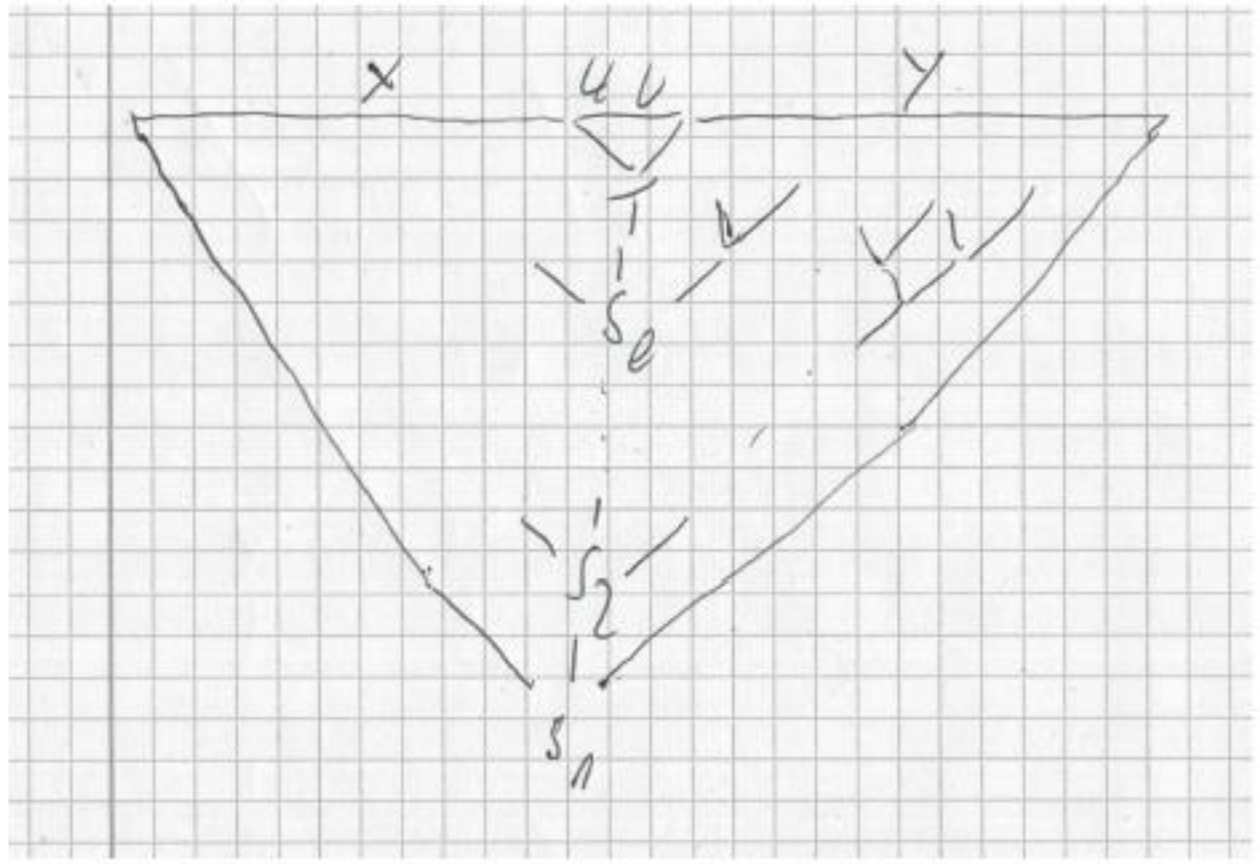

- turn tree into leftmost reduction.
- $xuvy$ is valid, $(|x|, T \to uv)$ is handle

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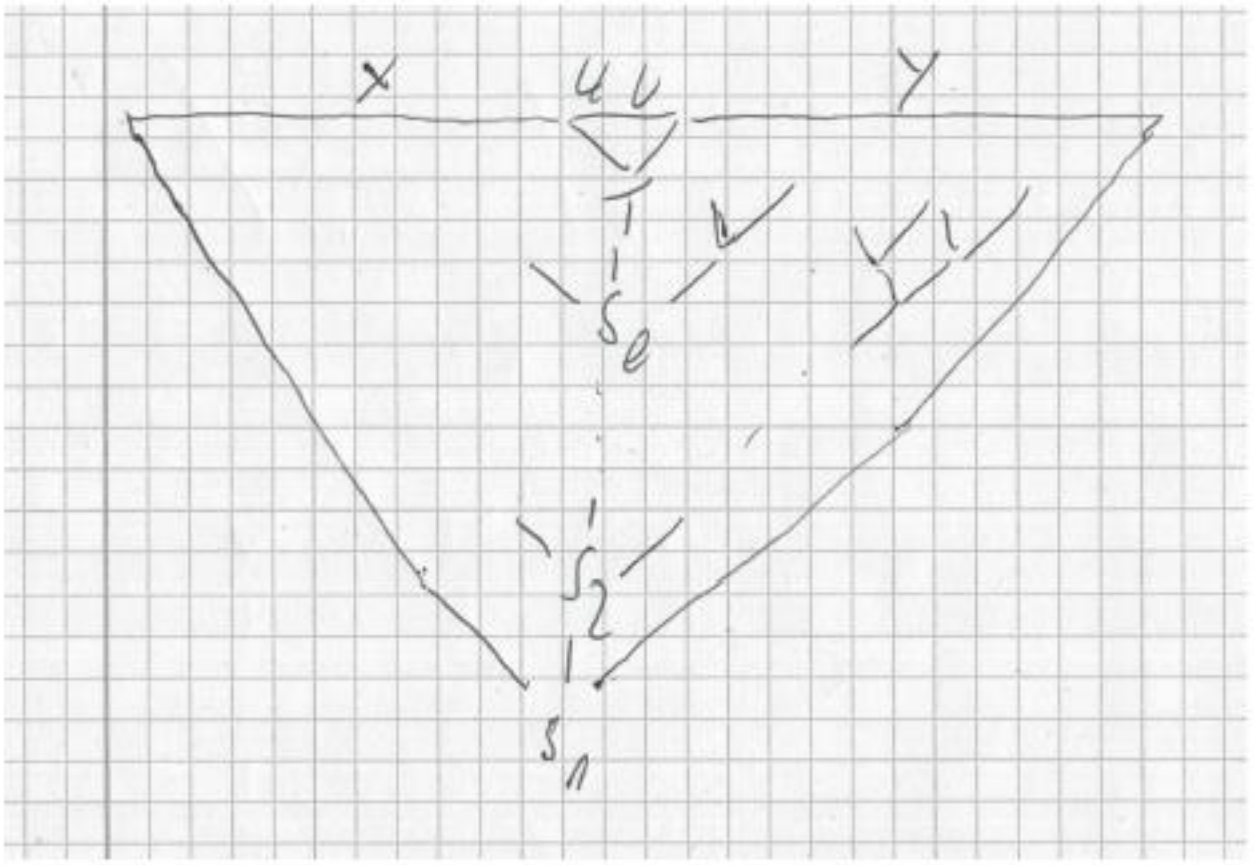
$\leftarrow$:

- derivation tree of $xuvy$ with handle $(|x|, T \rightarrow uv)$ has path

$$(T, S_\ell, \ldots, S_1)$$

as shown in figure 4.



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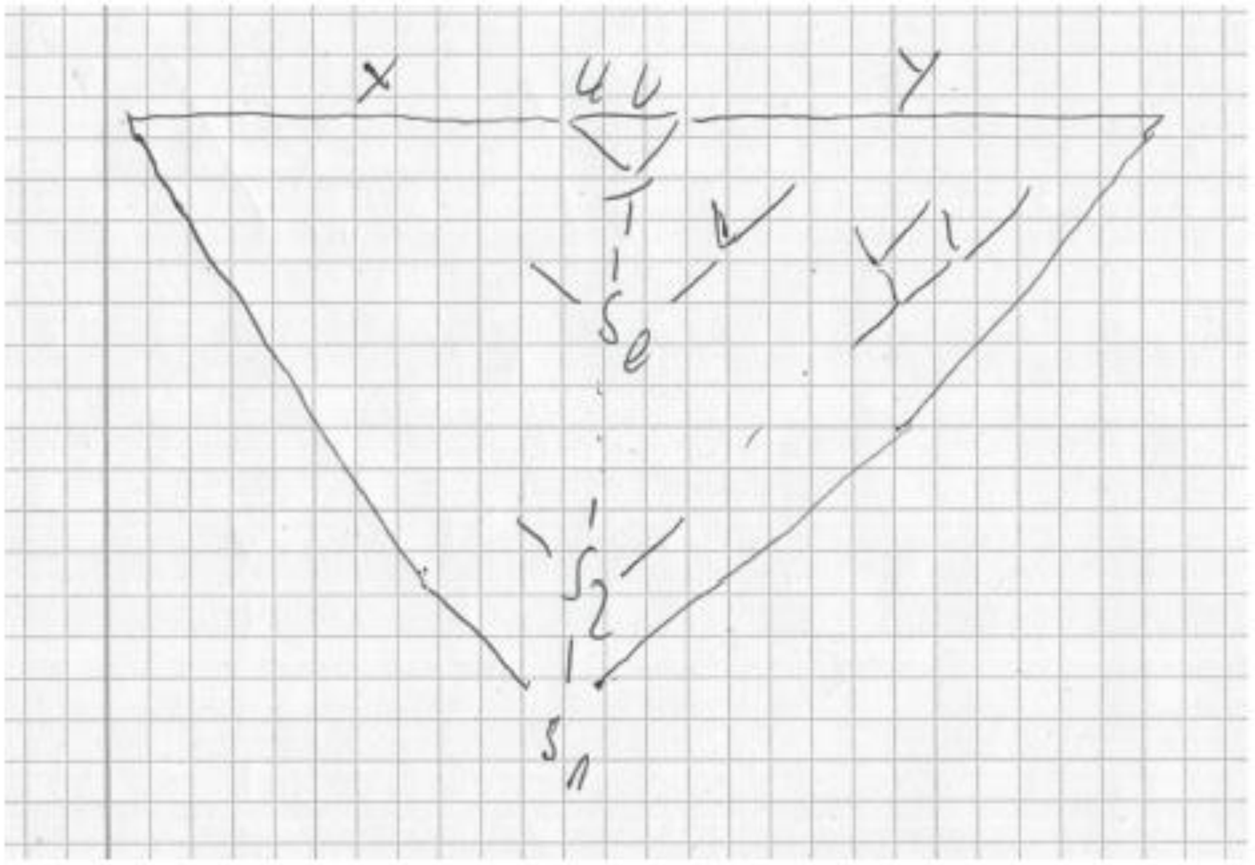

