CS 330 : Discrete Computational Structures
Fall Semester, 2015
ASSIGNMENT #9
Due Date: Tuesday, Nov 17
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Suggested Reading: Rosen Section 2.5; LLM Chapter 7.1

These are the problems that you need to turn in. For more practice, you are encouraged to work on the other problems. **Always explain your answers and show your reasoning.**

1. **[16 Pts]** Show that the following sets are countably infinite, by defining a bijection between $\mathcal{N}$ (or $\mathcal{Z}^+$) and that set. You do not need to prove that your function is bijective.

   (a) **[8 Pts]** the set of positive integers divisible by 5
   Answer:

   f: $Z^+ \to \mathcal{S}$, $f(x) = 5x$, which is a bijection.

   Onto: Assume $m \in \mathcal{S}$, so $m = 5k$ where $k \in \mathcal{Z}^+$. $f(k) = 5k = m$. $\square$
   One-To-One: Assume $f(m) = f(n)$, where $m, n \in \mathcal{Z}^+$, $5m = 5n \to m = n$. $\square$

   (b) **[8 Pts]** $\{1, 2, 3\} \times \mathcal{Z}$

2. **[16 Pts]** Determine whether the following sets are countable or uncountable. Prove your answer.
   Answer:

   (a) **[8 Pts]** the set of real numbers with decimal representation consisting of all 1's (1.11 and 11.111... are such numbers).
   Answer:

   Let A = the set defined above.

   First restrict the problem to the positive reals because there is a disjoint union of the positive and negative values and there is a bijection between them.

   Let S = the set of all positive values in A.

   $f : \mathcal{N} \times \mathcal{N} \to \mathcal{S} : (n, m) \to \sum_{i=1}^{n} 10^i + \sum_{i=1}^{m} \frac{1}{10^i}$

   (n, m) maps to n $1's$ to the left of the dot and m $1's$ of the right side. f is a bijection, $\mathcal{N} \times \mathcal{N}$ is countable and $\mathcal{S}$ is countable therefore A is also countable.

(b) **[8 Pts]** the set of real numbers with decimal representation consisting of 1's and 9's
Answer:

Let us enumerate the values between 0 and 1.

0.**1**111111191199...
0.9**1**19191199191...
0.19**9**9119199119...
0.199**1**111911911...
0.9191**9**19199111...
0.91919**1**1911199...
...

Taking the numbers bolded we get 0.119191. This could not appear in our list because it would differ by at least one digit. We could not possibly enumerate all of the values between 0 and 1, so this set in uncountable.

3. **[9 Pts]** Give an example of two uncountable sets A and B such that $A \cap B$ is (a) finite, (b) countably infinite, (c) uncountably infinite.
Answer:

1)

Let $A = \{1 \leq x \leq 2 | x \in \mathcal{R}\}$
Let $B = \{2 \leq x \leq 3 | x \in \mathcal{R}\}$

A and B are uncountable and $A \cap B = 2 \rightarrow$ finite. $\square$

2)

Let $A = \mathcal{Q} \cup \mathcal{R}^-$
Let $B = \mathcal{Q} \cup \mathcal{R}^+$

A and B are uncountable and $A \cap B = \mathcal{Q} \rightarrow$ countably infinite. $\square$

3)

Let $A = \{0 \leq x \leq 2 | x \in \mathcal{R}\}$
Let $B = \{1 \leq x \leq 3 | x \in \mathcal{R}\}$

A and B are uncountable and $A \cap B = [0, 1] \rightarrow$ uncountably infinite. $\square$

4. [**9 Pts**] Prove that the set of functions from $\mathcal{N}$ to $\{0,1\}$ is uncountable, by using a diagonalization argument.
Answer:

Assume $\mathcal{N} \rightarrow \{0,1\}$ is countable and can be enumerated by $\mathcal{S} = \{f_1, f_2, ...\}$.

$f(x) = 0$ if $f_x(x) = 1$, 1 if $f_x(x) = 0$.

So $f \neq f_x$ for any value x, because f differs from f_x in the nth position. The enumeration of the functions cannot completely list all of the functions. $\square$