CS 330 : Discrete Computational Structures

Cahlen Brancheau Homework 2

Fall Semester, 2015
ASSIGNMENT #2
Due Date: Friday, Sept 11

Suggested Reading: Rosen Sections 1.4 - 1.6; Lehman et al. Chapter 3

These are the problems that you need to turn in. For more practice, you are encouraged to work on the other problems. Always explain your answers and show your reasoning.

1. **[9 Pts]** Let $C(x)$ be the statement that x has a cat, let $D(x)$ be the statement that x has a dog, and let $P(x)$ be the statement that x has a parrot. Let the universe be all the students in class. Express each of these statements in terms of $C(x)$, $D(x)$ and $P(x)$.

   (a) Every student has a cat and either a dog or a parrot.
   Answer: $\forall x(C(x) \wedge (D(x) \vee P(x)))$

   (b) There is exactly one student who has both a dog and a parrot.
   Answer: $\exists x((D(x) \wedge P(x)) \wedge \forall z((z \neq x) \rightarrow \neg(D(z) \wedge P(z))))$

   (c) There are at least two students who have all three pets.
   Answer: $\exists x \exists y((x \neq y) \wedge (D(x) \wedge P(x) \wedge C(x)) \wedge (D(y) \wedge P(y) \wedge C(y)))$

2. **[5 Pts]** Are $\forall x(P(x) \leftrightarrow Q(x))$ and $\forall x P(x) \leftrightarrow \forall x Q(x)$ logically equivalent? If yes, give a proof. If no, give a counterexample.

$$\begin{array}{ll} Q(1) = T & Q(2) = F \\ P(1) = T & P(2) = F \end{array}$$

In $U = \{1, 2\}$ with predicates defined above $\forall x(P(x) \leftrightarrow Q(x))$ is a Tautology, but $\forall x P(x) \leftrightarrow \forall x Q(x)$ can be false. Specifically in the cases $\forall x P(1) \leftrightarrow \forall x Q(2)$ or $\forall x P(2) \leftrightarrow \forall x Q(1)$.

3. **[15 Pts]** Some players are placed on two teams A and B. Let all the players be your universe. Define predicates P, Q and R to represent the sentences in parts (a), (b) and (c). You can use the given predicates or any predicate you have already defined. You can use the predicate P to derive predicates Q and R, for example.

   Let $A(x)$ be the predicate that player x is placed on team A. Let $B(x)$ be the predicate that player x is placed on team B. $E(x, y)$ says that x was selected before y.

   (a) Student x was the first player chosen to Team A.
   Answer: $\exists x(A(x) \wedge \forall z(((x \neq z) \wedge A(z)) \rightarrow E(x, z))$

   (b) Student x was the last player to be picked to either team.
   Answer: $\exists x((A(x) \vee B(x)) \wedge \forall z((x \neq z) \rightarrow E(z, x))$

   (c) Student x was the second player chosen to Team A.
   Answer: $\exists x \exists y(((x \neq y) \wedge A(x) \wedge A(y) \wedge E(y, x)) \wedge \forall z(((z \neq x) \wedge (z \neq y) \wedge A(z)) \rightarrow E(x, z)))$

4. **[15 Pts]** Prove the following by defining predicates and using the appropriate rules of inference:

   (a) Jane, a student in class, knows JAVA. Everyone who knows JAVA will get a summer internship. Therefore, someone in class will get a summer internship.
   Answer:

$J(x)$: x knows Java
$C(x)$: x is in this class
$S(x)$: x has a summer internship
Hypothesis 1: $\forall x(J(x) \rightarrow S(x))$
Hypothesis 2: $C(Jane) \wedge J(Jane)$
Conclusion: $\exists x(C(x) \wedge S(x))$

