CPSC 221: Data Structures
Lecture #5
Branching Out

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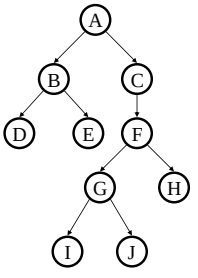
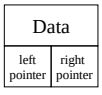
Today's Outline

- Binary Trees
- Dictionary ADT
- Binary Search Trees
- Deletion
- Some troubling questions

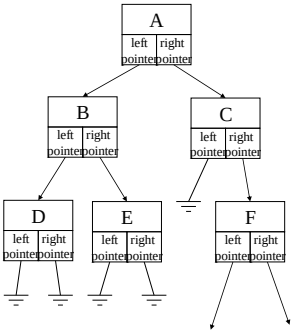
Binary Trees

- Binary tree is either
  - empty (NULL for us), or
  - a datum, a left subtree, and a right subtree
- Properties
  - max # of leaves:
  - max # of nodes:
- Representation:

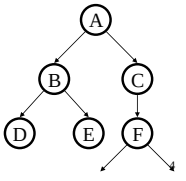




Representation




```
struct Node {
  KTYPE key;
  DTYPE data;
  Node * left;
  Node * right;
};
```



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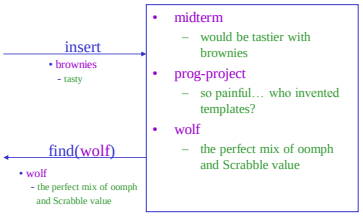
What We Can Do So Far

- Stack
  - Push
  - Pop
- Queue
  - Enqueue
  - Dequeue
- List
  - Insert
  - Remove
  - Find
- Priority Queue
  - Insert
  - DeleteMin

What's wrong with Lists?

Dictionary ADT

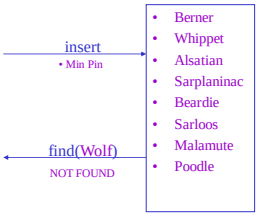
- Dictionary operations
  - create
  - destroy
  - insert
  - find
  - delete



- Stores *values* associated with user-specified *keys*
  - values may be any (homogenous) type
  - keys may be any (homogenous) comparable type

Search/Set ADT

- Dictionary operations
  - create
  - destroy
  - insert
  - find
  - delete



- Stores keys
  - keys may be any (homogenous) comparable
  - quickly tests for membership

A Modest Few Uses

- Arrays and "Associative" Arrays
- Sets
- Dictionaries
- Router tables
- Page tables
- Symbol tables
- C++ Structures

Desiderata

- Fast insertion
  - runtime:
- Fast searching
  - runtime:
- Fast deletion
  - runtime:

Naïve Implementations

insert find delete

- Linked list
- Unsorted array
- Sorted array

so close!

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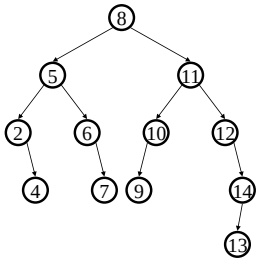
Binary Search Tree Dictionary Data Structure

- Binary tree property
  - each node has ≤ 2 children
  - result:
    - storage is small
    - operations are simple
    - average depth is small
- Search tree property
  - all keys in left subtree smaller than root's key
  - all keys in right subtree larger than root's key
  - result:
    - easy to find any given key



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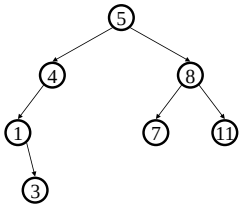
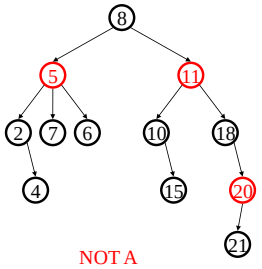
Getting to Know BSTs

Example and Counter-Example



BINARY SEARCH TREE



NOT A BINARY SEARCH TREE

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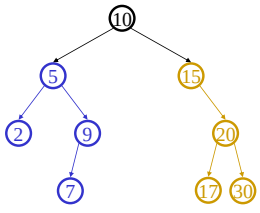
Getting to Know All About BSTs

