## (a) Independent (Unpaired) Samples Test with Unequal Variance

**Assumptions:**

a. **Population and Samples are Normal Distributed, and**
b. **Variances are assumed to be Unequal**
c. **Sample A and Sample B are independent (e.g. Male versus Female)**

**Step 1**

Define Null Hypothesis $H_0$:

$H_0$ There is no difference between the 2 samples

**Step 2**

Calculate the T-Value (Welch's Test):

$$\frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

where $\bar{x}_1$ is the mean for column $x_1$

$\bar{x}_2$ is the mean for column $x_2$

$S_1$ is the standard deviation for $x_1$

$S_2$ is the standard deviation for $x_2$

$S_1^2$ is the variance for $x_1$

$S_2^2$ is the variance for $x_2$

$n_1$ is the number for $x_1$

$n_2$ is the number for $x_2$

**Step 3**

To find the Critical Value in t-distribution table (t-table), we first need to find df (degree of freedom) from t-Table for 95% confidence level:

$$df = n_1 + n_2 - 2$$

**Step 4**

Hypothesis Testing:

$H_0$ There is no difference between the two samples

- If it is lower than the Critical Value, then do not Reject
- If it is higher than the Critical Value, then Reject

**Step 5**

Write your conclusion

**Example:**

Using the dataset from Class Exercise/Tutorial:

| Sample A | Sample B |
|---|---|
| 17.00 | 17.80 |
| 17.10 | 17.80 |
| 17.90 | 17.00 |
| 17.70 | 18.60 |
| 17.50 | 17.00 |
| 16.70 | 17.70 |
| 16.80 | 17.70 |
| 17.20 | 18.10 |
| 17.50 | 17.50 |
| 17.60 | 17.50 |
| 17.40 | 17.70 |
| 17.00 | 17.40 |
| 17.40 | 17.40 |
| 16.90 | 17.40 |
| 17.10 | 16.70 |
| 16.80 | 17.80 |

## Step 1

Define Null Hypothesis $H_0$:

$H_0$: There is no statically significant difference between Sample A and Sample B

## Step 2

We calculate the mean and standard deviations (and variances):

| | iPhone/Class A | iPad/Class B |
|---|---|---|
| | 17.00 | 17.80 |
| | 17.10 | 17.80 |
| | 17.90 | 17.00 |
| | 17.70 | 18.60 |
| | 17.50 | 17.00 |
| | 16.70 | 17.70 |
| | 16.80 | 17.70 |
| | 17.20 | 18.10 |
| | 17.50 | 17.50 |
| | 17.60 | 17.50 |
| | 17.40 | 17.70 |
| | 17.00 | 17.40 |
| | 17.40 | 17.40 |
| | 16.90 | 17.40 |
| | 17.10 | 16.70 |
| | 16.80 | 17.80 |
| Mean | **17.2250** | **17.5688** |
| StDev | **0.3568** | **0.4512** |
| n | **16.00** | **16.00** |

Use the formula to calculate T-Value:

$$\frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

=

$$\frac{17.5688 - 17.2250}{\sqrt{\frac{(0.4512)^2}{16} + \frac{(0.3568)^2}{16}}}$$

=

$$\frac{17.5688-17.2250}{\sqrt{\frac{0.2036}{16}+\frac{0.1273}{16}}}$$

=

$$\frac{17.5688-17.2250}{\sqrt{0.0127+0.0080}}$$

=

$$\frac{0.3438}{0.1438}$$

= **2.3908 or 2.34**

**Step 3**

Find the Critical Value for T-test for 95% confidence level:

df = $n_1 + n_2 - 2$

df = 16 + 16 – 2 = 30

Thus, Critical Value is 2.042 (see next page)

