CS 331 HW10

Cahlen Brancheau

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[Thm 3:
$\exists\ a \in \mathbb{N}$ s.t. $\forall x \in \{0,1\}^*$, $C(x) \leq |x| + a$]

Prove: $C(x) \leq T(|x|)$

$$C(x) = |x| + a \leq T(|x|)$$

T grows so fast that for any arbitrary constant a there will be a point where $T(|x|) \geq |x| + a$ is true, and as T only grows and $\mathbb{N}$ is infinite from this point on there will be infinitely many $x \in \{0,1\}^*$ that are extremely compressible.

$>4)$

Let π_x be a string st $C(x) = |\pi_x|$ and $U(\pi_x) = x$
Let π_y be a string st $C(y) = |\pi_y|$ and $U(\pi_y) = y$
Without loss of generality Let $C(x) = \min\{C(x), C(y)\}$

Proof

Let M be a TM st for every π and π' where $U(\pi)=x$ and $U(\pi')=y$, $M(\langle\pi,\pi'\rangle) = \langle x, y\rangle$

$$C(\langle x,y\rangle) \leq C_m(\langle x,y\rangle) + \tilde{c}_m \text{, by optimality}$$
$$\leq |\langle \pi_x, \pi_y\rangle| + \tilde{c}_m$$
$$= 2|\langle \pi_x, \pi_y\rangle| + \tilde{c}_m$$
$$= C(x) + C(x) + C(y) + 1 + \tilde{c}_m$$
$$= C(x) + C(y) + \min\{C(x), C(y)\} + c$$
where $c = 1 + \tilde{c}_m$

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$\text{Max}\{C(x), C(y)\} \le C(\langle x, y\rangle + c$

Show:

$C(x) \le C(\langle x, y\rangle) + c'$
$C(y) \le C(\langle x, y\rangle) + c''$

Let $\pi_{\langle x,y\rangle}$ be a min length program for $\langle x, y\rangle$
$(U(\pi_{\langle x,y\rangle}) = \langle x, y\rangle$ and $C(\langle x,y\rangle) = |\pi_{\langle x,y\rangle}|)$
Let M be a TM st for all $\pi \in \{0,1\}^*$, st
$U(\pi) = \langle x,y\rangle$, $M(\pi) = x$

Proof

TM M

1 input π
2 run $U(\pi) = \langle x, y\rangle$
3 output x

$C(x) = C_m(x) + c'$, by optimality
$\le |\pi_{\langle x,y\rangle}| + c'$
$= C(\langle x,y\rangle) + c'$

by a similar method the same can be done with y, so

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$\text{Max}\{C(x), C(x)\} \le C(\langle x,y\rangle) + c$

where $c = \text{Max}\{c', c''\}$

constant from doing the above for y.

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$x \in PAL$ will have 1 of 2 forms either it will be odd or even. We create a TM to act in the following maner

Case 1: $x \in PAL \wedge$ Even

$M(x) = x x^R$

Case 2: $x \in PAL \wedge$ odd

$M(x) = x x^R_{[0 \cdots \frac{|x|-1}{2}]}$

Case 1 Proof

$C(x) = C_m(x) + c'$, by optimality
$\leq \frac{|x|}{2} + c'$

Case 2 Proof

$c(x) = C_m(x) + c''$, by optimality
$\leq \frac{|x|}{2} + 1 + c''$

So,

$$C(x) \leq \frac{|x|}{2} + c,$$

where $c = \max\{c', c''\} + 1$

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