CS221: Algorithms and Data Structures Lecture #3.5 Sorting Takes Priority

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2014W1

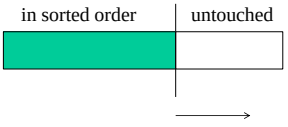
Today's Outline

- Sorting with Priority Queues, Three Ways

Quick Review of Sorts

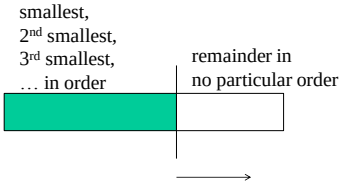
- Insertion Sort: Keep a list of already sorted elements. One by one, insert new elements into the right place in the sorted list.
- Selection Sort: Repeatedly find the smallest (or largest) element and put it in the next slot (where it belongs in the final sorted list).
- Merge Sort: Divide the list in half, sort the halves, merge them back together. (Base case: length ≤ 1.)

Insertion Sort Invariant



each iteration moves the line right one element

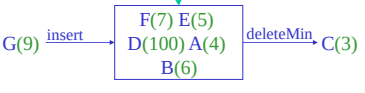
Selection Sort Invariant



each iteration moves the line right one element

How Do We Sort with a Priority Queue?

You have a bunch of data.
You want to sort by priority.
You have a priority queue.
WHAT DO YOU DO?



"PQSort"
(Super-Ridiculously Vague Pseudocode)

```
Sort(elts):
  pq = new PQ
  for each elt in elts:
    pq.insert(elt); // or all at once if that's easier
  sortedElts = new array of size elts.length
  for i = 0 to elts.length - 1:
    sortedElts[i] = pq.deleteMin  // or all at once
  return sortedElts
```

What sorting algorithm is this?

a. Insertion Sort
b. Selection Sort
c. Heap Sort
d. Merge Sort
e. None of these

Reminder: Naïve Priority Q Data Structures

- Unsorted array (or linked list):
  - *insert:* worst case O(1)
  - *deleteMin:* worst case O(n)
- Sorted array:
  - *insert:* worst case O(n)
  - *deleteMin:* worst case O(1)

"PQSort" deleteMaxes with Unsorted List **MAX**-PQ

0	1	2	3	4	5	6	7	8	9	10	11	12	13
5	9	4	8	1	6	10	12	13	2	3	14	20	7

How long does inserting all of these elements take?

And then the deletions…

"PQSort" deleteMaxes with Unsorted List MAX-PQ

0	1	2	3	4	5	6	7	8	9	10	11	12	13
5	9	4	8	1	6	10	12	13	2	3	14	20	7

PQ

"PQSort" deleteMaxes with Unsorted List MAX-PQ

0	1	2	3	4	5	6	7	8	9	10	11	12	13
5	9	4	8	1	6	10	12	13	2	3	14	20	7

PQ

5	9	4	8	1	6	10	12	13	2	3	14	7	20

PQ Result

"PQSort" deleteMaxes with Unsorted List MAX-PQ

0	1	2	3	4	5	6	7	8	9	10	11	12	13
5	9	4	8	1	6	10	12	13	2	3	14	20	7

PQ

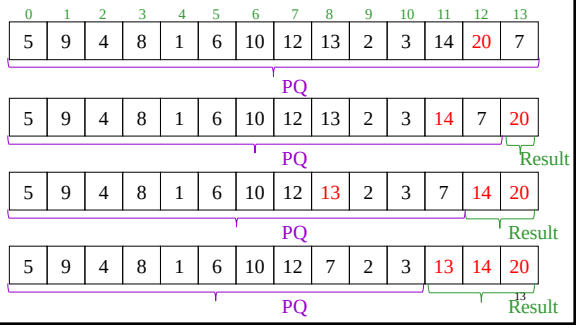
5	9	4	8	1	6	10	12	13	2	3	14	7	20

PQ Result

5	9	4	8	1	6	10	12	13	2	3	7	14	20

PQ Result

"PQSort" deleteMaxes with Unsorted List MAX-PQ



Two PQSort Tricks

1) Use the array to store both your results and your PQ. No extra memory needed!

2) Use a max-heap to sort in increasing order (or a min-heap to sort in decreasing order) so your heap doesn't "move" during deletions.

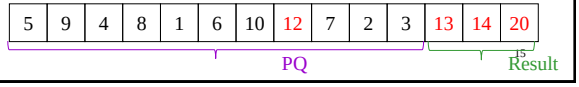
"PQSort" deleteMaxes with Unsorted List MAX-PQ

How long does "build" take? No time at all!

