Useful summation formulas and rules

$\sum_{i=l}^{u} 1 = 1+1+\ldots+1 = u - l + 1$, *In particular,* $\sum_{i=1}^{n} 1 = n-1+1 = n \in \Theta(n)$

$\sum_{i=1}^{n} i = 1+2+\ldots+n = \frac{n(n+1)}{2} \approx \frac{n^2}{2} \in \Theta(n^2)$

$\sum_{i=1}^{n} i^2 = 1^2 + 2^2 +\ldots+ n^2 = \frac{n(n+1)(2n+1)}{6} \approx \frac{n^3}{3} \in \Theta(n^3)$

$\sum_{i=0}^{n} a^i = 1 + a +\ldots+ a^n = \frac{a^{n+1}-1}{a-1}$ *for any* $a \neq 1 \in \Theta(a^n)$

In particular, $\sum_{i=0}^{n} 2^i = 2^0 + 2^1 +\ldots+ 2^n = 2^{n+1} - 1 \in \Theta(2^n)$

$\sum(a_i \pm b_i) = \sum a_i \pm \sum b_i$, $\sum c a_i = c \sum a_i$,

$\sum_{i=l}^{u} a_i = \sum_{i=l}^{m} a_i + \sum_{i=m+1}^{u} a_i$