- portion x left of this path is directly derived from the S_i; otherwise $(|x|, T \to uv)$ is not handle.

$$\begin{aligned} S_i &\to u_i S_{i+1} v_i \quad \text{for } i < \ell \\ S_\ell &\to u_\ell T v_\ell \\ T &\to uv \end{aligned}$$

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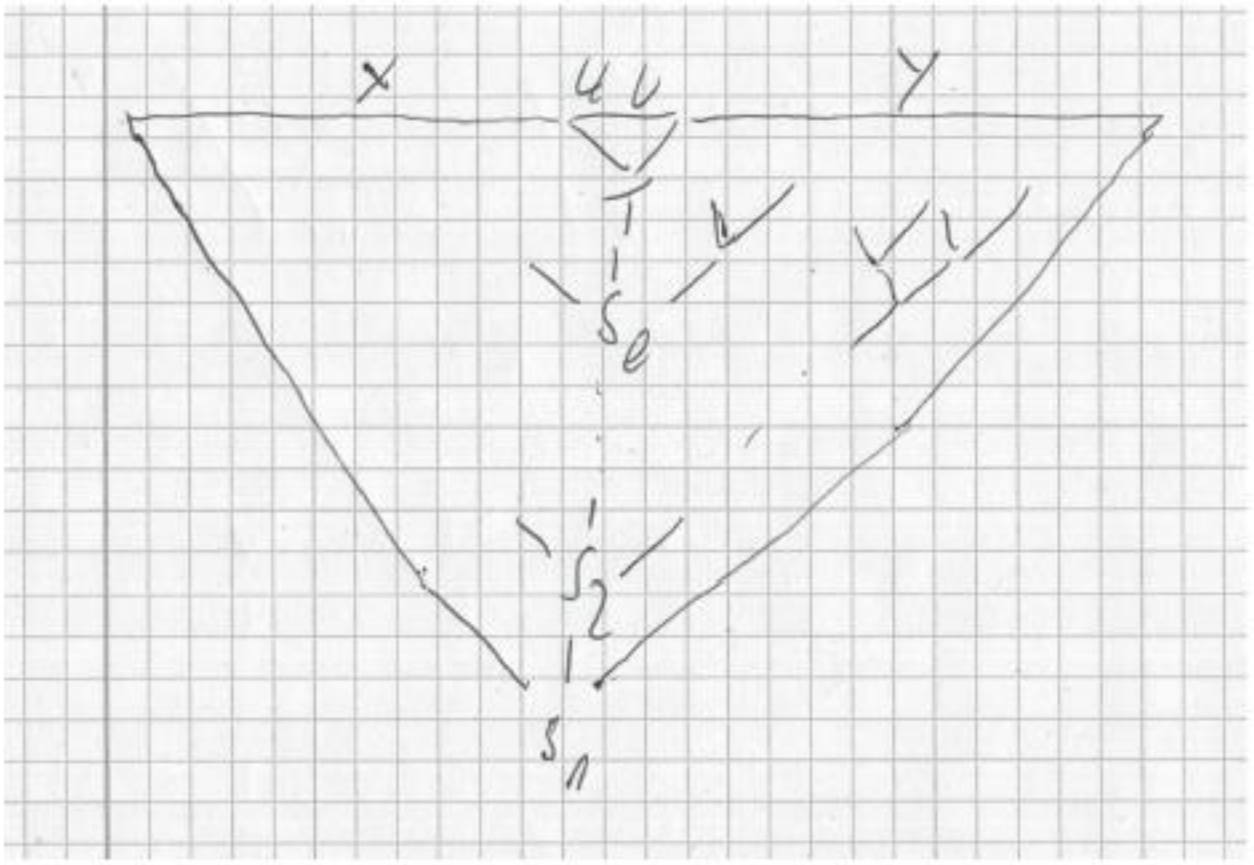

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- path of K with input $z = xu$

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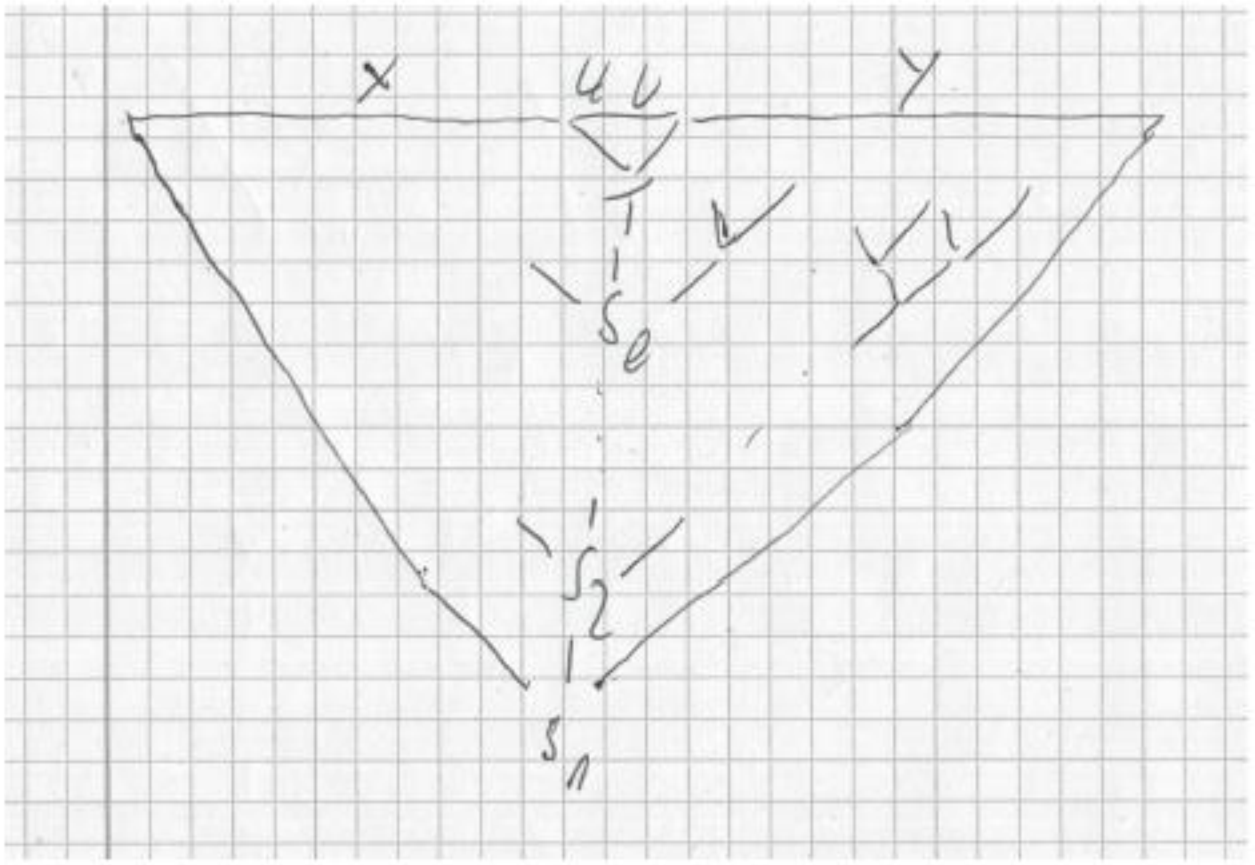

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Lemma 3. *K started with z can reach state $\langle T \to h.\rangle$ iff $z = xh$ and $(|x|, T \to h)$ is handle off a valid string xhy.*

Proof. Lemma 2 with $u = h, v = \varepsilon$ and handle$(|x|, T \to h)$. $\square$

Get fa DK from nfa K by power set construction .

