Com S 331

Spring, 2016

Final Exam

Name Cahlen Brancheau

This is a closed-book, closed-notes, no-calculator, no-cellphone, individual-effort examination. All answers should be explained, at least briefly. Please do all your work on these pages.

There are ten problems of equal weight.

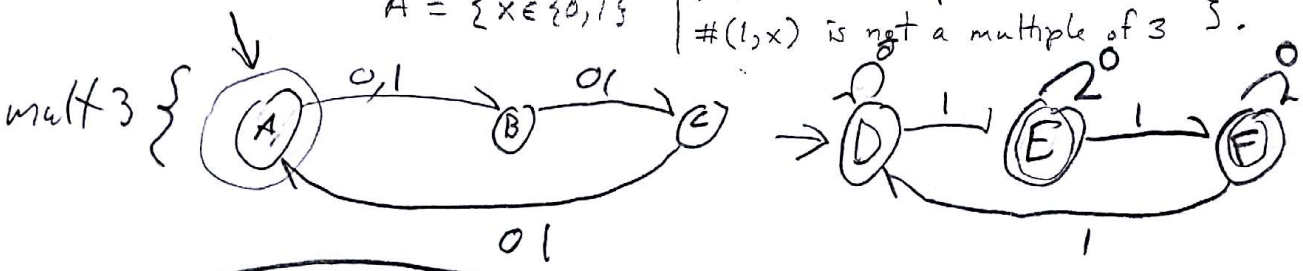
1	30
2	22
3	30
4	16
5	30
6	9
7	9
8	9
9	7
10	30
Total	162

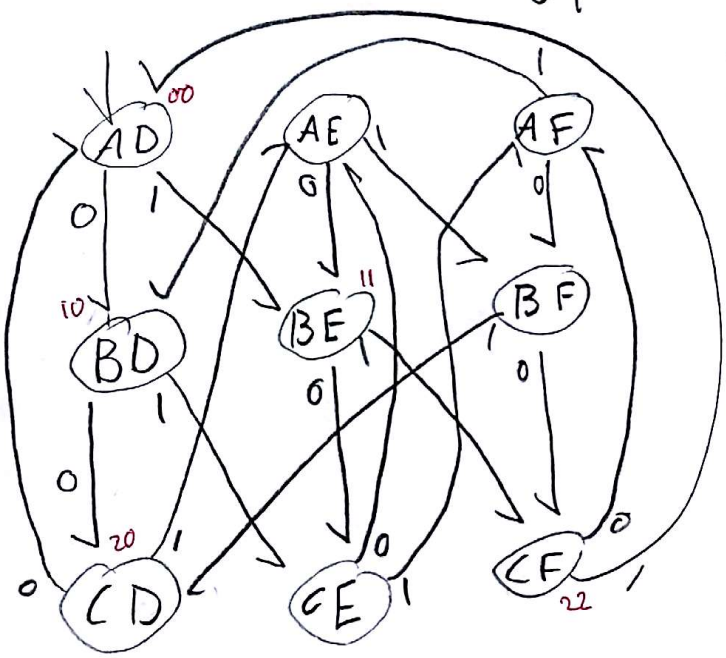
This is the best class I have ever taken! Thank you for being a great Prof, and Thank you Adam for being a great TA!

Name Cahlen Branchcau

1. Design a DFA that decides the language

$$A = \{x \in \{0,1\}^* \mid |x| \text{ is a multiple of 3 and } \#(1,x) \text{ is not a multiple of 3}\}.$$





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	0	1
→ AD	BD	BE
BD	CD	CE
BE	CE	CF
CE	AE	AF
CF	AF	AD
(AE)	BE	BF
(AF)	BF	BD
BF	CF	CD
CD	AD	AE

Accept = {AE, AF}

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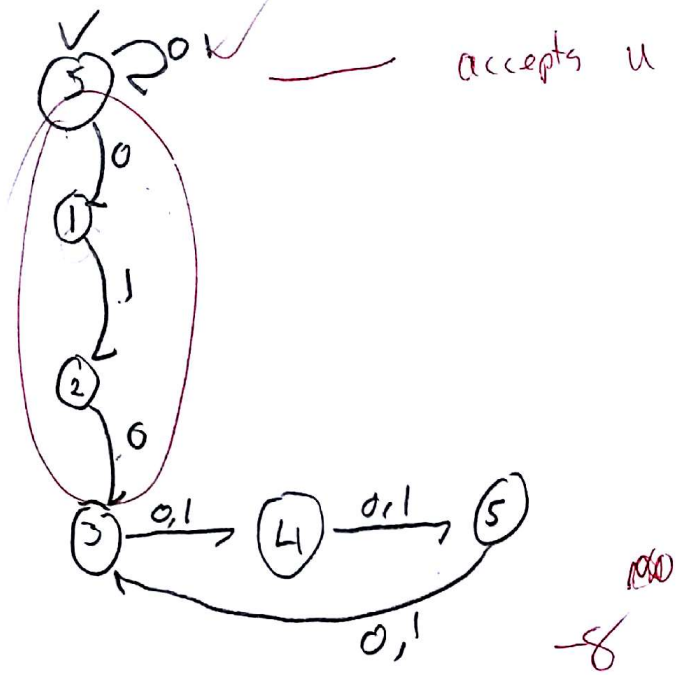
2. Design an NFA that decides the language

$$B = \{ u101v \mid u \in \{0,1\}^* \text{ and } v \in \{0,1\}^3 \}.$$



accepts u correctly

loop v

Accept = {3, 5}

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3. Prove: If α and β are regular expressions, then there is a regular expression r such that $L(r) = L(\alpha) \setminus L(\beta)$.

