CS 330 : Discrete Computational Structures

Fall Semester, 2015

ASSIGNMENT #4

Due Date: Sunday, Sept 27

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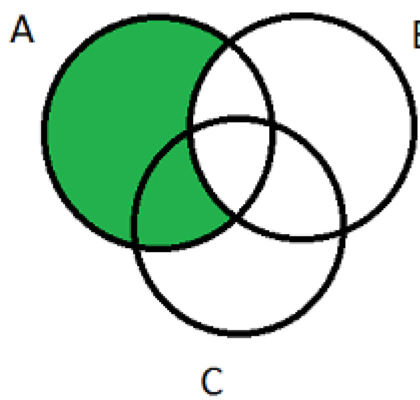
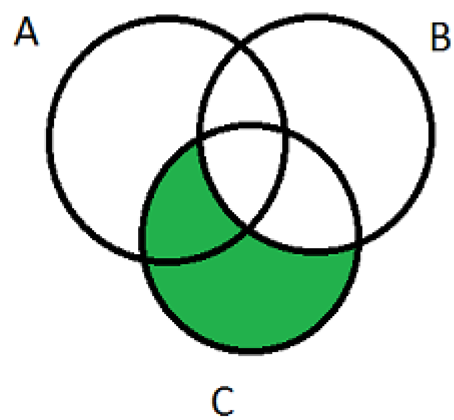
Suggested Reading: Rosen Sections 2.2 - 2.3; Lehman et al. Chapter 4.1, 4.3, 4.4

These are the problems that you need to turn in. For more practice, you are encouraged to work on the other problems. Always explain your answers and show your reasoning.

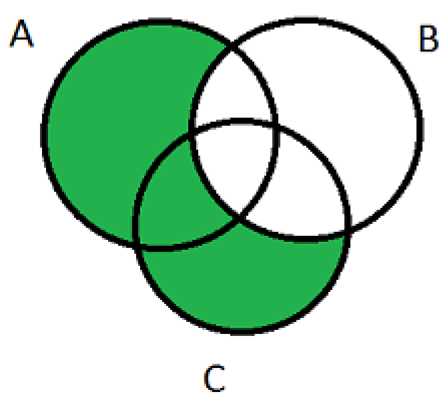
1. **[4 Pts]** Use Venn Diagrams to prove that $(A - B) \cup (C - B) = (A \cup C) - B$.
ANSWER:



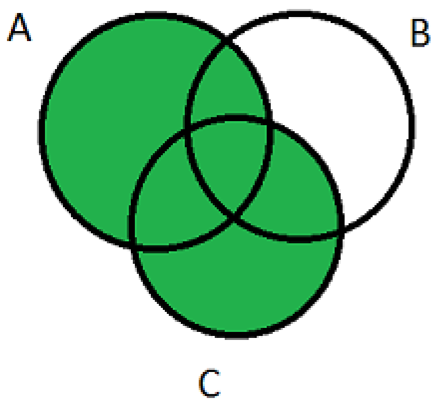
A-B



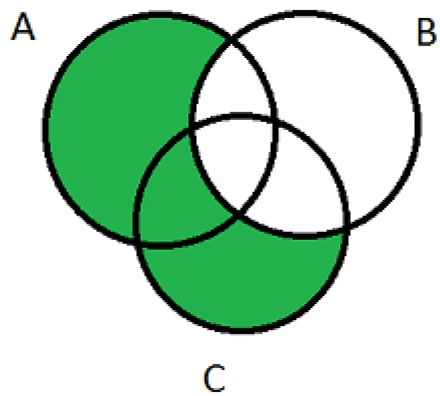
C-B



(A-B)U(C-B)



AUC



(AUC)-B

2. **[4 Pts]** Use an iff argument to prove that $(A - B) \cup (C - B) = (A \cup C) - B$. You may use logical equivalences in your proof.
ANSWER:

$$
\begin{array}{llll}
x \in (A \cup C) - B & \leftrightarrow & x \in (A \cup C) \cap \overline{B} & \text{Def of Subtraction} \\
 & \leftrightarrow & x \in (A \cup C) \wedge \overline{B} & \text{Def of } \cap \\
 & \leftrightarrow & x \in (A \cup C) \wedge \neg B & \text{Def of Compliment} \\
 & \leftrightarrow & x \in (A \vee C) \wedge \neg B & \text{Def of } \cup \\
 & \leftrightarrow & x \in (A \wedge \neg B) \vee (C \wedge \neg B) & \text{Def of Distributive} \\
 & \leftrightarrow & x \in (A \wedge \neg B) \cup (C \wedge \neg B) & \text{Def of } \cup \\
 & \leftrightarrow & x \in (A \wedge \overline{B}) \cup (C \wedge \overline{B}) & \text{Def of Compliment} \\
 & \leftrightarrow & x \in (A \cap \overline{B}) \cup (C \cap \overline{B}) & \text{Def of } \cap \\
 & \leftrightarrow & x \in (A - B) \cup (C - B) & \text{Def of Subtraction}
\end{array}
$$

3. **[4 Pts]** Prove by contradiction that $(A - C) \cap (C - B) = \emptyset$.
ANSWER:

Suppose $(A - C) \cap (C - B) \neq \emptyset$. So $x \in (A - C) \cap (C - B)$. Which means $x \in A$ and $x \notin C$ also $x \in C$ and $x \notin B$, contradiction. x cannot both be in and not in C. Therefore $(A - C) \cap (C - B) = \emptyset$.