- If DK is in state $s \in Z_{DK}$ after it has read read z, then

$$s = \{z \in Z_K \ : \ K \text{ can be in state } z \text{ after it has read } z\}$$

$$\begin{aligned} S &\rightarrow E\dashv \\ E &\rightarrow E+T \mid T \\ T &\rightarrow T\times a \mid a \end{aligned}$$

z_0

$$\langle S \rightarrow .E\dashv\rangle \xrightarrow{E} \langle S \rightarrow E.\dashv\rangle \xrightarrow{\dashv} \underline{\langle S \rightarrow E\dashv.\rangle}$$

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you can read this off directly from the grammar

$$\begin{aligned} S &\rightarrow E \dashv \\ E &\rightarrow E + T \mid T \\ T &\rightarrow T \times a \mid a \end{aligned}$$

z_0

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$$
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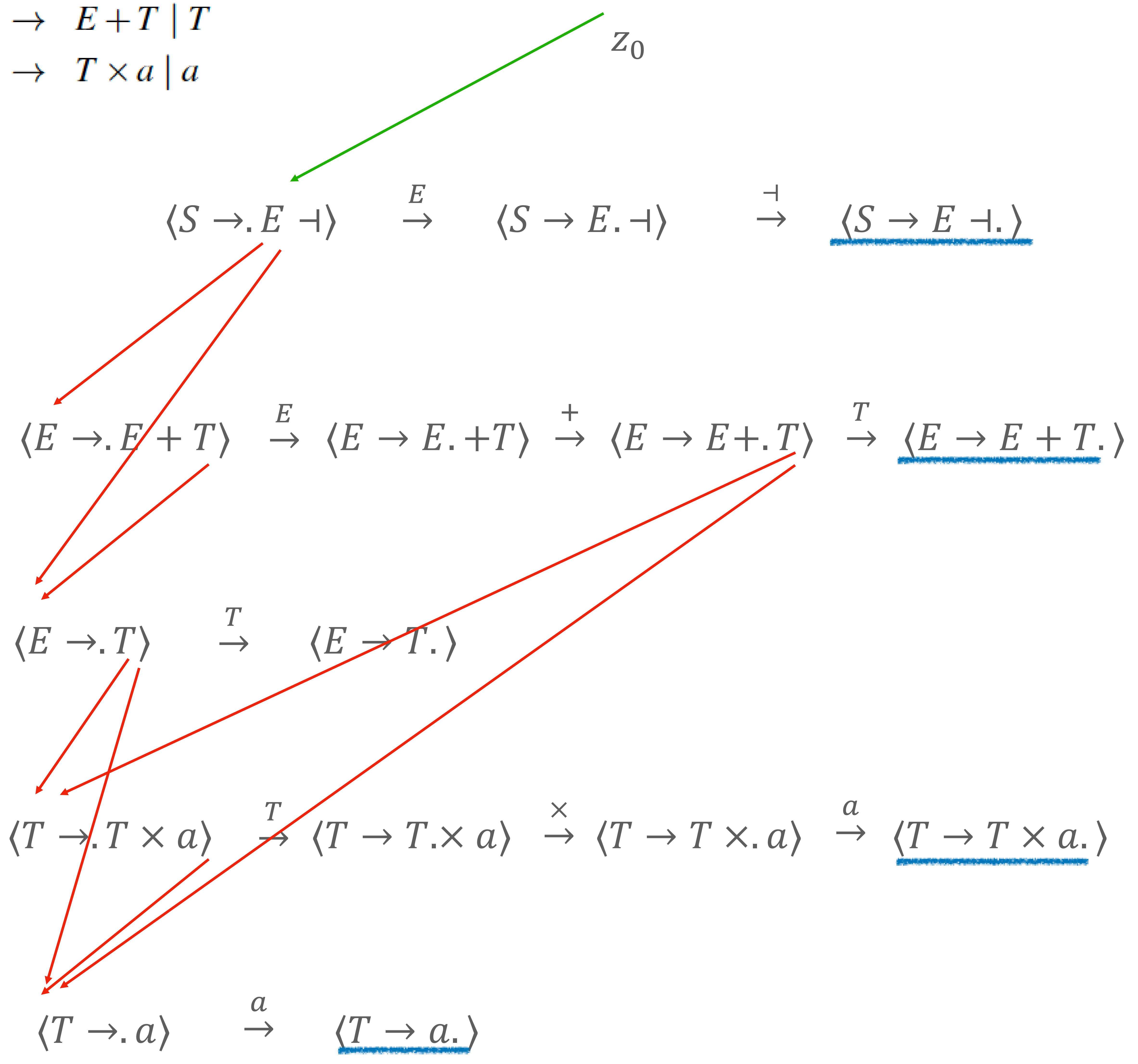
\xrightarrow{E}
\boxed{\begin{array}{c} \langle S \to E . \dashv \rangle \\ \langle E \to E . + T \rangle \end{array}}
$$

$$
\begin{aligned}
S &\to E \dashv \\
E &\to E + T \mid T \\
T &\to T \times a \mid a
\end{aligned}
$$



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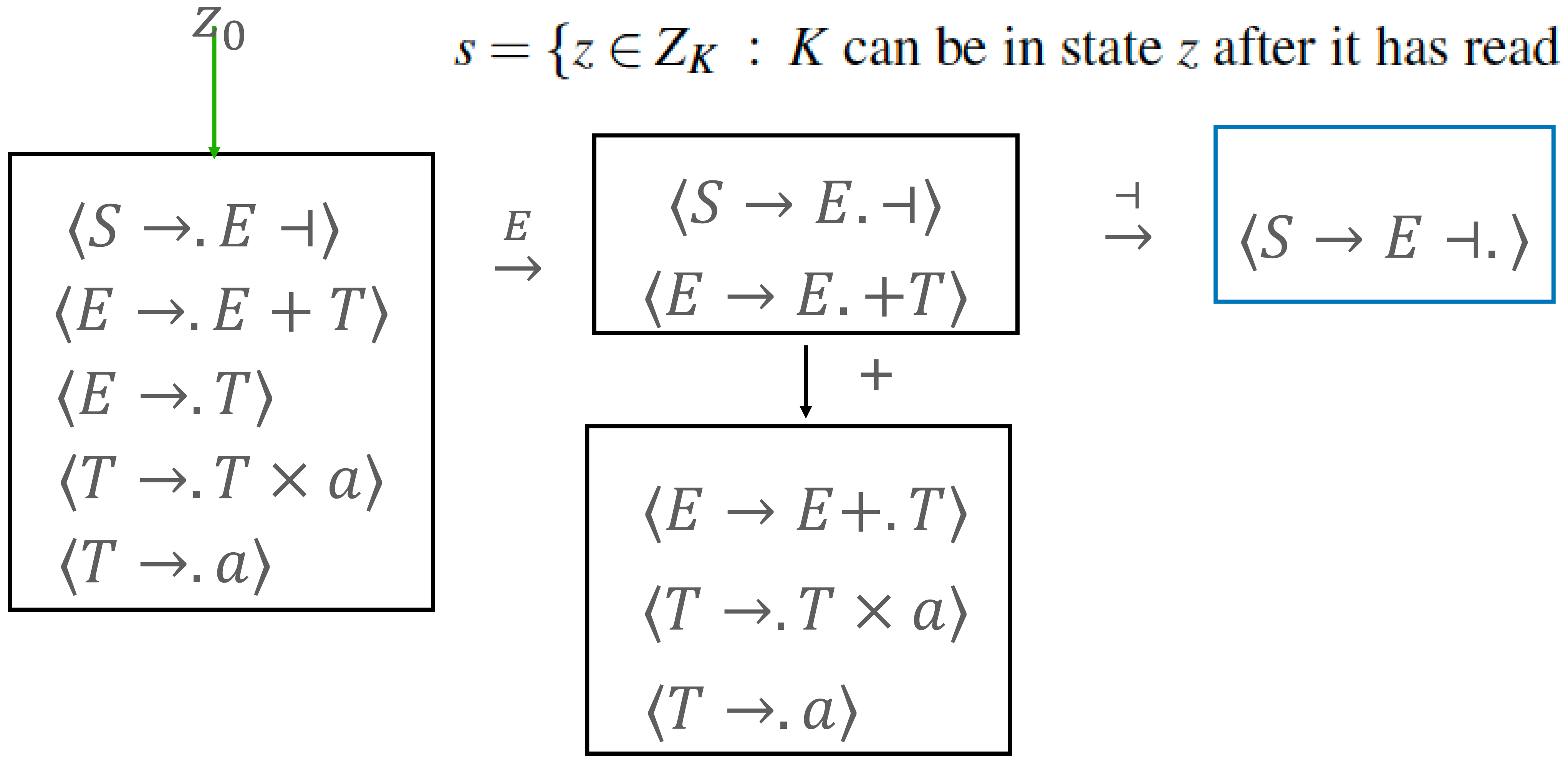