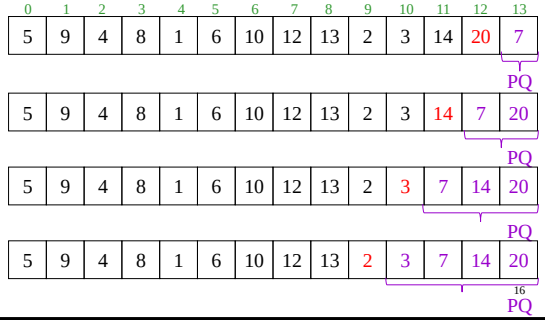
How long do the deletions take? Worst case: $O(n^2)$ ☹

What algorithm is this?

a. Insertion Sort
b. Selection Sort
c. Heap Sort
d. Merge Sort
e. None of these



"PQSort" insertions with Sorted List MAX-PQ



"PQSort" insertions with Sorted List MAX-PQ

How long does "build" take? Worst case: $O(n^2)$ ☹

How long do the deletions take?

What algorithm is this?

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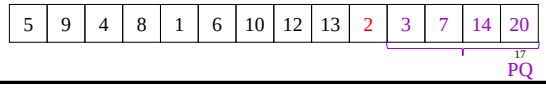
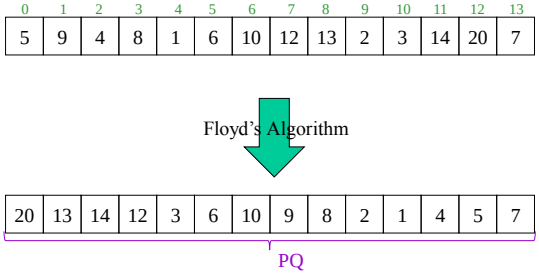


"PQSort" Build with Heap MAX-PQ



Takes only O(n) time!

"PQSort" deleteMaxes with Heap MAX-PQ

0	1	2	3	4	5	6	7	8	9	10	11	12	13
20	13	14	12	3	6	10	9	8	2	1	4	5	7

PQ

14	13	10	12	3	6	7	9	8	2	1	4	5	20

PQ

13	12	10	9	3	6	7	5	8	2	1	4	14	20

PQ

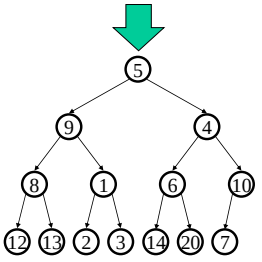
12	9	10	8	3	6	7	5	4	2	1	13	14	20

PQ

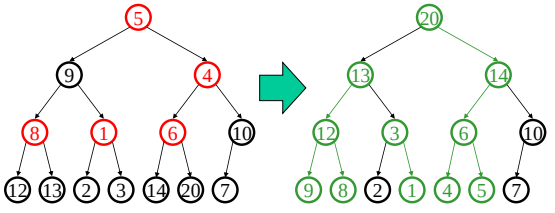
Totally incomprehensible as an array!

"PQSort" deleteMaxes with Heap MAX-PQ

5	9	4	8	1	6	10	12	13	2	3	14	20	7



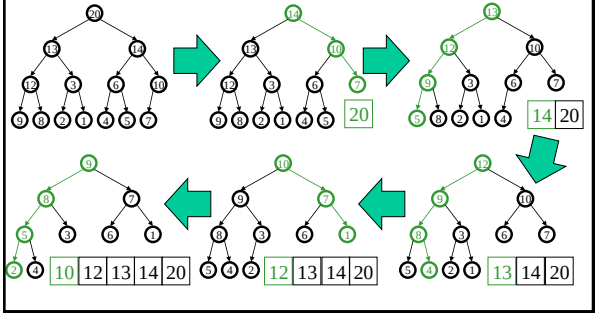
"PQSort" deleteMaxes with Heap MAX-PQ



Build Heap

Note: 9 ends up being perc'd down as well since its invariant is violated by the time we reach it.

"PQSort" deleteMaxes with Heap MAX-PQ



"PQSort" with Heap MAX-PQ

How long does "build" take? Worst case: O(n) ☺

How long do the deletions take? Worst case: O(n lg n) ☺

What algorithm is this?

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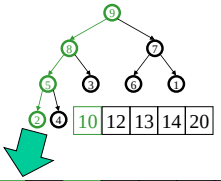

9	8	7	5	3	6	1	2	4	10	12	13	14	20

PQ | Result

"PQSort"

```
Sort(elts):
  pq = new PQ
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To Do

- Read: Epp Section 9.5 and KW Section 10.1, 10.4, and 10.7-10.10

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Coming Up

- More sorting!

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