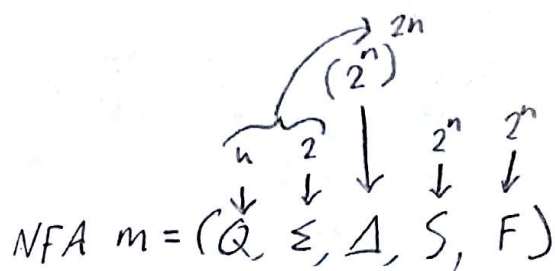


25)



$$(2^n)^{2^n} \cdot 2^n \cdot 2^n = 2^{2^n} (2^n)^{2^n}$$

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26)

Prove that there is a lang $A \subseteq \{0,1\}^*$ with both of the following properties:

i) For all $x \in A$, $|x| \leq 5$.

ii) Every NFA that decides A has more than 5 states.

$$|\mathcal{P}(\{0,1\}^{\leq 5})| = 2^{63}$$

Based on 25 the number of NFA's that decide A is

$$(2^5)^{2 \cdot 5} \cdot 2^{2 \cdot 5} = 2^{60}$$

There aren't enough NFA's with 5 states to go around, therefore

The number of states must be ≥ 6 because

$$(2^6)^{2 \cdot 6} \cdot 2^{2 \cdot 6} = 2^{84} \text{ and } 2^{84} > 2^{63}$$

27)

Every finite lang has some n number of strings. To prove a finite lang is regular you create n NFAs, one for each string in the lang, and each only accepting one string in the lang. The n NFA's together make one big NFA that decides the lang. An NFA can have multiple start states so each will run concurrently either one of them will be in an accept state at the end, accepting the string. Or none will, rejecting the string.

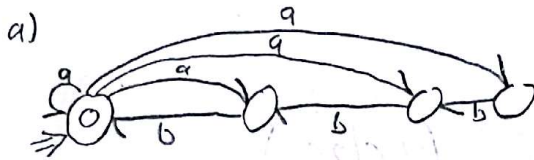
20/20

28)

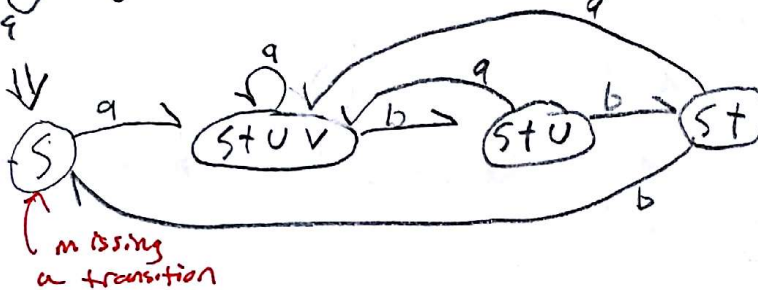
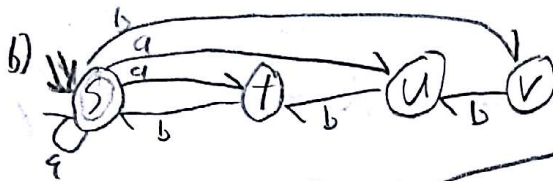
$$01||10 = \{0110, 0101, 1001, 1010\}$$

$$(01)^* || (10)^* = \{10, 01\}^*$$

30)



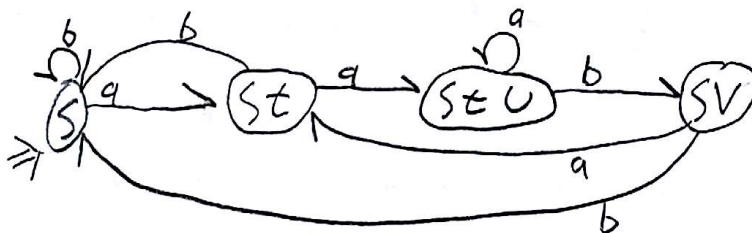
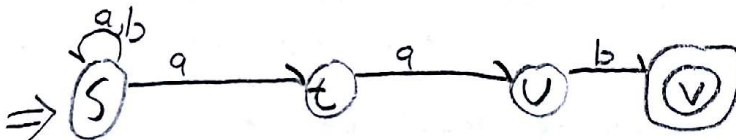
abbbb starts with a and is also rejected



$$F = \{S, ST, STU, STUV\}$$

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31)



$$F = \{SV\}$$

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An NFA with K states that accepts at least one string must have a path that leads from the start state to the accepting state(s) based on graph theory the longest path through a graph visits every node through $n-1$ vertices where n is the number of nodes in the graph. Therefore if an NFA has K states then the longest direct path from the start state to the accept state is $\leq K-1$.

If the number of transitions at the end of a string $\geq K$, then it can be known there were loops in the string, and therefore there must exist a string that takes the same path without the loops.

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29)

I don't know

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