**TABLE B: *t*-DISTRIBUTION CRITI**

| | Tail probability *p* | | | | | | |
|---|---|---|---|---|---|---|---|
| df | .25 | .20 | .15 | .10 | .05 | .025 | .02 |
| 1 | 1.000 | 1.376 | 1.963 | 3.078 | 6.314 | 12.71 | 15.89 |
| 2 | .816 | 1.061 | 1.386 | 1.886 | 2.920 | 4.303 | 4.849 |
| 3 | .765 | .978 | 1.250 | 1.638 | 2.353 | 3.182 | 3.482 |
| 4 | .741 | .941 | 1.190 | 1.533 | 2.132 | 2.776 | 2.999 |
| 5 | .727 | .920 | 1.156 | 1.476 | 2.015 | 2.571 | 2.757 |
| 6 | .718 | .906 | 1.134 | 1.440 | 1.943 | 2.447 | 2.612 |
| 7 | .711 | .896 | 1.119 | 1.415 | 1.895 | 2.365 | 2.517 |
| 8 | .706 | .889 | 1.108 | 1.397 | 1.860 | 2.306 | 2.449 |
| 9 | .703 | .883 | 1.100 | 1.383 | 1.833 | 2.262 | 2.398 |
| 10 | .700 | .879 | 1.093 | 1.372 | 1.812 | 2.228 | 2.359 |
| 11 | .697 | .876 | 1.088 | 1.363 | 1.796 | 2.201 | 2.328 |
| 12 | .695 | .873 | 1.083 | 1.356 | 1.782 | 2.179 | 2.303 |
| 13 | .694 | .870 | 1.079 | 1.350 | 1.771 | 2.160 | 2.282 |
| 14 | .692 | .868 | 1.076 | 1.345 | 1.761 | 2.145 | 2.264 |
| 15 | .691 | .866 | 1.074 | 1.341 | 1.753 | 2.131 | 2.249 |
| 16 | .690 | .865 | 1.071 | 1.337 | 1.746 | 2.120 | 2.235 |
| 17 | .689 | .863 | 1.069 | 1.333 | 1.740 | 2.110 | 2.224 |
| 18 | .688 | .862 | 1.067 | 1.330 | 1.734 | 2.101 | 2.214 |
| 19 | .688 | .861 | 1.066 | 1.328 | 1.729 | 2.093 | 2.205 |
| 20 | .687 | .860 | 1.064 | 1.325 | 1.725 | 2.086 | 2.197 |
| 21 | .686 | .859 | 1.063 | 1.323 | 1.721 | 2.080 | 2.189 |
| 22 | .686 | .858 | 1.061 | 1.321 | 1.717 | 2.074 | 2.183 |
| 23 | .685 | .858 | 1.060 | 1.319 | 1.714 | 2.069 | 2.177 |
| 24 | .685 | .857 | 1.059 | 1.318 | 1.711 | 2.064 | 2.172 |
| 25 | .684 | .856 | 1.058 | 1.316 | 1.708 | 2.060 | 2.167 |
| 26 | .684 | .856 | 1.058 | 1.315 | 1.706 | 2.056 | 2.162 |
| 27 | .684 | .855 | 1.057 | 1.314 | 1.703 | 2.052 | 2.158 |
| 28 | .683 | .855 | 1.056 | 1.313 | 1.701 | 2.048 | 2.154 |
| 29 | .683 | .854 | 1.055 | 1.311 | 1.699 | 2.045 | 2.150 |
| 30 | .683 | .854 | 1.055 | 1.310 | 1.697 | 2.042 | 2.147 |
| 40 | .681 | .851 | 1.050 | 1.303 | 1.684 | 2.021 | 2.123 |

## Step 4

$H_0$: There is no difference between Sample A and Sample B

- If t-value is lower than the Critical Value, then do not Reject
- If t-value is higher than the Critical Value, then Reject

As the T-Value (**2.34**) is higher than Critical Value (**2.04**), then **Reject the Null Hypothesis**.

## Step 5

Write your conclusion based on the context: for instance, there is a difference between using iPad and iPhone to revise before an exam.

Using Excel, you will get:

| t-Test: Two-Sample Assuming Unequal Variances | | |
|---|---|---|
| | | |
| | *Sample A* | *Sample B* |
| Mean | 17.225 | 17.56875 |
| Variance | 0.127333333 | 0.203625 |
| Observations | 16 | 16 |
| Hypothesized Mean Difference | 0 | |
| df | 28 | |
| t Stat | -2.390099812 | |
| P(T<=t) one-tail | 0.011907154 | |
| t Critical one-tail | 1.701130934 | |
| P(T<=t) two-tail | 0.023814309 | |
| t Critical two-tail | **2.048407142** | |

## (b) Independent (Unpaired) Samples Test with Equal Variance

**Assumptions:**

a. **Population and Samples are Normal Distributed, and**
b. **Variances are assumed to be Equal**

**Formula looks something like:**

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

**Results:**

Using Excel, you will get:

| t-Test: Two-Sample Assuming Equal Variances | | |
|---|---|---|
| | | |
| | *Sample A* | *Sample B* |
| Mean | 17.225 | 17.56875 |
| Variance | 0.127333333 | 0.203625 |
| Observations | 16 | 16 |
| Pooled Variance | 0.165479167 | |
| Hypothesized Mean Difference | 0 | |
| df | 30 | |
| t Stat | -2.390099812 | |
| P(T<=t) one-tail | 0.011663312 | |
| t Critical one-tail | 1.697260887 | |
| P(T<=t) two-tail | 0.023326624 | |
| t Critical two-tail | 2.042272456 | |

**(c) Paired T-Test**

A paired t-test is used to compare two population means where you have two samples in which observations in one sample can be paired with observations in the other sample.

Examples of where Paired T-Test are used:

1. Pre- and post- implementation measures on the same individual (e.g. one person fills in a survey (say using iPhone) in day one, and the same person fills in a second survey (says using iPad) in day two).
2. Cross-over trial
3. Matched pairs

Formula:

$$SE(\bar{d}) = \frac{S_d}{\sqrt{n}}$$

$$t = \frac{\bar{d} - 0}{SE(\bar{d})} = \frac{\bar{d}}{SE(\bar{d})} \; with \quad n-1 \quad df$$

Calculate:

The mean of the differences ($\bar{d}$)

And the standard deviation of the differences ($s_d$)

***/OR/***

$$t = \frac{\bar{d}}{\frac{S_d}{\sqrt{n}}} \qquad with \text{ n-1 } df$$

Where $\bar{d}$ = Mean Difference between the two samples

S = Standard Deviation of the differences

n = Number of Observations

**Step 1**

Calculate the difference between the two observations on each pair.

