HW 7 Com S 330

a) Prove by MI $5^i \in S$

Base: $k=1$, $5^1 = 5 \in S$ by the base definition of S.

Inductive: Assume 5^k for $k \in N$, we need to prove $5^{k+1} \in S$

We know $5 \in S$ from the base case and 5^k from our inductive hypothesis.

We can form $5^{k+1} = 5^k \cdot 5 \in S$.

) We know $5 \in S$ and 5 is a power of 5. This means $5 \in A$.

Inductive: Let $a, b \in S$. Using IH, assume $a, b \in A$.

We also know $a \cdot b \in S$. We need to prove $a \cdot b \in A$.

Since $a, b \in A$, $a = 5^x$ and $b = 5^y$ $x, y \in Z^+$

So, $a \cdot b = 5^x \cdot 5^y = 5^{x+y}$. Since x, y are integers, this proves $a, b \in A$. So $S \subseteq A$.

Bit strings are strings over the alphabet $\Sigma = \{0,1\}$

Base: ε (empty string) is a palindrome

Inductive: If w is a pal. and x is a bit ($x \in \Sigma$), then xwx is a pal.

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a) base: $(0,0) \in L'$

Recursive: if $(x,y) \in L'$, then $(a-1, b-1)$, $(a+1, b+1)$ $(a, b+3)$, $(a, b-3)$ $\in L'$

b) Every ordered pair (a,b) by the ind. def. of L holds the property $a-b$ is divisible by 3.

<u>Base</u> $0-0=3(0)$

We can make:

$a-b=3k$

$(a-1,b-1)$, $(a+1,b+1)$, $(a,b+3)$, $(a,b-3)$

$(a-1)-(b-1)$ and $(a+1)-(b+1)$ both $=a-b$ which is divisible by 3.

$a-(b+3)=3k-3=3(k-1)$, divisible by 3.

$a-(b-3)=3k+3=3(k+1)$, divisible by 3.

Each case, the recursive step produces ordered pairs that satisfy L.

c) Let $(a,b) \in L$ and $a-b=3k$. Let $b=-3k+a$