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CS331 HW11

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82)

Let $c \in \mathbb{N}$, Choose $m \in \mathbb{Z}^+$ st $C(1^m) > c$,
and let $x = 0^m$ then,

$$C(Y^A_{x,1}) = C(1^m) > c + \log(1) \quad \square$$

83)

$$A = \{ w \in \{0,1\}^* \mid |w| = n^2, n \in \mathbb{N} \}$$

$K^2 \in A$, next elt in $A = (K+1)^2 = K^2 + 2K + 1$.

Let $c \in \mathbb{N}$, Let $m \in \mathbb{Z}^+$ st $C(0^{2m+1}) > C + \log 2$, Let $x = 0^{m^2}$

then

$$C(Y^A_{x,2}) = C(0^{2m+1}) > c + \log(2) \quad \square$$

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81) There are infinitly many x such that

$$T(c(x)) < |x|$$

$$M(S_n) = 0^{T(n)}$$

$$c(0^{T(n)}) = C_m(0^{T(n)}) + \tilde{c}_m, \text{ by optimality}$$
$$\leq |S_n| + \tilde{c}_m$$
$$= \lfloor \log(n+1) \rfloor + \tilde{c}_m$$

Therfore

$$T(c(0^{T(n)})) \leq T(\lfloor \log(n+1) \rfloor + \tilde{c}_m)$$

and

$$|0^{T(n)}| = T(n)$$

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Prove

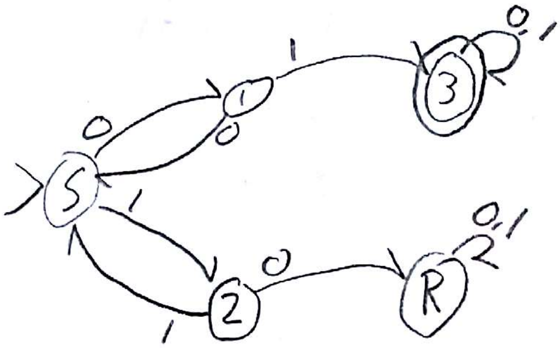
$$T(\lfloor \log(n+1) \rfloor + \tilde{c}_m) < T(n)$$

T is a strictly increasing function, a $|n|$ grows at a faster rate than $|\lfloor \log(n+1) \rfloor|$, so given an arbitrary $\tilde{c}_m$, $T(\lfloor \log(n+1) \rfloor + \tilde{c}_m) < T(n)$ will eventually become true, and will remain so for every n after that point. $\square$

86)

B is regular $(00+11)^*01(0+1)^*$ 20/20

PF



$\square$

87)

A is nonregular

Let $c \in \mathbb{N}$, Let $m \in \mathbb{Z}^+$ st $C(10^m) > c$, Let $x = 0^m$, then

$C(Y^A_{x,1}) = C(10^m) > c + \log(1)$ $\square$

88)

a)

$|PAL \cap \{0,1\}^n| = 2^{\lfloor \frac{n+1}{2} \rfloor}$

Case 1: n is odd

$xax^R \in PAL \cap \{0,1\}^n$, where $x \in \{0,1\}^*$, $a \in \{0,1\}$, and $n = 2|x|+1$.

xax^R can be arranged $2^{\frac{n-1}{2}} \cdot 2^1$ ways, becaus x^R depends only on x.

$$2^{\frac{n-1}{2}+1} = 2^{\frac{n-1}{2}+\frac{2}{2}} = 2^{\frac{n+1}{2}} = 2^{\lfloor \frac{n+1}{2} \rfloor}$$

case 2: n is even

$xx^R \in PAL \cap \{0,1\}^n$, where $x \in \{0,1\}^*$ and $n = 2|x|$

xx^R can be arranced $2^{\frac{n}{2}}$ ways, because x^R depends only on x.

$$2^{\frac{n}{2}} = 2^{\lfloor \frac{n+1}{2} \rfloor}$$, because n is even □

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b)

I don't know

84)

I don't know

85)

I don't know 6/20