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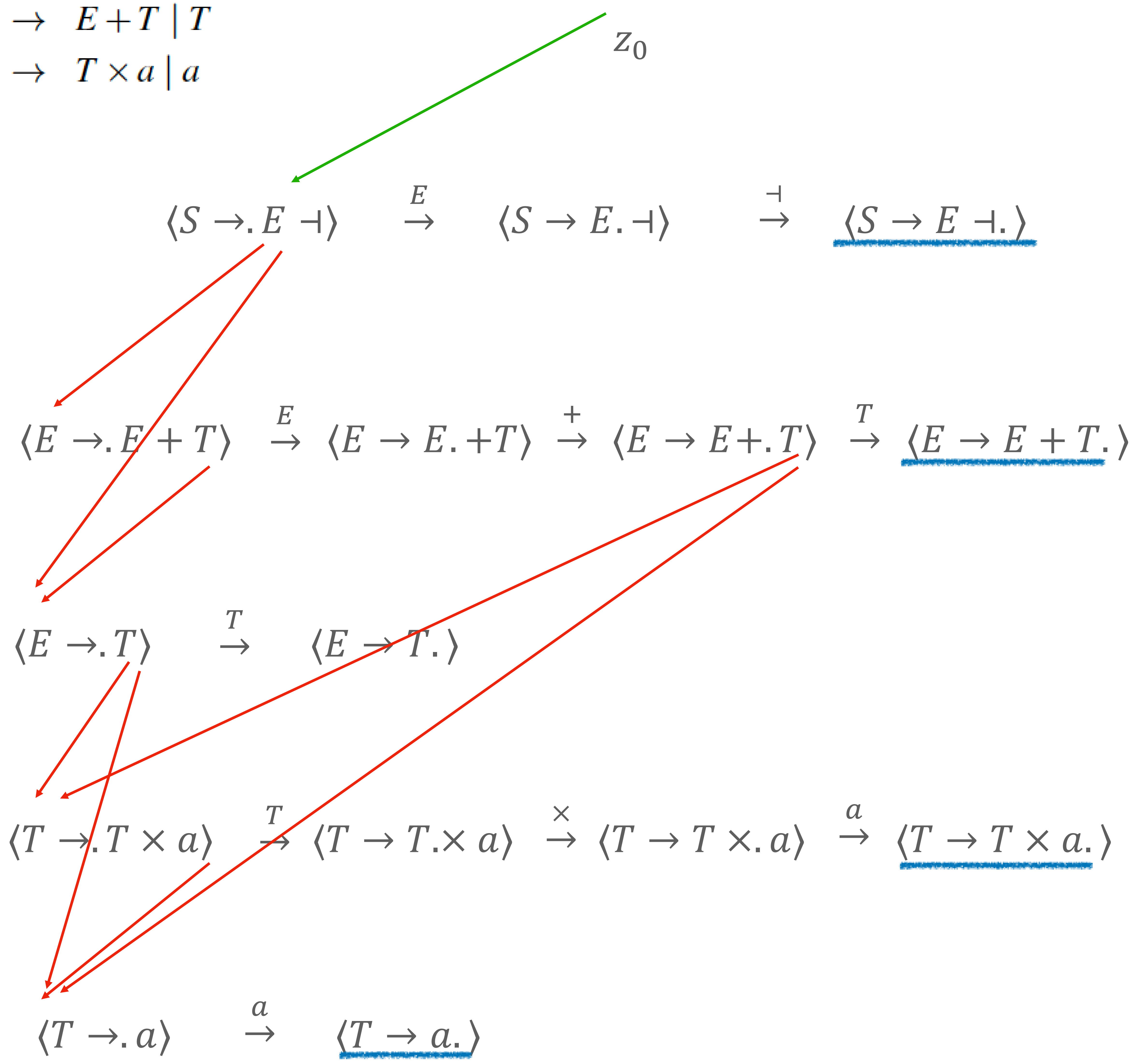
\begin{aligned}
S &\to E \dashv \\
E &\to E + T \mid T \\
T &\to T \times a \mid a
\end{aligned}
$$



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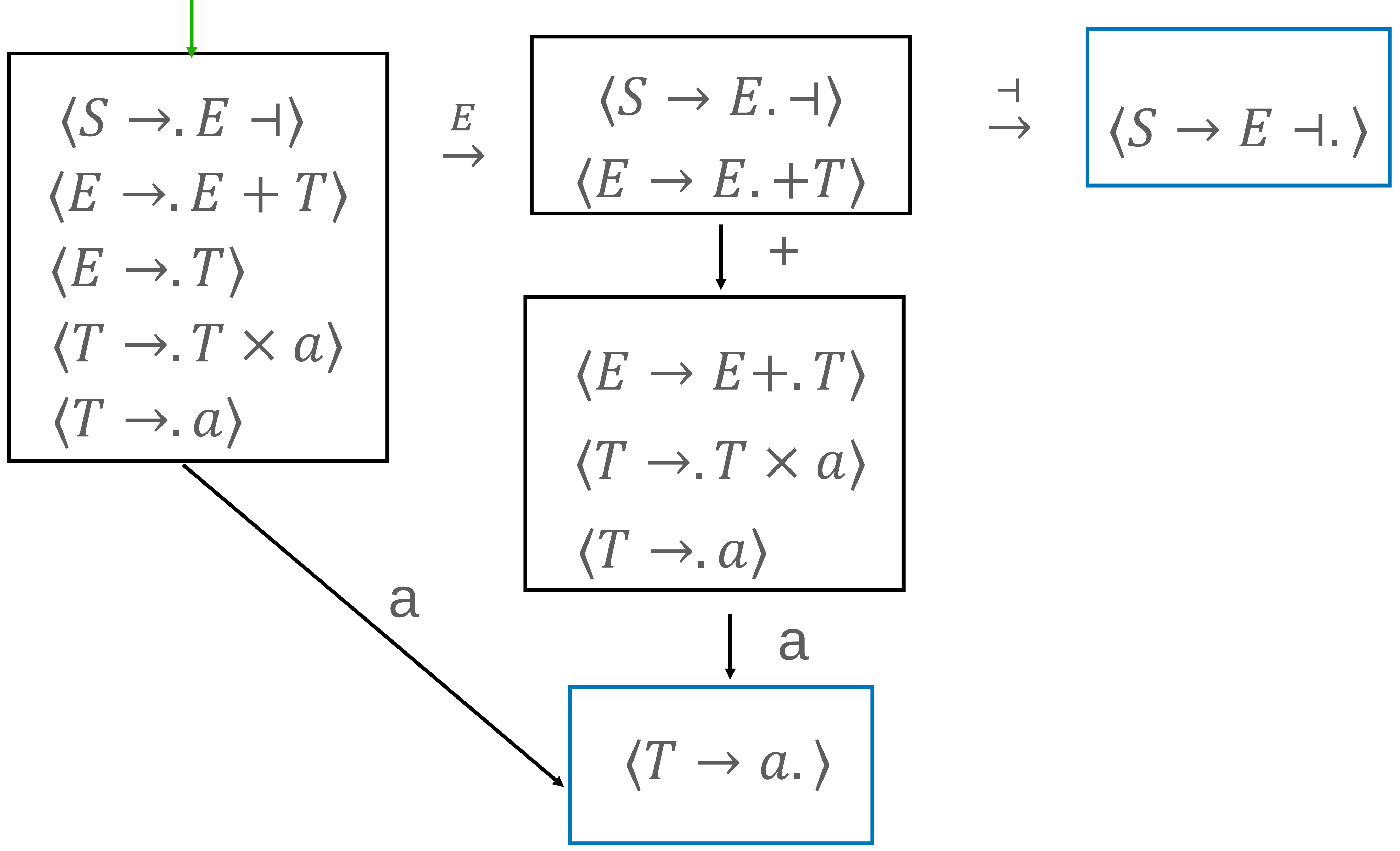

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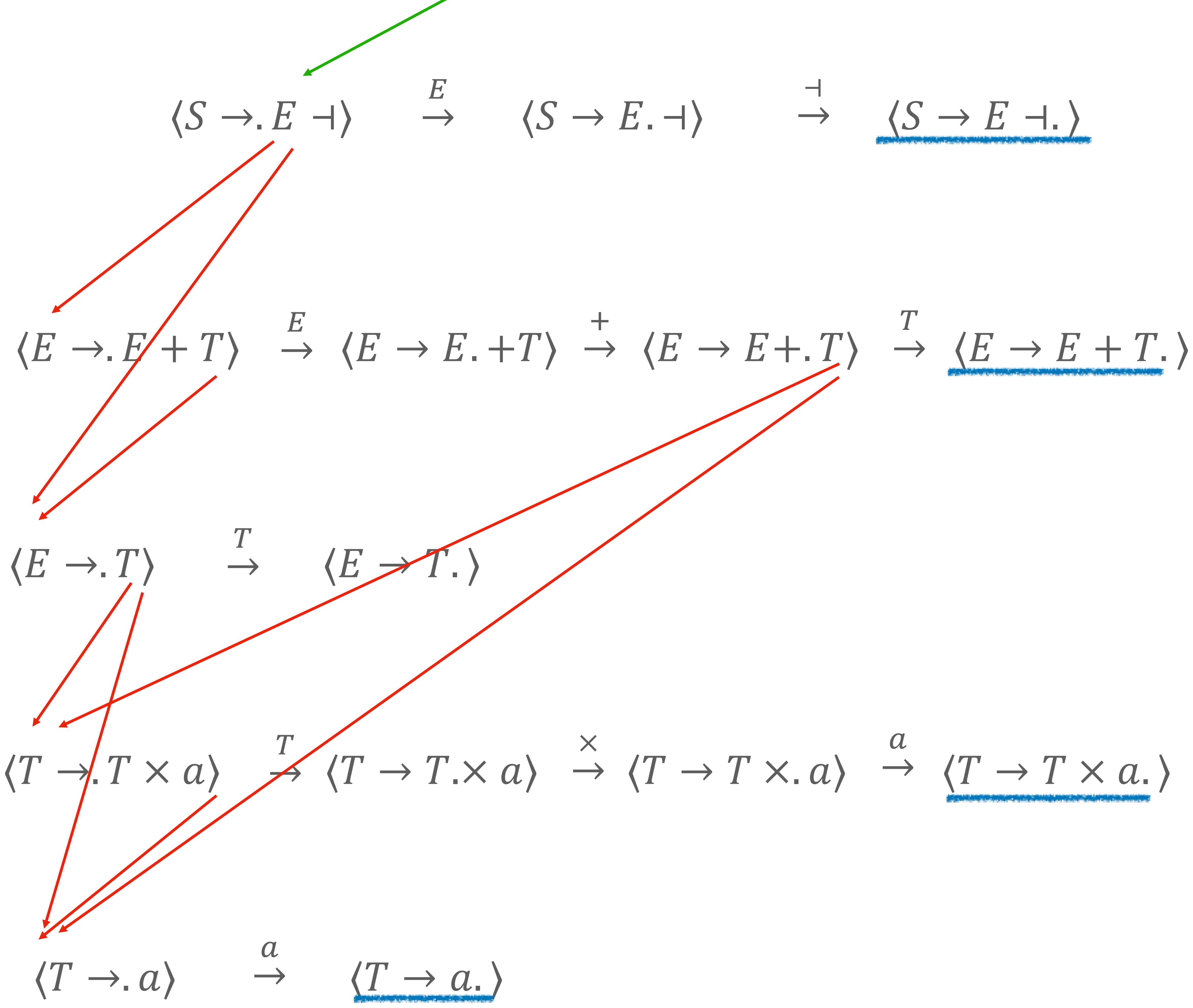

