Cahlen Brancheau
CS 331 HW 9

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6a)

1)

$A \leq_m A \sqcup B$

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$f(x) = x0$

$x \in A \Rightarrow f(x) \in A \sqcup B$, $f(x) = x0$

$x \notin A \Rightarrow f(x) \notin A \sqcup B$, this is trivial why?

2) $B \leq_m A \sqcup B$

same as above but $f(y) = y1$. $\square$

7b)

$K = \{ e \in \mathbb{N} \mid M_e(e) \downarrow \}$, is C.E. K^c is not C.E.

Proof (using 6a)

case 1

Assume $K \sqcup K^c$ is C.E.

$K \leq_m K \sqcup K^c$ ✓

$K^c \leq_m K \sqcup K^c$ x contradiction K^c is not C.E.

case 2

Assume $K \sqcup K^c$ is coCE

$K^c \leq_m K \sqcup K^c$ ✓

$K \leq_m K \sqcup K$ x contradiction K is C.E. $\square$

Scanned by CamScanner

A lang $A \subseteq \{0,1\}$ is $\leq_m$-hard for CE
if $\forall L \in CE$, $L \leq_m A$

72)

Show:

- $\forall L \in CE$, $A \leq_m \{k \mid (\forall \ell) M_k(\ell)\downarrow\}$?
- $\exists$ TM M_L st $L(M_L) = L$

MF

1. input x
2. construct TM M^x
3. Find k st $M_k = M^x$
4. output K

M^x

1. input ℓ
2. Run M_L on X
3. If M_L Accepts x
4. then halt
5. else run forever

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If $x \in L$ then M_L will accept then M^x will halt causing $(\forall \ell) M_k(\ell)\downarrow$

If $x \notin L$ then M_L will run forever making M^x run forever

so $x \in L \Leftrightarrow \{k \mid (\forall \ell) M_k(\ell)\downarrow\}$

7 (1)

$\{k \mid M_k(3)\uparrow\}$ is coCE

$\forall\ L \in coCE,\ L \leq_m \{k \mid M_k(3)\uparrow\}$

$L^c \in C.E.$, so $\exists$ TM M_{L^c} st $L(M_{L^c}) = L^c$

MF	M^x
1. input x	1. input n
2. construct TM M^x	2. Run M_{L^c} on x
3. Find k st $M_k = M^x$	3. If M_{L^c} Accepts and $n==3$
4. Output k	4. Then halt
	5 Else run forever

If $x \in \underbrace{L}_{\text{coCE Lang.}}$ and $n==3$ then M_{L^c} will run forever making M^x to run forever satisfying $M_k(3)\uparrow$

If $x \notin L$ and $n==3$ then M_{L^c} will accept then M^x will halt causing $M_k(3)$ to halt

so $x \in L \iff \{k \mid M_k(3)\uparrow\}$ for coCE $\square$

Good argument but you need to show that $\{k \mid M_k(3)\uparrow\}$ is coCE.

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[redacted]

65-68)

Prove $A \Rightarrow D \Rightarrow C \Rightarrow B \Rightarrow A$

65) $A \Rightarrow D$

I don't know

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66) $D \Rightarrow C$

I don't know

67) $C \Rightarrow B$

case 1: A is finite

Trivial

case 2:

If there is a compatable onto function then the existence of a compatable function is obvious and this is trivial

68) $B \Rightarrow A$

case 1: $A = \phi$

Trivial

case 2:

Explain why it is trivial

If there is a compatable function that enumerates A obviously implies that A is computably enumerable

You need to use Jack's original definition for c.e., which does not use enumerators.

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