4. [**6 Pts**] Disprove the statements below.

   (a) If $A \cup C = B \cup C$ then $A = B$.

   ANSWER:
   $A = \{1, 2, 3\}$
   $B = \{1, 2, 3, 4\}$
   $C = \{1, 2, 3, 4\}$

   $A \cup C = \{1, 2, 3, 4\}$
   $B \cup C = \{1, 2, 3, 4\}$
   $A \neq B$

   (b) If $A \cap C = B \cap C$ then $A = B$.

   ANSWER:
   $A = \{a, b, c\}$
   $B = \{a, b\}$
   $C = \{a, b\}$

   $A \cap C = \{a, b\}$
   $B \cap C = \{a, b\}$
   $A \neq B$

5. [**6 Pts**] Prove that if $A \cup C = B \cup C$ and $A \cap C = B \cap C$ then $A = B$.

   ANSWER:
   If $A = \{a, b\}$, $B = \{a, b\}$, $C = \{b, c\}$
   Then $A \cup C = \{a, b, c\}$ and $B \cup C = \{a, b, c\}$
   Then $A \cap C = \{b\}$ and $B \cup C = \{b\}$
   Therefore $A = B$

6. **[8 Pts]** Prove by subset argument (in both directions) that $\overline{A \cup B} = \overline{A} \cap \overline{B}$. You *may not* use logical equivalences in your proof.

ANSWER:
(1) $\overline{A \cup B} \subseteq \overline{A} \cap \overline{B}$
(2) $\overline{A} \cap \overline{B} \subseteq \overline{A \cup B}$

(1) Let $x \in \overline{A \cup B}$. We prove $x \in \overline{A} \cap \overline{B}$.
So we have $x \notin A \cup B$ by the definition of complement.
(a) Suppose $x \in A$ then $x \in A \cup B$, contradiction. So $x \in \overline{A}$.

(b) Suppose $x \in B$ then $x \in A \cup B$, contradiction. So $x \in \overline{B}$.
By definition of $\cap$ we have $\overline{A} \cap \overline{B}$.
Therefore $\overline{A \cup B} \subseteq \overline{A} \cap \overline{B}$

(2) Let $x \in \overline{A} \cap \overline{B}$. We prove $x \in \overline{A \cup B}$.
So we have $x \in \overline{A}$ and $x \in \overline{B}$ by the definition of $\cap$
and we have $x \notin A$ and $x \notin B$ by the definition of complement
Suppose $x \notin \overline{A \cup B}$ then by definition of complement we have $x \in A \cup B$
(a) $x \in A$, contradiction. $x \notin$ A
(B) $x \in B$, contradiction. $x \notin B$
Therefore $\overline{A \cup B}$.

7. **[4 Pts]** Consider the function f mapping $\mathcal{R}$ to $\mathcal{R}$, where $f(n) = 3n^2 - 8$.

   (a) Explain why f is neither one-to-one nor onto.
   ANSWER:
   $f(1) = 3(1)^2 - 8 = -5$
   $f(-1) = 3(-1)^2 - 8 = -5$
   but $1 \neq -1$ therefor f is not One-To-One

   (b) Now, restrict either the domain or co-domain to make f one-to-one.
   ANSWER:
   Restricting the domain to map from $\mathcal{R}^+$ to $\mathcal{R}$ removes negative numbers making f One-To-One.

   (c) Then, restrict either the domain or co-domain to make f onto.
   ANSWER:
   Restricting the co-domain to map from $\mathcal{R}^+$ to $\mathcal{R}^+$ removes negative numbers that cannot be mapped from the domain making f Onto.

8. **[4 Pts]** Prove that $f(n) = 3n + 5$ is one-to-one, where the domain and co-domain of f is $\mathcal{Z}^+$. Show that f is not onto.
   ANSWER:

$$\begin{aligned} 3x + 5 &= 3y + 5 \\ 3x &= 3y \\ x &= 3y + 5 \end{aligned}$$

Therefore f is One-To-One

$$\begin{aligned} 3x + 5 &= 1 \\ 3x &= -4 \\ x &= \tfrac{-4}{3} \end{aligned}$$

$1 \in \mathcal{Z}^+$ therefore in the co-domain, but $\frac{-4}{3} \notin \mathcal{Z}^+$ therefore not in the domain. So there does not exist an x that maps to 1. So f is not Onto.

9. **[4 Pts]** Prove that $f(m,n) = m + n - 2$ is onto, where the domain of f is $\mathcal{Z} \times \mathcal{Z}$ and the co-domain of f is $\mathcal{Z}$. Show that f is not one-to-one.
   ANSWER:
   $f(2,-2) = f(-2,2) = -2$ but $(2,-2) \neq (-2,2)$ therefore f is not Ont-To-One

   Let $t = m + n - 2$. Find $m, n \in \mathcal{Z}$ S.T. $m + n - 2 = t$
   $x = t$, $y = 2$
   $x = 2$, $y = t$
   Therefore f is Onto

10. **[6 Pts]** Prove that $f(m,n) = (m+n, m-n)$ is one-to-one and onto, where the domain and co-domain of f is $\mathcal{R} \times \mathcal{R}$. Give the inverse function of f.