CS 330 : Discrete Computational Structures

Fall Semester, 2015
ASSIGNMENT #1 SOLUTIONS

1. **[Ghazaleh 10 Pts]** Let p, q and r be the propositions "It is snowing", "It is freezing" and "Classes are canceled" respectively. Express each of these propositions as an English sentence.

   (a) $p \wedge \neg q$ **it's snowing and it's not freezing.**

   (b) $\neg p \to \neg q$ **if it's not snowing then it's not freezing**

   (c) $p \vee q \to r$ **if it's snowing or it's freezing then classes are canceled.**

   (d) $p \leftrightarrow q$ **it's snowing if and only if it's freezing.**

   (e) $(p \wedge \neg r) \to \neg q$ **if it's snowing and classes are not canceled then it's not freezing.**

2. **[Ghazaleh 10 Pts]** Write each of these statements in the form "if p then q".

   (a) It is necessary to attend all lectures to get an A in the class.
   If you got an A in the class then you have attended all the lectures.

   (b) Ann will get caught whenever she cheats.
   If Ann cheats then Ann will get caught.

   (c) You will finish the race unless it gets too hot.
   If it doesn't get too hot then you will finish the race. OR if you won't finish the race then it was too hot.

   (d) Eating candy everyday is a sufficient condition for getting tooth decay.
   If you eat candy everyday then you will get tooth decay.

   (e) I will miss lecture only if I am sick.
   If I miss the lecture then I'm sick. OR if I'm not sick then I wont miss the lecture.

3. **[Ghazaleh 5 Pts]** Show that $(p \vee q) \wedge (\neg p \vee r) \to (q \vee r)$ is a tautology.

p	q	r	$(p \vee q)$	$(\neg p \vee r)$	$(p \vee q) \wedge (\neg p \vee r)$	$(q \vee r)$	$(p \vee q) \wedge (\neg p \vee r) \to (q \vee r)$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T
T	F	T	T	F	F	T	T
T	F	F	T	F	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T
F	F	T	F	T	F	T	T
F	F	F	F	T	F	F	T

4. **[Peter 15 Pts]** On an island, there are three kinds of people, knights, knaves and spies. Knights always tell the truth, knaves always lie and spies can do either. You meet three people, A, B and C, where one is a knight, one is a knave and one is a spy. For the following problems, state all solutions (there may be no unique solution) or state that there are no solutions. Explain your reasoning.

   (a) A says 'I am the knight', B says 'I am the knave' and C says 'I am the spy'.

   Neither B nor C can be the knight, since they would be lying, so A must be the knight. B can't be the knave, because then B would be telling the truth, so B must be the spy. The only role left for C is the knave.
   A: Knight B: Spy C: Knave

   (b) A says 'I am the knave', B says 'A is not a knave' and C says 'I am a knight'.

   A can't be the knave, because then A would be telling the truth; A can't be the knight, because then A would be lying. Thus, A is the spy. B is telling the truth, so B can't be the knave; since the spy is taken, B must be the knight. The only role left for C is the knave.
   A: Spy B: Knight C: Knave

   (c) A says 'C is the spy', B says 'I am not the spy' and C says 'I am not the spy'.

   Neither B nor C can be the knave, as they would then be telling the truth, so A must be the knave. This means the statement "C is the spy" is a lie. Since the knave is taken, the only role left for C is the knight, which means B is the spy.
   A: Knave B: Spy C: Knight

5. **[Peter 10 Pts]** Prove that $(p \to r) \wedge (q \to r)$ and $(p \vee q) \to r$ are logically equivalent by (a) truth tables, and (b) by deduction using the logical equivalences studied in class (don't use this logical equivalence!).

a.

p	q	r	$(p \to r) \wedge (q \to r)$	$(p \vee q) \to r$
T	T	T	T	T
T	T	F	F	F
T	F	T	T	T
T	F	F	F	F
F	T	T	T	T
F	T	F	F	F
F	F	T	T	T
F	F	F	T	T

b.

$$\begin{aligned}
(p \to r) \wedge (q \to r) &\equiv (\neg p \vee r) \wedge (\neg q \vee r) && \text{Logical equivalence of implication} \\
&\equiv (\neg p \wedge \neg q) \vee r && \text{Distributive property} \\
&\equiv \neg(p \vee q) \vee r && \text{DeMorgan's Law} \\
&\equiv (p \vee q) \to r && \text{Logical equivalence of implication}
\end{aligned}$$