Brancheau CS331 - HW1

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1)

If $A = \{0^n 1^n \mid n \in \mathbb{N}\}$, then $A^* = A$

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false. Counter example:

000111 and 01 are both elements in A and so the combination of the two 01000111 are in A^*, but this new string does not the critiria for elements in A. So A^* cannot equal A. □

2)

If $B = \{x \in \{0,1\}^* \mid \#(0,x) = \#(1,x)\}$, then $B^* = B$.

B is all strigs with equal #'s of 1's and 0's irrespective of position in the string. Given this $B^* = B$ is trivially true. Taking any number of strigs with equal number of 1's and 0's and concatenating them together will result in a new string also containing an equal number of 1's and 0's

3)

Prove $\sum_{k=1}^{n} \frac{1}{k^2} \leq 2 - \frac{1}{n}$

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Base case: 1

$$\sum_{k=1}^{1} \frac{1}{k^2} \leq 2 - \frac{1}{n} = \frac{1}{1} \leq 2 - \frac{1}{1} = 1 \leq 1$$

Inductive Step:

assume $\sum_{k=1}^{n+1} \frac{1}{k^2} \leq 2 - \frac{1}{(n+1)}$

You don't assume this... This is what your trying to prove...

by IH $2 - \frac{1}{n} + \frac{1}{(n+1)^2} \leq 2 - \frac{1}{(n+1)}$

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$$-\frac{1}{2} + \frac{1}{(n+1)^2} \leq 2 - \frac{1}{(n+1)}$$

$$\frac{-n^2 - n - 1}{n(n+1)^2} \leq \frac{-1}{n+1}$$

$$-n^2 - n - 1 \leq -n(n+1)$$

$$-1 \leq 0 \quad \square$$

What are you trying to show here?

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If A^* contains every possible combination of the elements in A from λ to strings whose length continues to ∞, then any elts in A^{**} would necessarily already exist in A^*.

so $A^{**} = A^*$ $\square$

Why? Need more explanation...

5) $S = \{0,1\}$, $T \subseteq \{0,1\}^*$

$S^* = T^* \Rightarrow S \subseteq T$

for $T \subseteq \{0,1\}^*$ and $S^* = T^*$ then T must contain at least $\{0,1\}$, so $S \subseteq T$ holds $\square$

why? You don't provide a proof.

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6)

$\{0,1\} \subseteq S \subsetneq T \subseteq \{0,1\}^*$

where

$S = \{0,1\}$ and $T = \{0,1,01\}$ $\square$?

7)

for S to be prefix repetitive then the sequence must start with two identically fliped sub sequences.

Let $P(n)$ = Probability of two fliped sequences of size n.

Then,

$P(1) = 1/4 = 0.25$

$P(2) = 1/16 = 0.0625$

$P(3) = 1/64 = 0.015625$

$P(4) = 1/256 = 0.00390625$

$P(5) = 1/1024 = 0.0009765625$

If w is infinitly long then the probability of s being Prefix repetitive is so small that it is functionally 0. $\square$

8)

I dont know how to do this one

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