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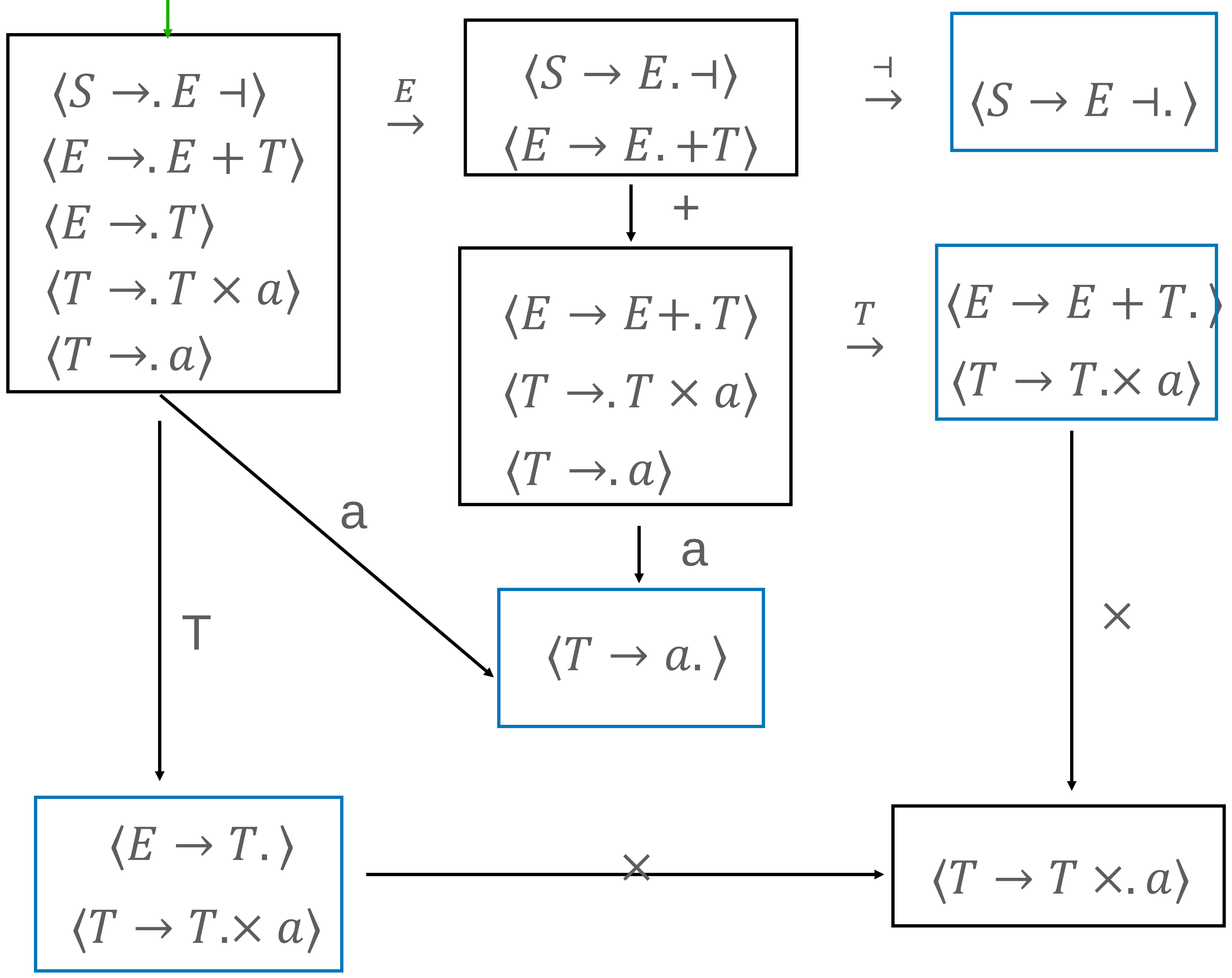

