Useful summation formulas and rules

$\sum_{i=l}^{u} 1 = 1+1+\ldots+1 = u - l + 1$, *In particular,* $\sum_{i=1}^{n} 1 = n-1+1 = n \in \Theta(n)$

$\sum_{i=1}^{n} i = 1+2+\ldots+n = \frac{n(n+1)}{2} \approx \frac{n^2}{2} \in \Theta(n^2)$

$\sum_{i=1}^{n} i^2 = 1^2 + 2^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6} \approx \frac{n^3}{3} \in \Theta(n^3)$

$\sum_{i=0}^{n} a^i = 1 + a + \ldots + a^n = \frac{a^{n+1} - 1}{a-1}$ *for any* $a \neq 1 \in \Theta(a^n)$

In particular, $\sum_{i=0}^{n} 2^i = 2^0 + 2^1 + \ldots + 2^n = 2^{n+1} - 1 \in \Theta(2^n)$

$\sum(a_i \pm b_i) = \sum a_i \pm \sum b_i$, $\sum c a_i = c \sum a_i$,

$\sum_{i=l}^{u} a_i = \sum_{i=l}^{m} a_i + \sum_{i=m+1}^{u} a_i$

Solve Summation

- $\sum_{i=0}^{n-1}(i^2+1)^2$ $(\sum_{i=1}^{n} i^k \approx \frac{1}{k+1}n^{k+1})$

$$
\begin{aligned}
\sum_{i=0}^{n-1}(i^2+1)^2 &= \sum_{i=0}^{n-1}(i^4+2i^2+1) \\
&\approx \sum_{i=0}^{n-1} i^4 = \sum_{i=1}^{n} i^4 + 0^4 - n^4 \\
&= \sum_{i=1}^{n} i^4 - n^4 \\
&\approx \frac{1}{4+1}n^{4+1} - n^4 \\
&= \frac{1}{5}n^5 - n^4 \\
&\approx \frac{1}{5}n^5 \in \Theta(n^5)
\end{aligned}
$$

- $\sum_{i=2}^{n-1} \lg i^2$ $(\sum_{i=1}^{n} \lg i \approx n \lg n)$

$$
\begin{aligned}
\sum_{i=2}^{n-1} lgi^2 &= \sum_{i=2}^{n-1} 2lgi \\
&= 2\sum_{i=2}^{n-1} lgi = 2\sum_{i=1}^{n} lgi - 2lg1 - 2lgn \\
&= 2\sum_{i=1}^{n} lgi - 2lgn \\
&\approx 2nlgn - 2lgn \approx nlgn \in \Theta(nlogn)
\end{aligned}
$$

Analyze the algorithm (Recursive)

1. *input size*
2. Identify *basic operation* (innermost loop)
3. *Worst*, *average*, and *best* cases?
4. Set up a Recurrence Relation with an appropriate initial condition expressing the number of times the basic operation is executed – (Recurrence Relation Calculation)
5. Solve the recurrence to establish its *order of growth.*

Divide-and-Conquer

- Sorting
  - Brute Force: Selection Sort; Bubble Sort
  - Decrease and Conquer: Insertion Sort;
  - Divide and Conquer: **Merge Sort ; Quick Sort**
- Closet-pair Problem