In Order Listing



In order listing:
2→5→7→9→10→15→17→20→30

```
struct Node {
  KTYPE key;
  DTYPE data;
  Node * left;
  Node * right;
};
```

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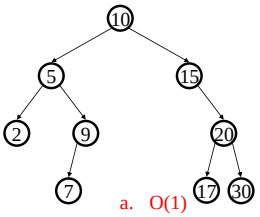
Getting to Like BSTs

Finding a Node



runtime:
a. O(1)
b. O(lg n)
c. O(n)
d. O(n lg n)
e. None of these

```
Node *& find(Comparable key,
             Node *& root) {
  if (root == NULL)
    return root;
  else if (key < root->key)
    return find(key,
                root->left);
  else if (key > root->key)
    return find(key,
                root->right);
  else
    return root;
}
```

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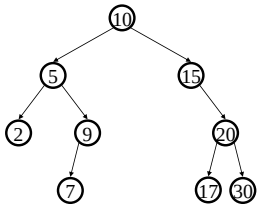
Getting to Like BSTs

Finding a Node



WARNING: Much fancy footwork with refs (&) coming. You can do ***all*** of this without refs... just watch out for special cases.

```
Node *& find(Comparable key,
             Node *& root) {
  if (root == NULL)
    return root;
  else if (key < root->key)
    return find(key,
                root->left);
  else if (key > root->key)
    return find(key,
                root->right);
  else
    return root;
}
```

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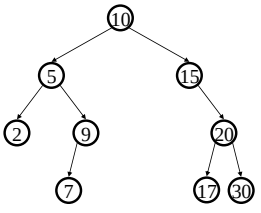
Getting to Hope BSTs Like You

Iterative Find

```
Node * find(Comparable key,
            Node * root) {
  while (root != NULL &&
         root->key != key) {
    if (key < root->key)
      root = root->left;
    else
      root = root->right;
  }

  return root;
}
```



Look familiar?

(It's trickier to get the ref return to work here.)

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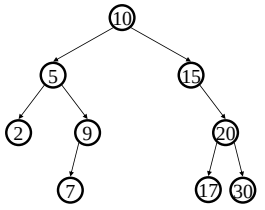
Insert




```
// Precondition: key is not
// already in the tree!
void insert(Comparable key,
            Node * root) {
  Node *& target(find(key,
                      root));
  assert(target == NULL);

  target = new Node(key);
}
```

runtime:

Funky game we can play with the *& version.

Digression: Value vs. Reference Parameters

- Value parameters (Object foo)
  - copies parameter
  - no side effects
- Reference parameters (Object & foo)
  - shares parameter
  - can affect actual value
  - use when the value needs to be changed
- Const reference parameters (const Object & foo)
  - shares parameter
  - cannot affect actual value
  - use when the value is too big for copying in pass-by-value

BuildTree for BSTs

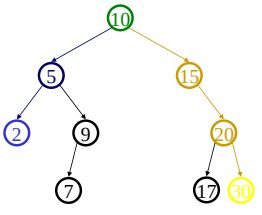
- Suppose the data 1, 2, 3, 4, 5, 6, 7, 8, 9 is inserted into an initially empty BST:
  - in order
  - in reverse order
  - median first, then left median, right median, etc.

Analysis of BuildTree

- Worst case: $O(n^2)$ as we've seen
- Average case assuming all orderings equally likely turns out to be O(n lg n).

Bonus: FindMin/FindMax

- Find minimum
- Find maximum

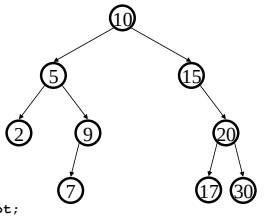


Double Bonus: Successor

Find the next larger node in this node's subtree.

```
Node *& succ(Node *& root) {
  if (root->right == NULL)
    return root->right;
  else
    return min(root->right);
}

Node *& min(Node *& root) {
  if (root->left == NULL) return root;
  else return min(root->left);
}
```