**Step 2**

Calculate the mean difference, i.e. $\bar{d}$.

**Step 3**

Calculate the Standard deviation of the differences, i.e. $s_d$.

| | iPhone/Class A | iPad/Class B | Differences |
|---|---|---|---|
| | 17.00 | 17.80 | -0.80 |
| | 17.10 | 17.80 | -0.70 |
| | 17.90 | 17.00 | 0.90 |
| | 17.70 | 18.60 | -0.90 |
| | 17.50 | 17.00 | 0.50 |
| | 16.70 | 17.70 | -1.00 |
| | 16.80 | 17.70 | -0.90 |
| | 17.20 | 18.10 | -0.90 |
| | 17.50 | 17.50 | 0.00 |
| | 17.60 | 17.50 | 0.10 |
| | 17.40 | 17.70 | -0.30 |
| | 17.00 | 17.40 | -0.40 |
| | 17.40 | 17.40 | 0.00 |
| | 16.90 | 17.40 | -0.50 |
| | 17.10 | 16.70 | 0.40 |
| | 16.80 | 17.80 | -1.00 |
| Mean | 17.2250 | 17.5688 | -0.3438 |
| StDev | 0.3568 | 0.4512 | 0.6000 |
| n | 16.00 | 16.00 | 16.00 |

**Step 4**

Calculate the standard error of the mean difference:

$$SE(\bar{d}) = \frac{S_d}{\sqrt{n}}$$

$$= \frac{0.6000}{\sqrt{16}}$$

**= 0.15**

**Step 5**

Calculate the t-value:

$$t = \frac{\bar{d}}{\frac{s_d}{\sqrt{n}}}$$

$$t = \frac{-0.3438}{0.15}$$

**t = -2.292**

**Step 6**

Degree of freedom = n - 1

df = 16 – 1

df = **15**

**Step 7**

In t-distribution table, find the critical value for the paired t-test

| | Tail probability $p$ | | | | | | |
|---|---|---|---|---|---|---|---|
| df | .25 | .20 | .15 | .10 | .05 | .025 | .02 |
| 1 | 1.000 | 1.376 | 1.963 | 3.078 | 6.314 | 12.71 | 15.89 |
| 2 | .816 | 1.061 | 1.386 | 1.886 | 2.920 | 4.303 | 4.849 |
| 3 | .765 | .978 | 1.250 | 1.638 | 2.353 | 3.182 | 3.482 |
| 4 | .741 | .941 | 1.190 | 1.533 | 2.132 | 2.776 | 2.999 |
| 5 | .727 | .920 | 1.156 | 1.476 | 2.015 | 2.571 | 2.757 |
| 6 | .718 | .906 | 1.134 | 1.440 | 1.943 | 2.447 | 2.612 |
| 7 | .711 | .896 | 1.119 | 1.415 | 1.895 | 2.365 | 2.517 |
| 8 | .706 | .889 | 1.108 | 1.397 | 1.860 | 2.306 | 2.449 |
| 9 | .703 | .883 | 1.100 | 1.383 | 1.833 | 2.262 | 2.398 |
| 10 | .700 | .879 | 1.093 | 1.372 | 1.812 | 2.228 | 2.359 |
| 11 | .697 | .876 | 1.088 | 1.363 | 1.796 | 2.201 | 2.328 |
| 12 | .695 | .873 | 1.083 | 1.356 | 1.782 | 2.179 | 2.303 |
| 13 | .694 | .870 | 1.079 | 1.350 | 1.771 | 2.160 | 2.282 |
| 14 | .692 | .868 | 1.076 | 1.345 | 1.761 | 2.145 | 2.264 |
| 15 | .691 | .866 | 1.074 | 1.341 | 1.753 | 2.131 | 2.249 |
| 16 | .690 | .865 | 1.071 | 1.337 | 1.746 | 2.120 | 2.235 |
| 17 | .689 | .863 | 1.069 | 1.333 | 1.740 | 2.110 | 2.224 |

**The critical value is 2.131**

**Step 8**

$H_0$: There is no difference between Sample A and Sample B

- If t-value is lower than the Critical Value, then do not Reject
- If t-value is higher than the Critical Value, then Reject

As the t-value (**2.292**) is higher than Critical Value (**2.131**), then (technically) **Reject the Null Hypothesis**.

| t-Test: Paired Two Sample for Means | | |
|---|---|---|
| | | |
| | *Sample A* | *Sample B* |
| Mean | 17.225 | 17.56875 |
| Variance | 0.127333333 | 0.203625 |
| Observations | 16 | 16 |
| Pearson Correlation | -0.090049519 | |
| Hypothesized Mean Difference | 0 | |
| df | 15 | |
| t Stat | -2.291799298 | |
| P(T<=t) one-tail | 0.018401372 | |
| t Critical one-tail | 1.753050356 | |
| P(T<=t) two-tail | 0.036802745 | |
| t Critical two-tail | 2.131449546 | |