A company has set a goal of developing a battery that lasts over 5 hours (300 minutes) in continuous use. A random test of 12 of these batteries measured the following lifespans (in minutes): 321, 295, 332, 351, 281, 336, 311, 253, 270, 326, 311, and 288. The distribution of battery lifespans is roughly unimodal and symmetric.

these

a) Is there evidence that the company has met its goal? Test at a 5% significance level.
b) Find a 90% confidence interval for the mean lifespan of this type of battery.
c) If interested in a 95% confidence interval for the mean lifespan, compared with the interval in (b), keeping all else constant, the 95% confidence interval would be:
    a. The same
    b. Narrower
    c. Wider

a) $H_0: \mu = 300$ batteries have a true mean lifetime of 300 min.

$H_A: \mu > 300$ batteries have true mean lifespan greater than 300 min

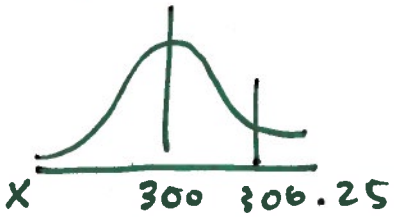
μ_0

Random sample. Reasonable that batteries are independent.

Told battery distribution roughly unimodal + symmetric.

$n = 12$, unknown σ^2

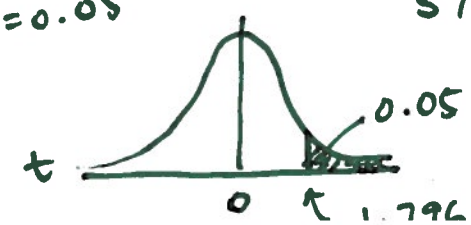
$\bar{x} = 306.25$ min, $s = 29.31$ min.



→ one sample t-test.

$$t_{obs} = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{306.25 - 300}{29.31/\sqrt{12}} = 0.7387.$$

$\alpha = 0.05$



rejection region.

$t_{0.05, df=11} = 1.796$ (table)

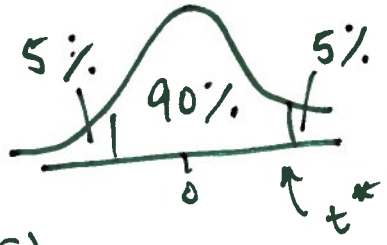
$t_{obs} < t_{\alpha, df=11}$

$0.7387 < 1.796$ t_{obs} does not lie in critical region.

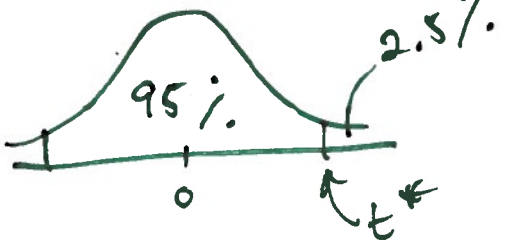
Fail to reject H_0.
No evidence to suggest the mean battery lifetime exceeds 300 min.

b) $\bar{x} \pm t^*_{11} SE(\bar{x}) = 306.25 \pm 1.796 \left(\frac{29.31}{\sqrt{12}}\right)$

$= (291.05, 321.45)$



c)



$\bar{x} \pm t^* SE(\bar{x})$

$(287.6272, 324.9)$

A sample of 79 comparable companies was selected at random and annual sales (x, in \$100K) and profits (y, in \$100K) were recorded. A regression line was fitted by least squares, this being predicted y = −176.644 + 0.09249x.
The estimated standard errors for the estimates of the intercept and slope are 61.16 and 0.0075 respectively. $SE(\hat{\beta}_1) = 0.0075$

Test the null hypothesis that there is no linear relationship between profit and sales for such companies. Test at a 5% significance level.

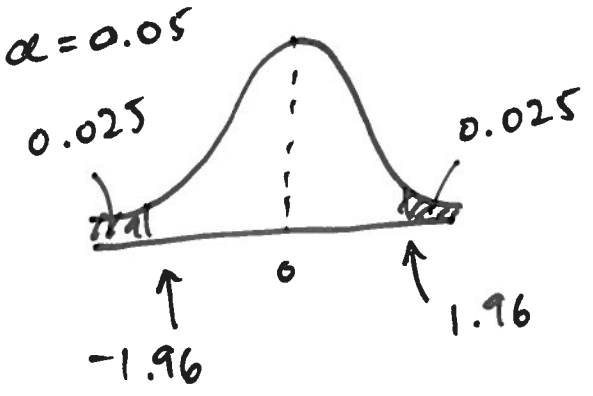
$H_0: \beta_1 = 0$

vs $H_A: \beta_1 \neq 0$

$$\frac{\hat{\beta}_1 - 0}{SE(\hat{\beta}_1)} = \frac{0.09249}{0.0075} = 12.332$$

$\sim t_{79-2=77}$

$t_{77} \approx N(0,1)$



Since 12.332 lies in the rejection region we reject the null hypothesis that there is no linear relationship.