THE UNIVERSITY of EDINBURGH

Student projects

Legendrian knot theory

Project supervisor: Nick Sheridan

Project type

dissertation

Suitable degrees

Mathematics (BSc, MA & MMath-Y5)

Project description

Contact and symplectic geometry arose in the study of optics and mechanics, but have seen an explosion of activity in recent decades due to the advent of new invariants motivated by string theory. In general these invariants are quite difficult to define and compute, but in certain low-dimensional cases they are much more tractable, and in fact reduce to combinatorics; this project focuses on one such case. The geometric objects of interest are "Legendrian knots in contact three-manifolds": a contact manifold is a smooth manifold equipped with a certain kind of one-form, and a Legendrian knot in a three-dimensional contact manifold is a smoothly embedded circle which is everywhere tangent to the kernel of this one-form. The simplest example of a contact three-manifold is three-dimensional Euclidean space, equipped with the one-form dz + ydx, and a knot is then Legendrian if it is everywhere orthogonal to the vector field (y,0,1). The project will cover the basics of Legendrian knot theory, then move on to the Chekanov-Eliashberg differential graded algebra, a combinatorial invariant which can be used to prove that there exist Legendrian knots which are isotopic as smooth knots, but not as Legendrian knots.

Prerequisites

Algebraic topology; Differentiable manifolds.

Recommended reading

John Etnyre, Legendrian and transversal knots.