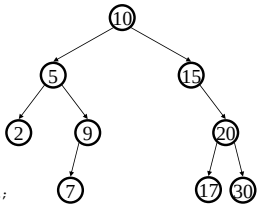


More Double Bonus: Predecessor

Find the next smaller node in this node's subtree.

```
Node *& pred(Node *& root) {
  if (root->left == NULL)
    return root->left;
  else
    return max(root->left);
}

Node *& max(Node *& root) {
  if (root->right == NULL) return root;
  else return max(root->right);
}
```



Today's Outline

- Some Tree Review (here for reference, not discussed)
- Binary Trees
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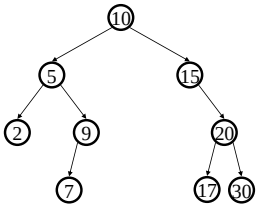
- Some troubling questions

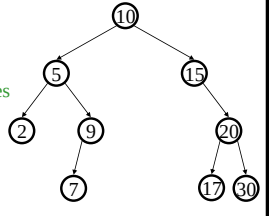
Deletion



Why might deletion be harder than insertion?

Lazy Deletion

- Instead of physically deleting nodes, just mark them as deleted (with a "tombstone")
  - \+ simpler
  - \+ physical deletions done in batches
  - \+ some adds just flip deleted flag
  - – small amount of extra memory for deleted flag
  - – many tombstones slow finds
  - – some operations may have to be modified (e.g., min and max)



Lazy Deletion

Delete(17)

Delete(15)

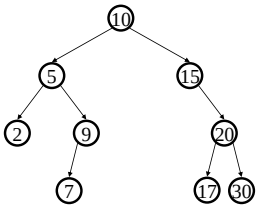
Delete(5)

Find(9)

Find(16)

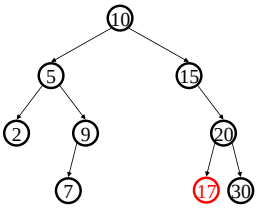
Insert(5)

Find(17)



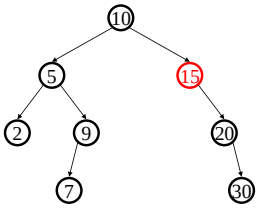
Deletion - Leaf Case

Delete(17)



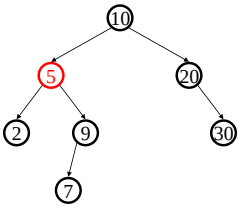
Deletion - One Child Case

Delete(15)

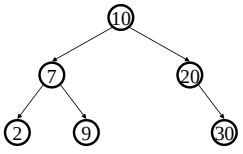


Deletion - Two Child Case

Delete(5)



Finally…



Delete Code

```
void delete(Comparable key, Node *& root) {
  Node *& handle(find(key, root));
  Node * toDelete = handle;
  if (handle != NULL) {
    if (handle->left == NULL) {          // Leaf or one child
      handle = handle->right;
    } else if (handle->right == NULL) { // One child
      handle = handle->left;
    } else {                             // Two child case
      Node *& successor(succ(handle));
      handle->data = successor->data;
      toDelete = successor;
      successor = successor->right;      // Succ has <= 1 child
    }
    delete toDelete;
  }
}
```

Refs make this short and "elegant"…
but could be done without them with a bit more work.

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  (here for reference, not discussed)
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Thinking about Binary Search Trees

- Observations
  - Each operation views two new elements at a time
  - Elements (even siblings) may be scattered in memory
  - Binary search trees are fast *if they're shallow*
- Realities
  - For large data sets, disk accesses dominate runtime
  - Some deep and some shallow BSTs exist for any data

One more piece of bad news: what happens to a balanced tree after *many* insertions/deletions?

Solutions?

- Reduce disk accesses?
- Keep BSTs shallow?

To Do

- Continue readings on website!

Coming Up

Spawns parallel task. Since we have only one classroom, one of these goes first!

- `cilk_spawn` Parallelism and Concurrency
- `cilk_spawn` Self-balancing Binary Search Trees
- `cilk_spawn` Priority Queues
- `cilk_spawn` Sorting (most likely!)
- **Huge** Search Tree Data Structure
- `cilk_join`
- `cilk_join`
- `cilk_join`
- `cilk_join`