1) Introduction

After studying the learning of simple conservative dynamics with a single pendulum, we extend the study to the double pendulum system.
The double pendulum is a chaotic system, showing extreme sensibility to initial conditions while still being governed by deterministic Hamiltonian dynamics. The goal is to explore the limits of the models used and trained before to predict the dynamics of the pendulum.

2) Physical system

2.1) Description

The system is described by the following state:

X = (θ□, θ□, ω□, ω□)

This is a conservative system.

2.2) Equations of motion

The equations of motion are derived using the Euler–Lagrange formalism.
Due to their length, they are directly implemented in code and validated by checking energy conservation. The mathematical calculation can be observed in the lnn.py python code in the section src.systems.double_pendulum.models.lnn.py.

The equation for the Hamiltonian model remains the same and are just done twice for both angle and angular velocity. The symplectic matrix is changed in 4x4 matrix to adapt the new system.

3) Dataset

3.1) Dataset Generation

The data were generated using the RK4 integration method. The parameters used for the simulations are the following:

- $g = 9.81$ m/s²
- $m = 1$ kg
- $l = 1$ m
- Simulation duration: $T = 5$ s
- Step time: $dt = 0.01$ s

Both of the pendulum had the same mass m and the same rod length l.

3.2) Dataset Validation

Before the training, the generated data were verified by checking the energy conservation, the coherence between θ, ω and dω and the shape of the trajectories and phase-space portraits.

4) Results

4.1) MLP vector field

Learning the vector field for the double pendulum is significantly harder than for the simple pendulum due to both the higher dimensional input size and the chaotic behavior of the system. As a consequence, the observed loss was higher than for the pendulum. However, the trained model was sometimes able to reproduce the qualitative behavior of the pendulum, mostly for low energy trajectories, probably because those are less chaotic than the high energy trajectories.

4.2) Hamiltonian Neural Network

Learning the Hamiltonian of the double pendulum was way harder than for the pendulum because of its complexity but the neural network was able to approximatively learn the Hamiltonian and to preserve the energy of the system and is able to reproduce the qualitive behavior for higher energy trajectories than the mlp model. However, the limits of his model are clear, despite trying to adjust hyperparameters, training the neural network on specific data (first low energy trajectories and then high energy trajectories), the neural network wasn't able to accurately reproduces the quantitative behavior of the double pendulum.

4.3) Lagrangian Neural Network

Despite the equation being more complex than for the pendulum, this model is able to perform way better than the previous models:

For most initial conditions, the Lagrangian Neural Network is able to accurately reproduce both the qualitative structure and the quantitative evolution of the double pendulum over several seconds.

For a subset of trajectories, a divergence appears after approximately 8 seconds of simulation. This behavior is expected due to the chaotic nature of the double pendulum, where small numerical errors grow exponentially over time.

Importantly, before divergence, the model preserves key physical properties such as energy conservation and realistic phase-space structure, indicating that the underlying dynamics are correctly learned.

Conclusion

This project investigated different strategies for learning nonlinear physical dynamics using neural networks, ranging from purely data-driven models to physics-informed architectures. The simple pendulum and the double pendulum were used as benchmark systems due to their increasing levels of complexity, nonlinearity first, and then the chaotic behavior.

For the simple pendulum, purely data-driven approaches such as direct state prediction and vector field learning were able to achieve acceptable short-term prediction but systematically failed to preserve the qualitative structure of the dynamics. In particular, these models introduced artificial energy dissipation, leading to incorrect long-term behavior despite low one-step prediction errors. This highlights the limitations of unconstrained neural networks.

Physics-informed models, such as Hamiltonian and Lagrangian Neural Networks, strongly improved both qualitative and quantitative performance. By embedding physical structure directly into the model, these approaches were able to conserve energy, preserve phase-space geometry, and maintain stable long-term rollouts. Moreover, the Hamiltonian model proved particularly useful in diagnosing inconsistencies in the training dataset, revealing physical errors that purely data-driven models were able to fit without exposing.

The goal of the extension to the double pendulum was to explore the limits of the physics informed models due to its chaotic nature and higher-dimensional state space. In this regime, vector-field-based MLPs struggled to learn a stable and accurate representation of the dynamics, even after extensive tuning. In contrast, the Lagrangian Neural Network demonstrated strong performance, successfully reproducing both the qualitative structure and quantitative evolution of the system for most trajectories over several seconds. Divergences observed after longer simulation times were consistent with the intrinsic sensitivity of chaotic systems to small modeling and numerical errors, and therefore do not indicate a failure of the model.

Overall, these experiments demonstrate that incorporating physical constraints to learning models is a must when modeling dynamical systems, especially in regimes where long-term stability and conservation laws matters. Physics-informed neural networks not only improve predictive performance but also provide valuable insight into the structure and consistency of the data itself.