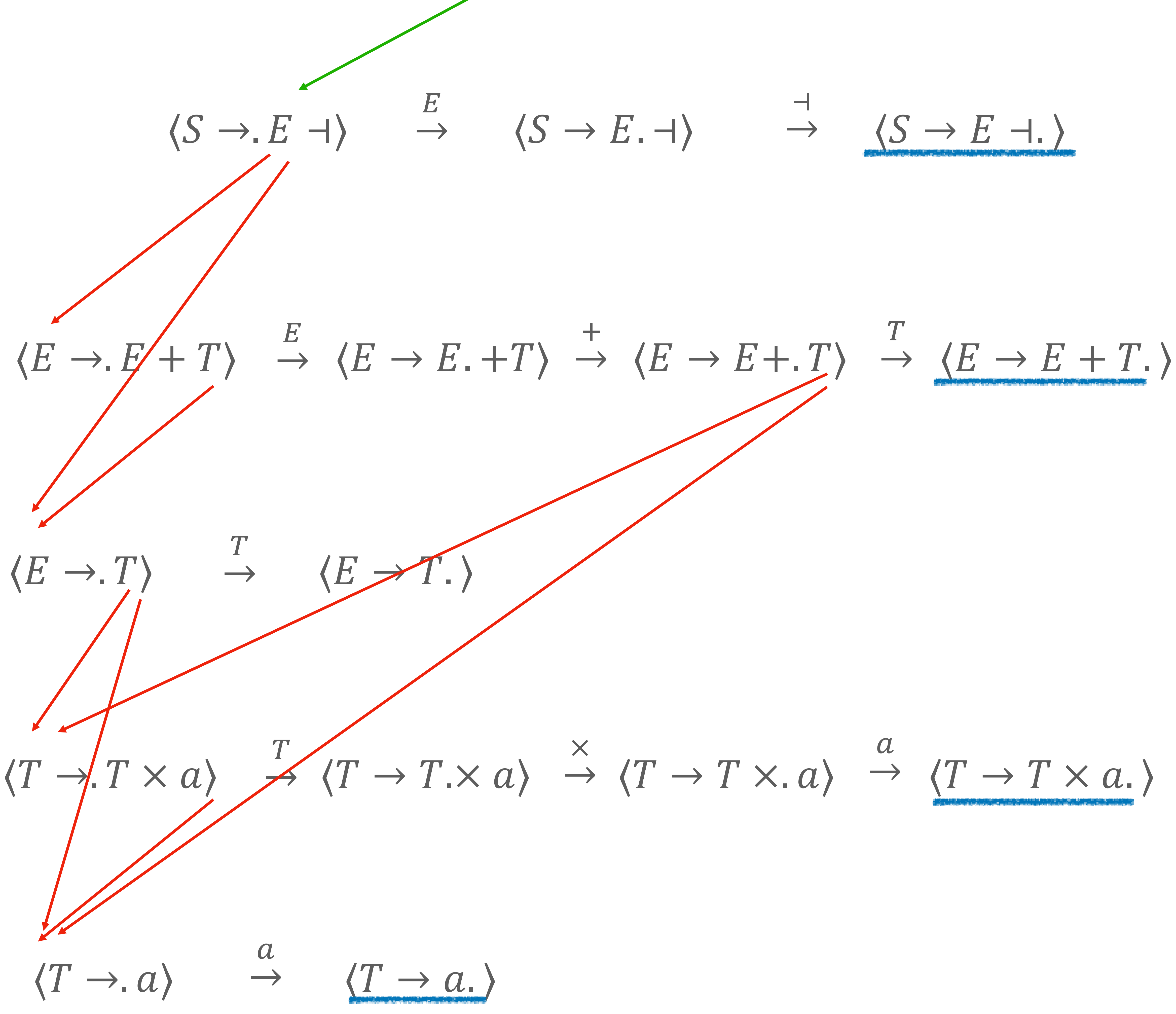

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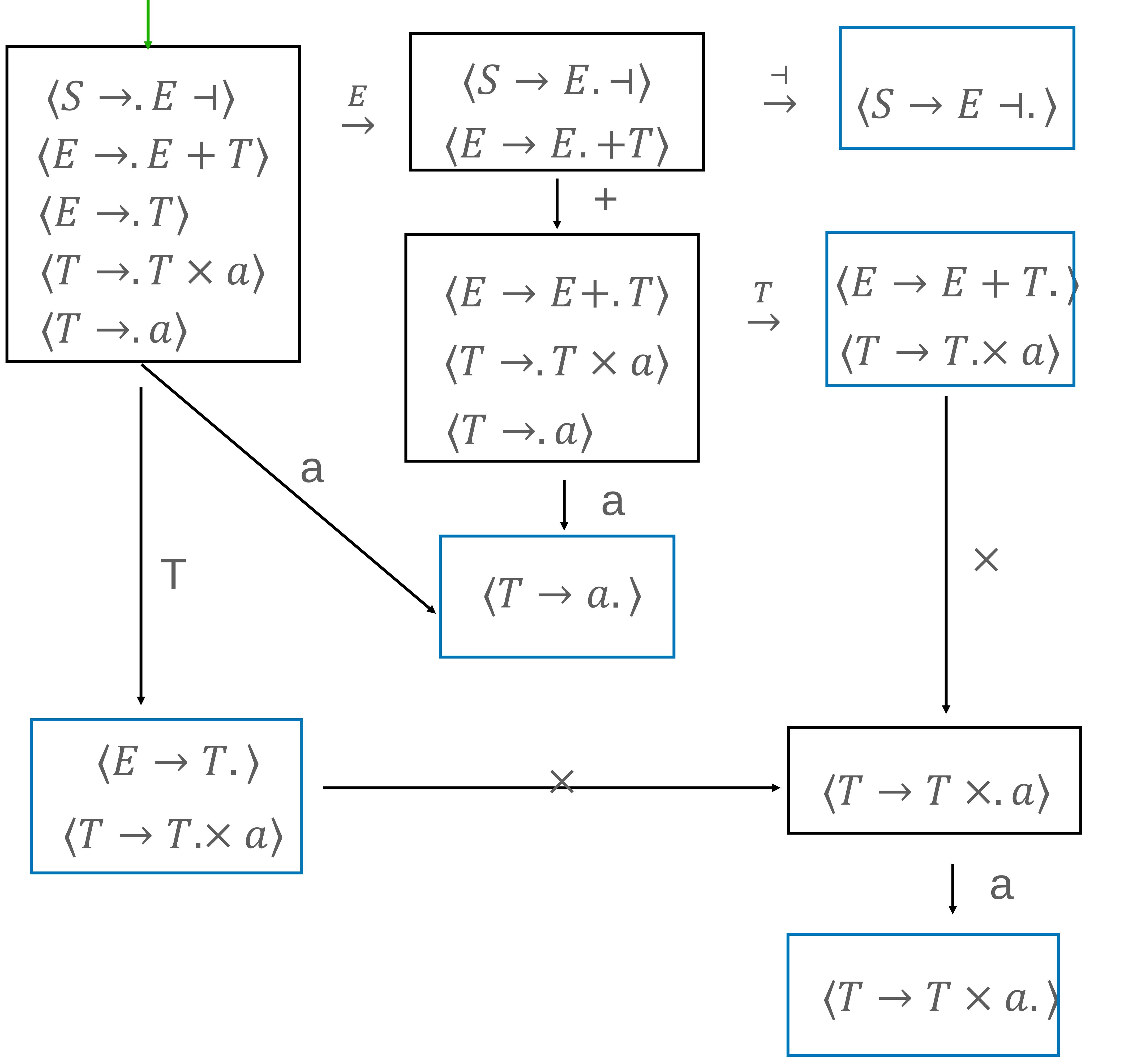

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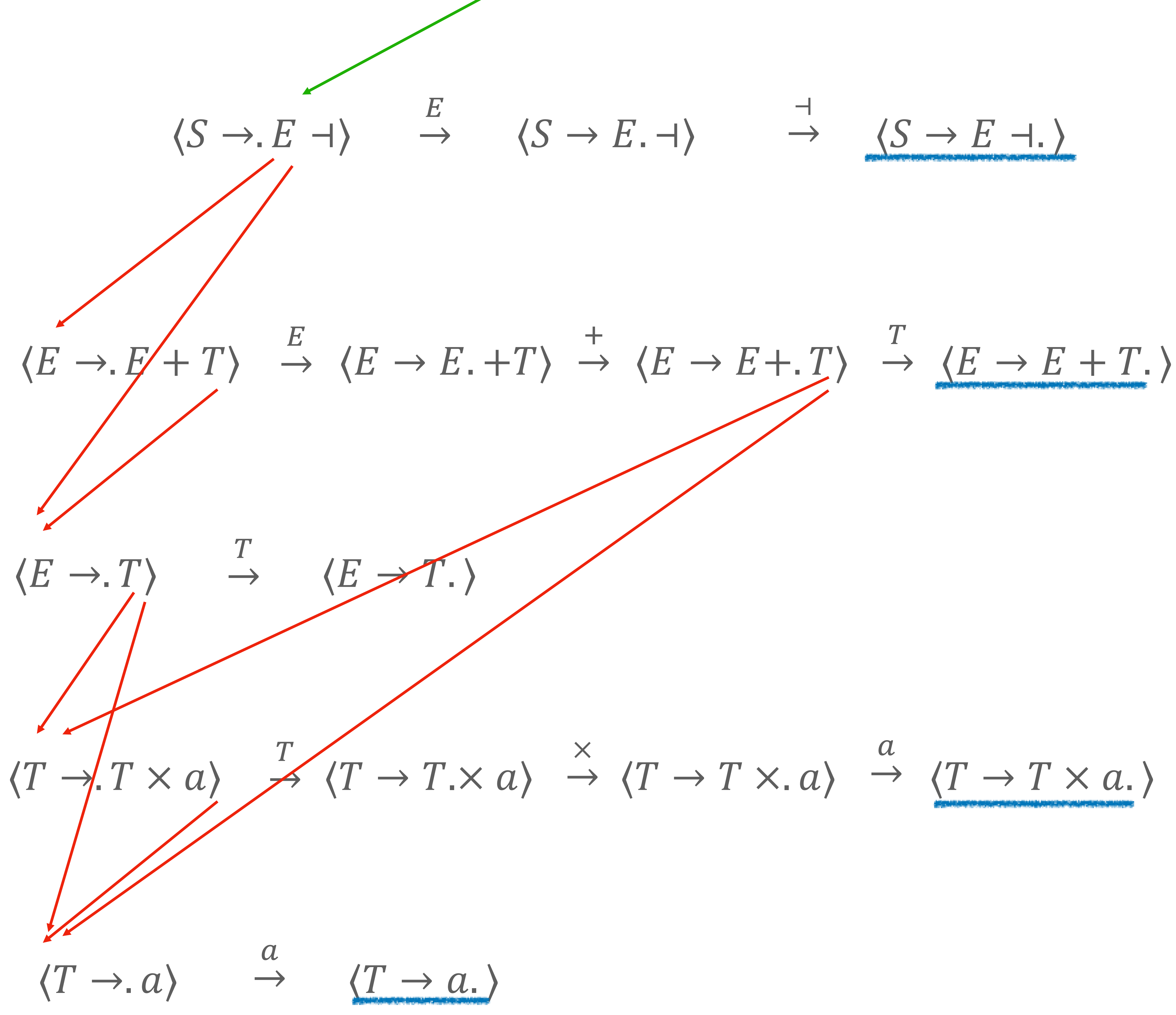

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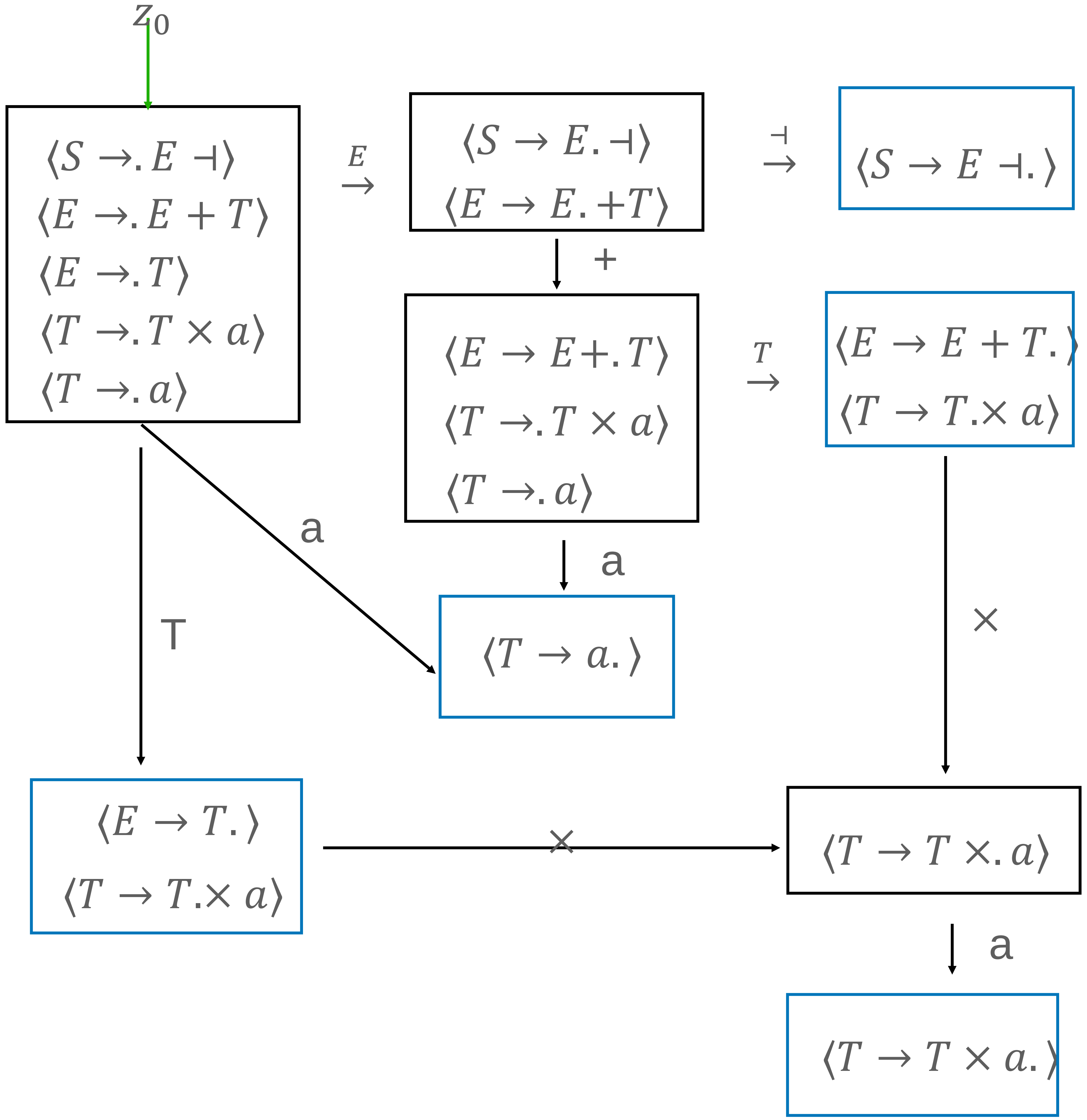

