Student projects

Determination of Measures

Project supervisor: Richard Gratwick

Project type

dissertation

Suitable degrees

Mathematics (BSc, MA & MMath-Y5)

Project description

A measure on a set is simply a way of assigning a notion of size to each subset, for example length, area, and volume are familiar measures in one-, two-, and three-dimensional Euclidean space respectively. There are many other possible measures, however. The main question of this project is: when is the value of a measure on every subset determined entirely by its values on balls only? For example, if the measure of every circle in the plane is pi times the radius squared, then is the measure equal to the usual definition of area for every other shape? This is a question that can be posed in any metric space, and to answer it we need to consider the way in which the metric and the measure interact. This leads us to discussions of so-called "covering theorems", and some very strange definitions of metric spaces. There are still some open questions, for the more ambitious.

Prerequisites

Honours Analysis.
Essentials in Analysis and Probability would be useful for measure theory.

Recommended reading

1. Geometry of Sets and Measures in Euclidean Spaces, P. Mattila, CUP, 1995.
2. Measures not Approximable or not Specifiable by Means of Balls, R. O. Davies, Mathematika 18, 1971, 157-160.
3. A survey of small ball theorems and problems, J. P. R. Christensen, Measure theory, Oberwolfach 1979 (Proc. Conf., Oberwolfach, 1979), pp. 24?30, Lecture Notes in Math., 794, Springer, Berlin, 1980.