CS 330 : Discrete Computational Structures
Fall Semester, 2015
ASSIGNMENT #11 [**Extra Credit**]
Due Date: Sunday, Dec 6

Suggested Reading: Rosen Sections 6.4 - 6.5

These are the problems that you need to turn in. Always explain your answers and show your reasoning. **Spend time giving a complete solution. You will be graded based on how well you explain your answers. Just correct answers will not be enough!**

1. [**5 Pts**] Prove, using a combinatorial argument, that $C(m + n, 2) = C(m, 2) + C(n, 2) + mn$, where $m, n \geq 2$.

2. [**10 Pts**] Prove that $C(n, 2k)C(2k, k) = C(n, k)C(n - k, k)$, where $n \geq 2k > 0$, by using (a) a combinatorial proof, (b) an algebraic proof.

3. [**5 Pts**] Prove, using a combinatorial argument, that

$$C(2n, n) = \sum_{k=0}^{n} [C(n, k)]^2.$$

Hint: Show that both sides count the number of ways to select a committee of size n from a group of n men and n women.

4. [**7 Pts**] A cookie shop sells 5 different kinds of cookies. How many different ways are there to choose 16 cookies if (a) you have no restrictions? (b) you pick at least two of each? (c) you pick at least 4 oatmeal cookies, at least 3 sugar cookies and at most 5 chocolate chip cookies?

5. [**4 Pts**] If I have 5 bananas, 3 oranges, and 8 apples, how many ways can I distribute these to 16 friends, if each friend gets one fruit?

6. [**9 Pts**] How many ways are there to pack 18 different books into 6 boxes with 3 books each if (a) all 6 boxes are sent to different addresses? (b) all 6 boxes are sent to the same address? (c) 3 of the boxes are shipped to three different addresses while 3 are left to be addressed later?

7. [**5 Pts**] How many ways are there to pack 5 different books into 5 identical boxes with no restrictions placed on how many can go in a box (some boxes can be empty)? What if the books are identical?

8. [**5 Pts**] How many ways can we place 8 books on a bookcase with 5 shelves if the books are (a) indistinguishable copies (b) all distinct? Note that the position of the books on the shelves matter.