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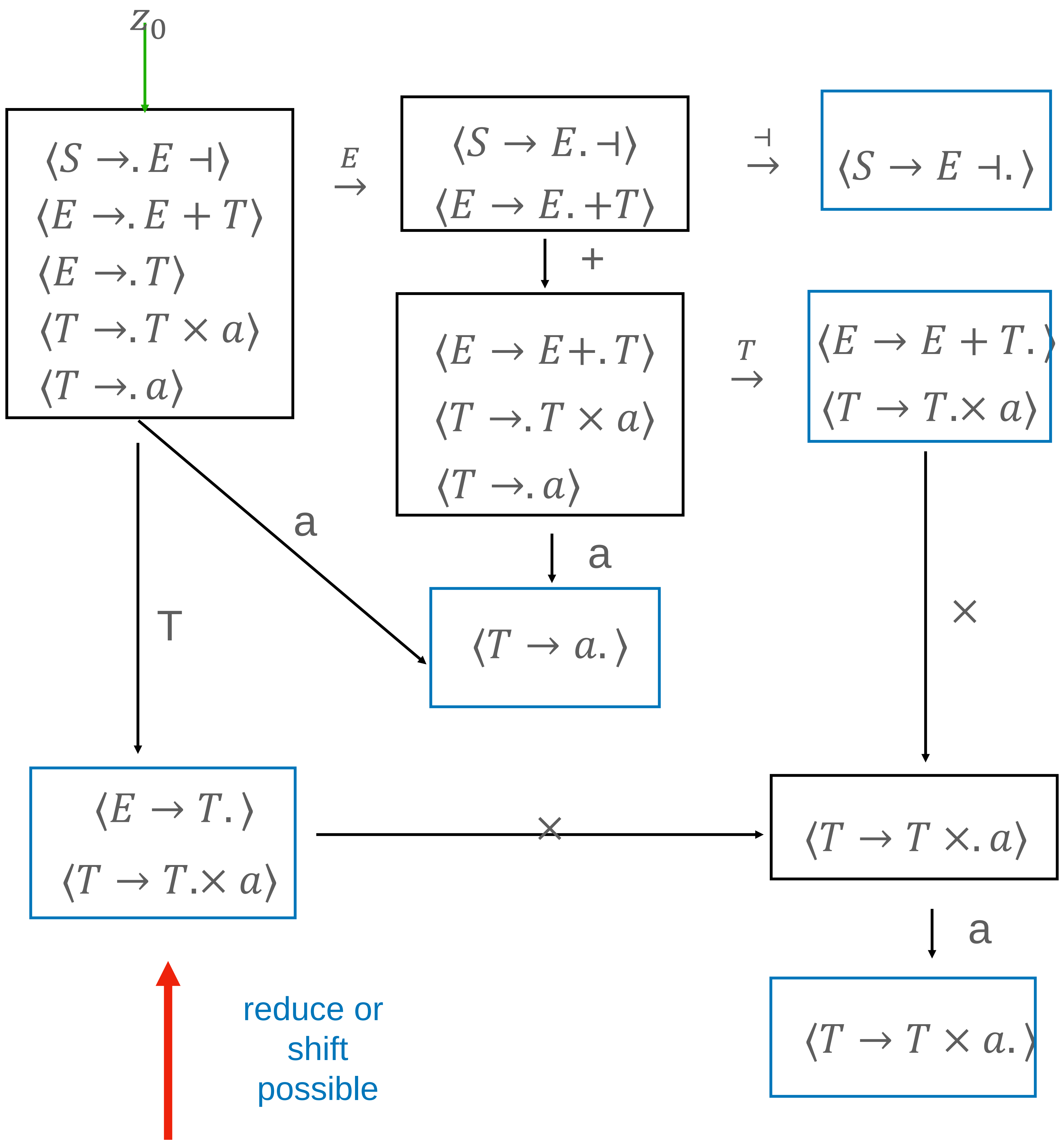

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- so C0 is not deterministic
- this *alone* won't work

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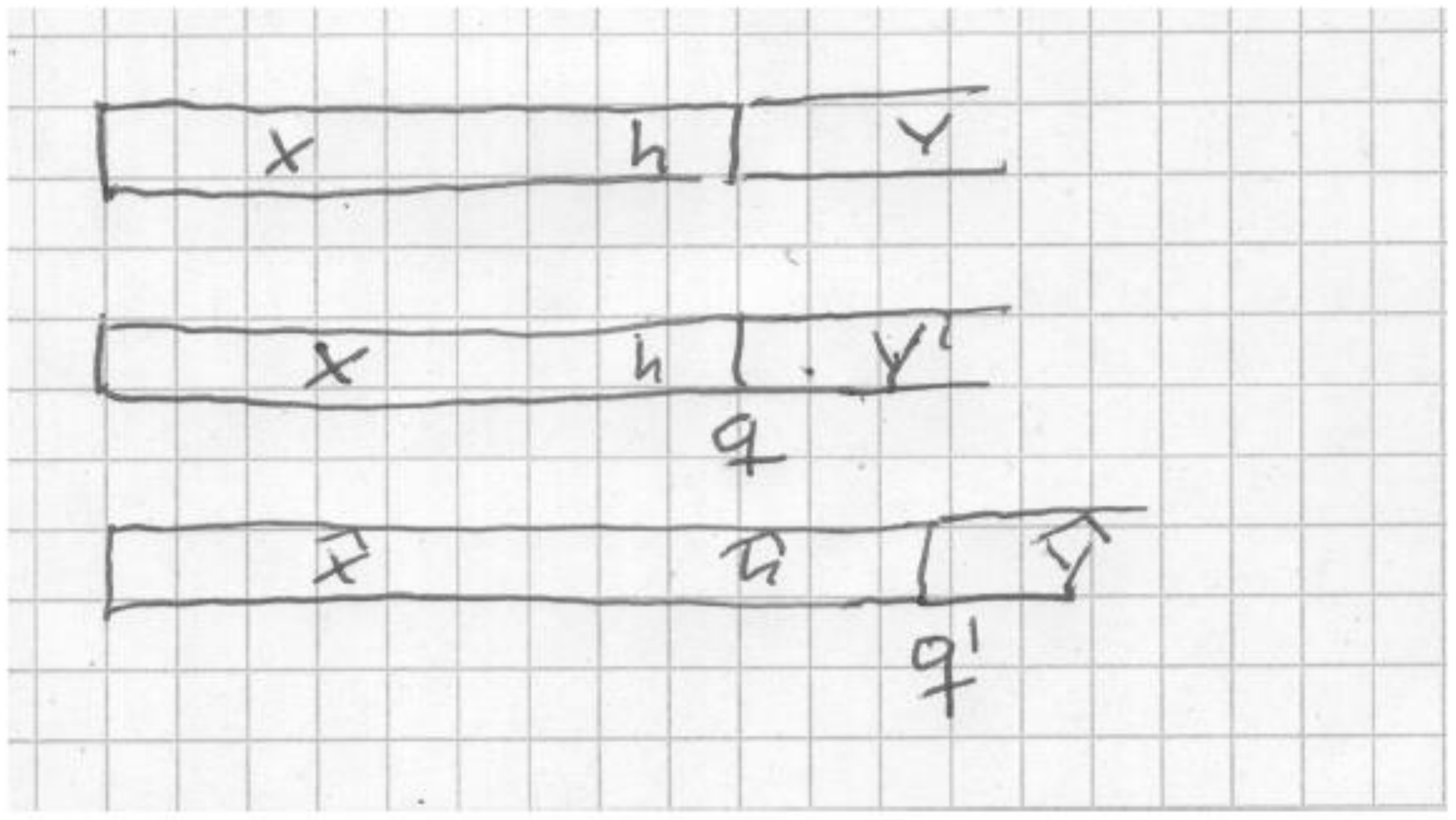

