

# Student projects

# Convexity in the calculus of variations

Project supervisor: Richard Gratwick

Project type

dissertation

## Suitable degrees

Maths with Management Maths and Stats

#### **Project description**

The calculus of variations is the search for minimal objects: shapes with the smallest surface area, curves defining a shortest path. Convexity of the underlying quantity turns out to be important in establishing the existence of solutions. But exactly what type of convexity? For multi-dimensional vector-valued problems, quasiconvexity is the key notion. But this is understood woefully poorly: we really struggle to identify quasiconvex functions. More easily spotted are rank-1 convex functions, and polyconvex functions. But these are all, in general, different things. This project will look at the direct method of the calculus of variations, and the importance of convexity to this. The main emphasis will then be on considering the different generalized notions of convexity, and what we know and do not know about them. There are plenty of open questions here for the ambitious and the inspired.

#### **Prerequisites**

SVCDE, Honours Analysis. General Topology and Linear Analysis might be useful at points (but we can discuss what exactly is needed).

### Recommended reading

- 1. Direct Methods in the Calculus of Variations (2ed), B. Dacorogna, Springer, 2008.
- 2. Variational models for microstructure and phase transitions, S. Muller, Max Planck Institute Lecture notes, 1998, sec 4 in particular.
- 3. The calculus of variations and materials science, J. M. Ball, Quart. Appl. Math. 56 (1998), no. 4, 719--740.