CS 330 : Discrete Computational Structures
Fall Semester, 2015
ASSIGNMENT #8
Due Date: Tuesday, Nov 3
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Suggested Reading: Rosen 9.1 and 9.5, LLM 9.4

These are the problems that you need to turn in. For more practice, you are encouraged to work on the other problems. **Always explain your answers and show your reasoning.**

1. **[18 Pts]** For each of these relations decide whether it is reflexive, anti-reflexive, symmetric, anti-symmetric and transitive. Justify your answers. R_1 and R_2 are over the set of real numbers and R_3 is over the set of all people.

   (a) $(x, y) \in R_1$ if and only if $xy = 4$
   Reflexive: No. $(1, 1) \notin R_1$
   Anti-Reflexive: No. $(2, 2) \in R_1$
   Symmetric: Yes. $xy = 4 \leftrightarrow xy = 4$ therefore $(x, y) \in R_1 \rightarrow (y, x) \in R_1$
   Anti-Symmetric: No. Because Symmetry
   Transitive: No. $(1, 4) \in R_1$ and $(4, 1) \in R_1$ but $(1, 1) \notin R_1$

   (b) $(x, y) \in R_2$ if and only if $x = 1$ or $y = 1$
   Reflexive: No. $(5, 5) \notin R_2$
   Anti-Reflexive: No. $(1, 1) \in R_2$
   Symmetric: Yes. $(x, y) \in R_2 \rightarrow (y, x) \in R_2$
   Anti-Symmetric: No. $(2, 1) \in R_2$ and $(1, 2) \in R_2$
   Transitive: No. $(2, 1) \in R_2$ and $(1, 2) \in R_2$ but $(2, 2) \notin R_2$

   (c) $(x, y) \in R_3$ if and only if x and y share a common biological parent
   Reflexive: Yes. Trivially true. $(x, x) \in R_3$
   Anti-Reflexive: No. $(x, x) \in R_3$
   Symmetric: Yes. $(x, y) \in R_3 \rightarrow (y, x) \in R_3$
   Anti-Symmetric: No. Because symmetry
   Transitive: No. Bob and Linda have baby a. Linda and Steve have baby b. Steve and Beth have baby c. $(a, b) \in R_3$ and $(b, c) \in R_3$ but $(a, c) \notin R_3$

2. **[12 Pts]** Consider relation R_4 on the set of positive real numbers where $(x, y) \in R_4$ if and only if $x/y \in \mathcal{Q}$. Decide whether it is (a) reflexive, anti-reflexive, symmetric, anti-symmetric and transitive and (b) show that this an equivalence relation. Describe the equivalence classes. (c) What is the equivalence class of 2? of π? Justify your answers.

   (a) **Reflexive**: Yes. $\frac{a}{a} = 1$ and $1 \in \mathbb{Q}$ therefore $(a, a) \in R_4$
   Anti-Reflexive: No. Because reflexivity
   Symmetric: Yes. $\forall x, y \in \mathbb{R}^+, \frac{x}{y} \in \mathbb{Q} \to \frac{y}{x} \in \mathbb{Q}$ therefore $(x, y) \in R_4$ and $(y, x) \in R_4$
   Anti-Symmetric: No. Because Symmetry
   Transitive: Yes. $(x, y) \in R_4 \to (y, x) \in R_4$ and $(x, x) \in R_4$

   (b) An equivalence relation is defined as a relation that is **Reflexive**, **Symmetric**, and **Transitive**. R_4 is all three. R_4 is an equivalence relation.

   (c) $[2]_R = \{\forall n \in \mathbb{R}+ | \frac{2}{n} or \frac{n}{2}\}$, $[\pi]_R = \{\forall n \in \mathbb{R}+ | \frac{\pi}{n} or \frac{n}{\pi}\}$

3. **[12 Pts]** Let R_5 be the relation on $\mathbb{Z} \times \mathbb{Z}$ where $((a, b), (c, d)) \in R_5$ if and only if $a - c = b - d$.

   (a) Prove that R_5 is an equivalence relation.

   Reflexive: Yes. $a - a = b - b \to ((a, b)(a, b)) \in R_5$
   Symmetric: Yes. $a - c = b - d \leftrightarrow c - a = d - b$ therefore $((a, b)(c, d)) \in R_5$ and $((c, d)(a, b)) \in R_5$
   Transitive: Yes. $((a, b)(c, d)) \in R_5$ and $((c, d)(e, f)) \in R_5$ gives us $a - c = b - d$ and $c - e = d - f$. $(a - c) - (c - f) = (b - d) - (d - f)$ which implies that $a - e = b - f$ therefore $((a, b)(e, f)) \in R_5$

   (b) Define a function f such that $f(a, b) = f(c, d)$ if and only if $((a, b), (c, d)) \in R$.

   $f(x, y) = x - y$

   (c) Define the equivalence class containing $(1, 1)$ and list 2 elements in the class.

   $[(1, 1)] = \{(x, x) | x \in \mathbb{Z}\}$

   Elements: $(2, 2)$, $(3, 3)$

   (d) Describe each equivalence class. How many classes are there and how many elements in each class?

   There are a countably infinite number of classes each with a countably infinite number of elements.

4. **[4 Pts]** Describe all the equivalence classes for the relation *congruence modulo 6* over $\mathcal{Z}$, using set-builder notation.
$[0] = \{6n | n \in \mathbb{Z}\}$
$[1] = \{6n + 1 | n \in \mathbb{Z}\}$
$[2] = \{6n + 2 | n \in \mathbb{Z}\}$
$[3] = \{6n + 3 | n \in \mathbb{Z}\}$
$[4] = \{6n + 4 | n \in \mathbb{Z}\}$
$[5] = \{6n + 5 | n \in \mathbb{Z}\}$

5. **[4 Pts]** LLM Problem 9.9 (c)
8.01 $\rightarrow$ 8.02 $\rightarrow$ 18.01 $\rightarrow$ 18.02 $\rightarrow$ 6.042 $\rightarrow$ 6.046 $\rightarrow$ 6.840 $\rightarrow$ 18.03 $\rightarrow$ 6.002 $\rightarrow$ 6.001 $\rightarrow$ 6.034 $\rightarrow$ 6.003 $\rightarrow$ 6.004 $\rightarrow$ 6.033 $\rightarrow$ 6.857