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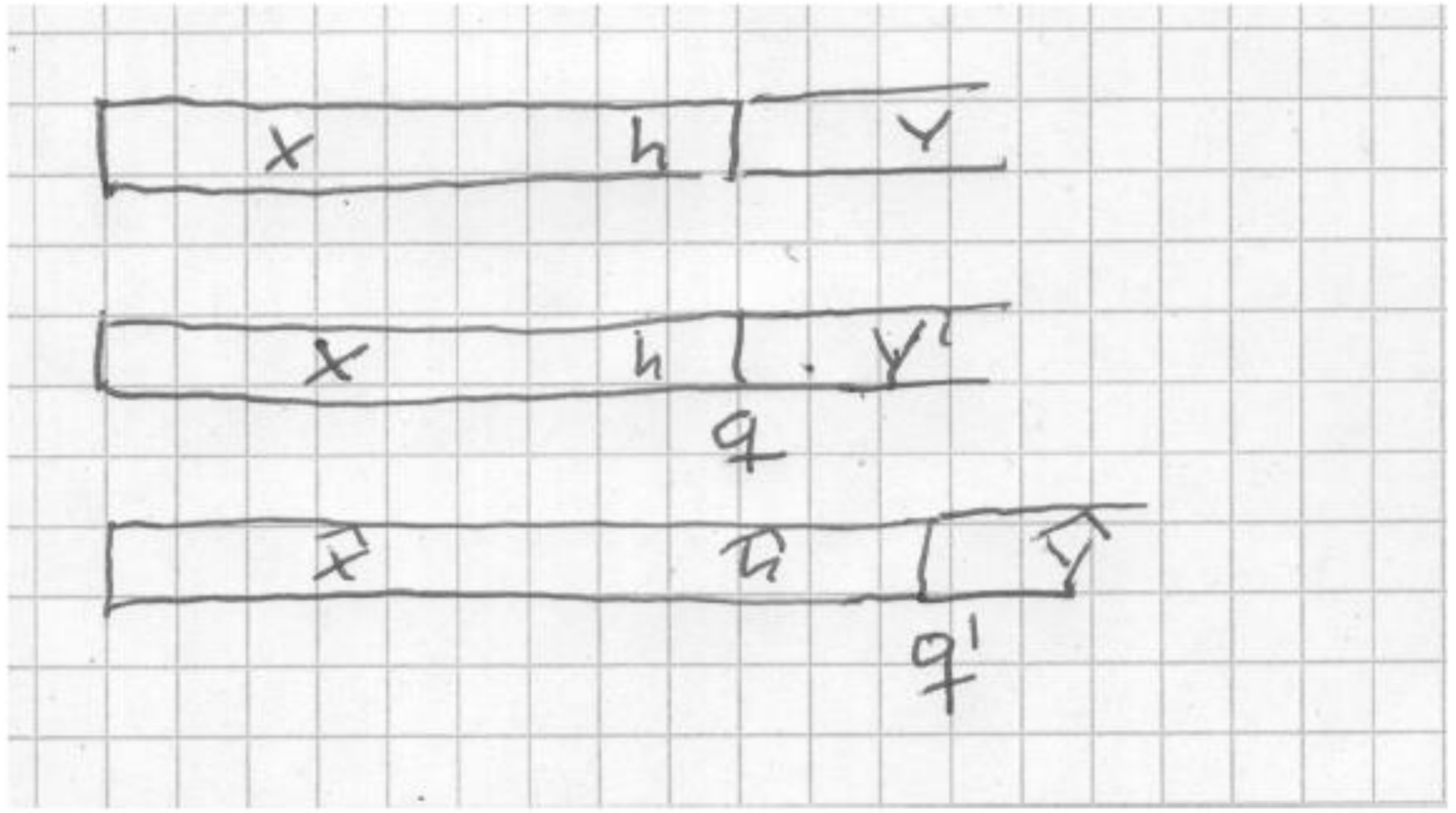

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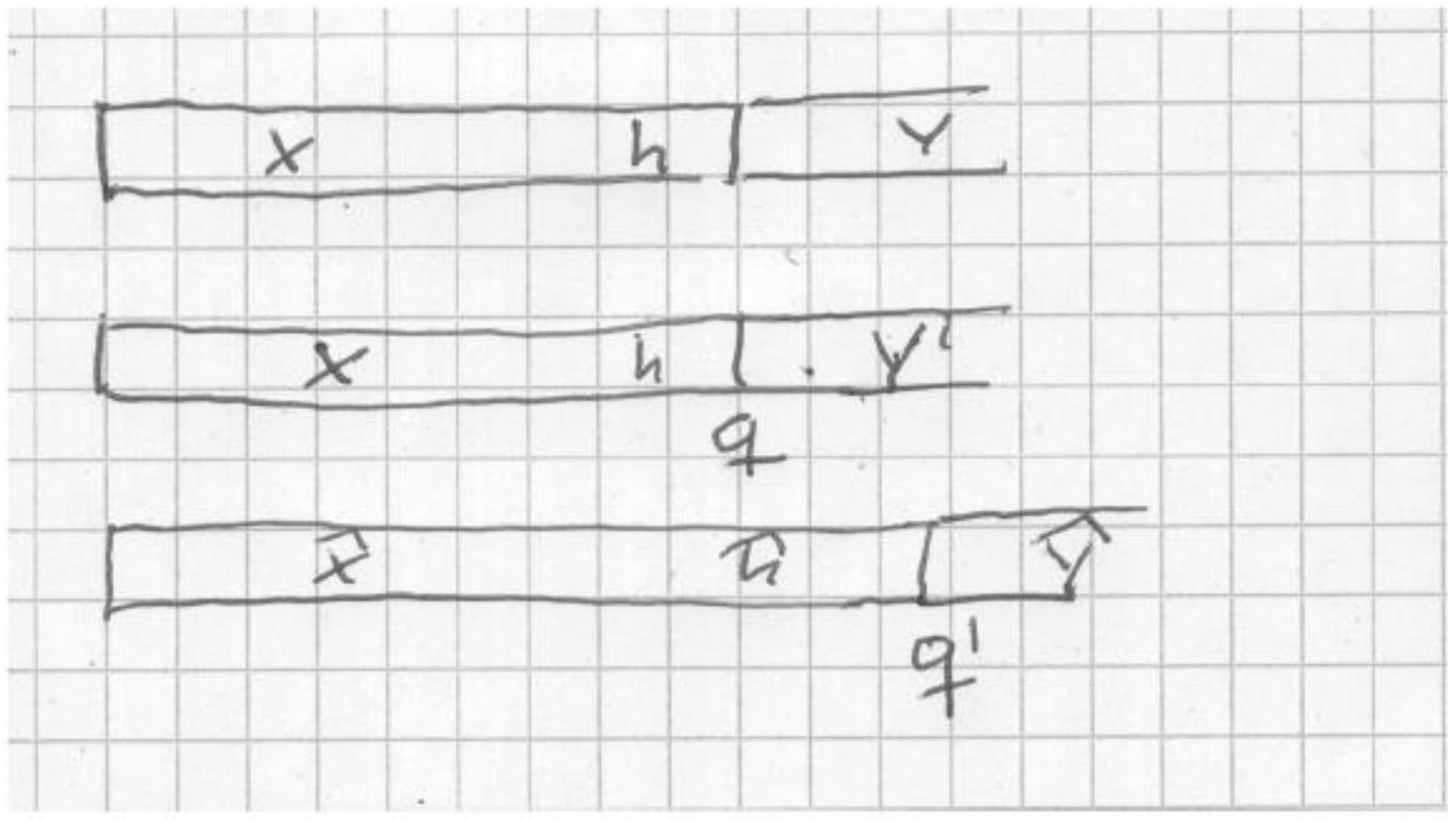

- $xh = \hat{x}\hat{h}$: Then $T \neq \hat{T}$. Test fails
- w.l.o.g xh prefix of $\hat{x}\hat{h}$: DK accepts xh in state q

There is path in DK from q to state q' leaving q with input symbol $y'_1 \in T$. Thus q contains a rule $B \to u.y'_1v$. Test fails.

4 Constructing leftmost derivations (and derivation trees) in linear time

Lemma 5. *If G passes the DK-test, then $L(G)$ is accepted by a dpda.*

naive solution in rounds i. Maintain valid strings v_i.
Let w be the input and $n = |w|$.

- round 0: v_0= input

- round $i > 0$: run DK on string v_i. If it finds handle $(n, T \to b)$ apply it to v_i to obtain v_{i+1}

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using the stack

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examples get complicated very quickly. DK is generated by programs; parser generators like 'yacc'.

does C0 grammar pass DK test? no

5 $LR(k)$ grammars

here treated for $k = 1$.

def: handle forced by lookahead Let $H = (|x|, T \to h)$ be handle of a valid string xhy. We say H is *forced by lookahead* k if if it is the unique handle of every valid string $xh\hat{y}$, where y and $\hat{y}$ agree on the first k symbols (if either string is shorter than k they must agree on the symbold of the shorter string).

$LR(k)$**-grammar:** a cfg, where the handle of each valid string is forced by lookahead k.

5.1 DK_1-automaton

constructed from nfa K_1:

states have the form

$$\langle B \to u.v \quad a \rangle \quad \text{with lookahead symbol } a \in T$$

starting From start state z_0 and all production $S_1 \to u$ and all $a \in T$

$$z_0 \xrightarrow{\varepsilon} \langle S_1 \to .u \quad a \rangle$$

shift moves for $x \in N \cup T$ and every production $B \to uav$ and all $a \in T$

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- read u
- part of handle uv
- if v follows u
- and a follows v

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R and R' are *consistent* if

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DK_1 test passed if no end state of DK_1 contains consistent dotted rules.

this works for example grammar and possibly C0 grammar

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Lemma 6. *If cfg G passes the DK_1-test, then*

- *G is an $LR(1)$ grammar*
- *$L(G)$ is recognized by a dpda*

- handles are identified as before independent of lookahead symbols
- pda reads 1 symbol ahead and stores it in its finite control, then disambiguates rules using this symbol.