	Step	Reason
1	$\forall x(J(x) \rightarrow S(x))$	Hypothesis 1
2	$C(Jane) \wedge J(Jane)$	Hypothesis 2
3	$C(Jane)$	Simplification (2)
4	$J(Jane)$	Simplification (2)
5	$J(Jane) \rightarrow S(Jane)$	Universal Instantiation (1)
6	$S(Jane)$	Modus Ponens (4), (5)
7	$C(Jane) \wedge S(Jane)$	Conjunction (3), (6)
8	$\exists x(C(x) \wedge S(x))$	Existential Generalization (7)

   (b) Steven Spielberg made a movie about Lincoln. All movies by Steven Spielberg win awards. Therefore, there is a movie about Lincoln that wins awards.
   Answer:

$U = \{$ All Movies $\}$
$S(x)$: x made by Spielberg
$L(x)$: x is about Lincoln
$A(x)$: x wins awards
Hypothesis 1: $\forall x(S(x) \rightarrow A(x))$
Hypothesis 2: $\exists x(S(x) \wedge L(x))$
Conclusion: $\exists x(L(x) \wedge A(x))$

	Step	Reason
1	$\forall x(S(x) \rightarrow A(x))$	Hypothesis 1
2	$\exists x(S(x) \wedge L(x))$	Hypothesis 2
3	$S(m) \wedge L(m)$	Existential Instantiation (2)
4	$L(m)$	Simplification (3)
5	$S(m)$	Simplification (3)
6	$S(m) \rightarrow A(m)$	Universal Instantiation (1)
7	$A(m)$	Modus Ponens (5), (6)
8	$L(m) \wedge A(m)$	Conjunction (4), (7)
9	$\exists x(L(x) \wedge A(x))$	Existential Generalization (8)

(c) Every computer science major takes discrete mathematics. Anyone who takes discrete mathematics understands logic. Anyone who understands logic can win arguments. Therefore, all computer science majors can win arguments.
Answer:

$U = \{$ All Students $\}$
$C(x)$: x is a CS student
$D(x)$: x has taken Discrete Maths
$L(x)$: x understands logic
$A(x)$: x wins arguments
Hypothesis 1: $\forall x(C(x) \rightarrow D(x))$
Hypothesis 2: $\forall x(Dx) \rightarrow L(x))$
Hypothesis 3: $\forall x(L(x) \rightarrow A(x))$
Conclusion: $\forall x(C(x) \rightarrow A(x))$

	Step	Reason
1	$\forall x(C(x) \rightarrow D(x))$	Hypothesis 1
2	$\forall x(D(x) \rightarrow L(x))$	Hypothesis 2
3	$\forall x(L(x) \rightarrow A(x))$	Hypothesis 3
4	$C(a) \rightarrow D(a)$	Universal Instantiation (1)
5	$D(a) \rightarrow L(a)$	Universal Instantiation (2)
6	$L(a) \rightarrow A(a)$	Universal Instantiation (3)
7	$C(a) \rightarrow L(a)$	Hypothetical Syllogism (4),(5)
8	$C(a) \rightarrow A(a)$	Hypothetical Syllogism (7),(6)
9	$\forall x(C(x) \rightarrow A(x))$	Universal Generalization (8)

5. **[6 Pts]** State whether the following arguments are correct. Explain your answer briefly.

Possibility	p	q	$p \to q$
1	0	0	T
2	0	1	T
3	1	0	F
4	1	1	T

(a) Everyone living in Texas believes in gun rights. Andrew believes in gun rights. Therefore, he lives in Texas.
Answer: p = you live in Texas. q = you believe in gun rights. If Andrew believes in gun rights then according to possibility 2 and 4 in the table above then Andrew may or may not live in Texas, so the argument is false. q implies nothing about p.

(b) All students at ISU live in dorms. Mary is not an ISU student. Therefore, Mary does not live in a dorm.
Answer: p = you are an ISU student. q = you live in the dorms. If Mary is not an ISU student then according to possibility 1 and 2 in the table above Mary may or may not live in the dorms, so the argument is false. $\neg p$ implies nothing about q.