1 Домашнее задание(Денис М.)

1. $\forall M, FM =_\beta MF$
   $F =_\beta \lambda m.mF$
   $F =_\beta (\lambda fm.mf)F$
   $F =_\beta Y(\lambda fm.mf)$
   $F =_\beta (\lambda f.(\lambda x.f(xx))(\lambda x.f(xx)))(\lambda fm.mf)$

2. $\forall MN, FMN =_\beta FN(MF)$
   $F =_\beta (\lambda mn.Fn(mF))$
   $F =_\beta (\lambda fmn.fn(mf))F$
   $F =_\beta Y(\lambda fmn.fn(mf))$
   $F =_\beta (\lambda f.(\lambda x.f(xx))(\lambda x.f(xx)))(\lambda fmn.fn(mf))$

3. $\forall M, FM =_\beta FM$
   F - любой терм, потому что для любого F, $I(FM) \rightarrow_\beta FM$, то есть $FM =_\beta I(FM) =_\beta FM$.

4. $\forall M, FM =_\beta F$
   $F =_\beta (\lambda m.F)$
   $F =_\beta (\lambda fm.f)F$
   $F =_\beta Y(\lambda fm.f)$
   $F =_\beta (\lambda f.(\lambda x.f(xx))(\lambda x.f(xx)))(\lambda fm.f)$

5. Для A, B, C найти F, G, H такие, что

   $$\begin{cases} F =_\beta AFGH \\ G =_\beta BFGH \\ H =_\beta CFGH \end{cases}$$

   $F =_\beta AFGH$
   $F =_\beta (\lambda f.AfGH)F$
   $F =_\beta Y(\lambda f.AfGH)$
   $F =_\beta (\lambda f.(\lambda x.f(xx))(\lambda x.f(xx)))(\lambda f.AfGH)$
   $F =_\beta (\lambda x.(\lambda f.AfGH)(xx))(\lambda x.(\lambda f.AfGH)(xx))$
   $F =_\beta (\lambda x.A(xx)GH)(\lambda x.A(xx)GH)$

   $G =_\beta BFGH$
   $G =_\beta B((\lambda x.A(xx)GH)(\lambda x.A(xx)GH))GH$
   $G =_\beta (\lambda g.B((\lambda x.A(xx)gH)(\lambda x.A(xx)gH))gH)G$
   $G =_\beta Y(\lambda g.B((\lambda x.A(xx)gH)(\lambda x.A(xx)gH))gH)$
   $G =_\beta (\lambda f.(\lambda x.f(xx))(\lambda x.f(xx)))(\lambda g.B((\lambda x.A(xx)gH)(\lambda x.A(xx)gH))gH)$
   $G =_\beta (\lambda y.B((\lambda x.A(xx)(yy)H)(\lambda x.A(xx)(yy)H))(yy)H)$
   $(\lambda y.B((\lambda x.A(xx)(yy)H)(\lambda x.A(xx)(yy)H))(yy)H)$

$H =_\beta Y(CFG)$
$H =_\beta (\lambda x.CFG(xx))(\lambda x.CFG(xx))$

$$\begin{cases} F =_\beta AFG(Y(CFG)) \\ G =_\beta BFG(Y(CFG)) \end{cases}$$

$G =_\beta (\lambda g.BFg(Y(CFg)))G$

$F =_\beta AF(Y(\lambda g.BFg(Y(CFg))))$
$F =_\beta (\lambda f.Af(Y(\lambda g.Bfg(Y(Cfg)))))F$
$F =_\beta Y(\lambda f.Af(Y(\lambda g.Bfg(Y(Cfg)))))$
$F =_\beta (\lambda f.(\lambda x.f(xx))(\lambda x.f(xx)))$
$(\lambda f.Af((\lambda f.(\lambda x.f(xx))(\lambda x.f(xx)))$
$(\lambda g.Bfg((\lambda f.(\lambda x.f(xx))(\lambda x.f(xx)))(Cfg)))))$