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α and β can be converted to DFA's and a product construction can be done with them where only the state's that contain an accept state from α are accept states in the new DFA. any state containing an accept state from β will not be an accept state in the new DFA.

This gives us a DFA for $\alpha \setminus \beta$ and all DFA's can be represented by a regex therefore there exists a regex r st

$L(r) = L(\alpha) \setminus L(\beta)$

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4. For each of the following, either give an example of an object with the given property or state that no such object exists. (No proofs are required for this problem.)

(a) A decidable language that is not regular.

$$A = \{0^n 1^n \mid n \in \mathbb{N}\}$$

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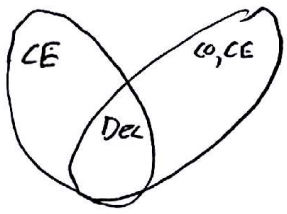
(b) A decidable language that is not c.e.

DNE



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Dec $\subseteq$ CE

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4 (continued).

(c) A trio A, B, C of languages such that $A \not\leq_m B$ and $A \not\leq_m C$, but $A \leq_m B \sqcup C$.

(Notes: $A \not\leq_m B$ means that A is not $\leq_m$-reducible to B, and

$$B \sqcup C = \{x0 \mid x \in B\} \cup \{y1 \mid y \in C\}$$

is the join of B and C.)

Do Not Grade 2

Name _____

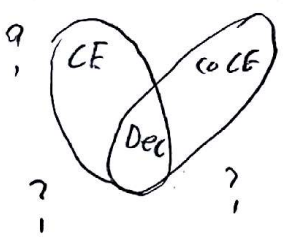
4 (continued).

(d) A pair of strings $x, y \in \{0,1\}^*$ such that $C(xy) < C(x) + C(y)$.

Do Not Grade 2

(e) A language that is neither c.e. nor co-c.e.

DNE



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5. Prove or disprove: If $A, B \subseteq \{0,1\}^*$ are both decidable, then $A \setminus B$ is decidable.

A, B are dec $\Rightarrow$ $\exists$ TM M_A st $L(M_A) = A$ and
$\exists$ TM M_B st $L(M_B) = B$

we construct $TM_{A \setminus B}$

$TM_{A \setminus B}$

1 input x
2 run M_A on x
If M_A rejects then REJECT
3 Else If M_A Accepts run M_B on x
If M_B Accepts then REJECT
4 Else If M_B rejects then ACCEPT

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6. Prove or disprove: If $A, B \subseteq \{0,1\}^*$ are both c.e., then $A \setminus B$ is c.e.

Do Not Grade

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7. Prove that the language

$$A = \{ x \in \{0,1\}^* \mid C(x) \leq \frac{|x|}{2} \}$$

is c.e.

Do Not Grade

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8. Let $A \subseteq \{0,1\}^*$ be a c.e. language that contains at most n strings of each length n. That is, for every $n \in \mathbb{N}$,

$$|A \cap \{0,1\}^n| \leq n.$$

Prove that there is a constant $c \in \mathbb{N}$ such that, for every $x \in A$,

$$C(x) \leq c + 2 \log |x|.$$

Do Not Grade

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9. Prove that the language

$$A = \{ uv0w \mid u,v,w \in \{0,1\}^* \text{ and } |u| = |v| = |w| \}$$

is not regular.

Let $c \in \mathbb{N}$, Let $m \in \mathbb{Z}^+$ st $C(0^m 00^m) > c$

Let $x = 0^m$ then what if $m = 13$? then $y_{x,i}^{A} = \lambda$

$$C(y_{x,i}^{A}) = C(0^m 00^m) > c + \log(1)$$ □

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10. Prove that the language A of problem 9 is decidable.

Let TM M_A st $L(M_A) = A$

TM_4

1 input x
2 If [|x|-1]%3 != 0 REJECT
3 find the last 0 ∈ x
If the length of x after the zero is less than
The length of x before the 0 divided by 2
4 Then find the next privious 0
5 goto 3
6 If the length of x after the 0 is == the
length of x before the 0 divided by 2
7 Then ACCEPT
8 If the length of x after the 0 is greater
than the length of x before the 0
9 Then REJECT
10 If The beginning of the string is reached
or there are no 0's then REJECT □

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