$G =_\beta (\lambda f.(\lambda x.f(xx))(\lambda x.f(xx)))$
$(\lambda g.B((\lambda f.(\lambda x.f(xx))(\lambda x.f(xx)))$
$(\lambda f.Af((\lambda f.(\lambda x.f(xx))(\lambda x.f(xx)))$
$(\lambda g.Bfg((\lambda f.(\lambda x.f(xx))(\lambda x.f(xx)))(Cfg))))))$
$g((\lambda f.(\lambda x.f(xx))(\lambda x.f(xx)))$
$(C((\lambda f.(\lambda x.f(xx))(\lambda x.f(xx)))$
$(\lambda f.Af((\lambda f.(\lambda x.f(xx))(\lambda x.f(xx)))$
$(\lambda g.Bfg((\lambda f.(\lambda x.f(xx))(\lambda x.f(xx)))(Cfg))))))g)))$
$H = (\lambda f.(\lambda x.f(xx))(\lambda x.f(xx)))$
$(C((\lambda f.(\lambda x.f(xx))(\lambda x.f(xx)))$
$(\lambda f.Af((\lambda f.(\lambda x.f(xx))(\lambda x.f(xx)))$
$(\lambda g.Bfg((\lambda f.(\lambda x.f(xx))(\lambda x.f(xx)))(Cfg))))))$
$((\lambda f.(\lambda x.f(xx))(\lambda x.f(xx)))(\lambda g.B((\lambda f.(\lambda x.f(xx))(\lambda x.f(xx)))$
$(\lambda f.Af((\lambda f.(\lambda x.f(xx))(\lambda x.f(xx)))$
$(\lambda g.Bfg((\lambda f.(\lambda x.f(xx))(\lambda x.f(xx)))(Cfg))))))$
$g((\lambda f.(\lambda x.f(xx))(\lambda x.f(xx)))$
$(C((\lambda f.(\lambda x.f(xx))(\lambda x.f(xx)))$
$(\lambda f.Af((\lambda f.(\lambda x.f(xx))(\lambda x.f(xx)))$
$(\lambda g.Bfg((\lambda f.(\lambda x.f(xx))(\lambda x.f(xx)))(Cfg))))))g)))))$

6. WHNF но не HNF: $\lambda x.(\lambda x.x)x$
   HNF но не NF: $x(\lambda x.x)x$
   не WHNF: $(\lambda x.x)x$

7. $\nexists F \forall MN, F(MN) =_\beta M$

   Доказательство:

   Допустим существует такой терм F, что $F(MN) =_\beta M$ для любого терма M и N.

   Тогда

$F((\lambda x.x)((\lambda xy.yx)(\lambda x.x))) =_\beta \lambda x.x$
$F((\lambda x.x)((\lambda xy.yx)(\lambda x.x)))$
$=_\beta F((\lambda xy.yx)(\lambda x.x)) =_\beta \lambda xy.yx$

Нормальная форма существует и не определена однозначно. Предположение, что существует такой F неверно.

8. Построим терм N' из M путём замены всех редексов по правилу

$$(\lambda x.A)B \to (i(\lambda x.A)B),$$

   где A и B некоторые подтермы M.

   Пусть $N = (\lambda i.N')$, тогда в N нет редексов, то есть он находится в нормальной форме и $NI = M$.

9. leftmost
   $= \lambda t.\text{if (isleaf } t)(\text{value } t)(\text{leftmost (left } t))$
   $= Y(\lambda ft.\text{if (isleaf } t)(\text{value } t)(f(\text{left } t)))$

10. count
    $= \lambda t.\text{if (isleaf } t)1(\text{plus (count (left } t))(\text{count (right } t)))$
    $= Y(\lambda ft.\text{if (isleaf } t)1(\text{plus } (f(\text{left } t))(f(\text{right } t))))$

11. sum
    $= \lambda t.\text{if (isleaf } t)(\text{value } t)(\text{plus (sum (left } t))(\text{sum (right } t)))$
    $= Y(\lambda ft.\text{if (isleaf } t)(\text{value } t)(\text{plus } (f(\text{left } t))(f(\text{right } t))))$

12.
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usedIn = fix(\f var term ->
    case term of
        Var v -> v = var
        App a b -> f var a || f var b
        Lam v b -> v = var || f var b
```