

Winter 2014/15 Term 1 STAT 241/251 Assignment #2

Please attach a cover sheet when you hand in the assignment.

Due: Noon on Wednesday November 19, 2014 in STAT 241/251 dropbox in ESB

Total Marks: 30

Make sure you show all your work and define all variables you use to receive full marks.

1. In each batch of manufactured items, let p be the proportion of defective items.
 - (a) From each batch, a random sample of nine is taken and each of the nine items are inspected. If two or more of the nine items are defective, the batch is rejected; otherwise, it is accepted. Show that the probability that a batch is accepted is $(1 - p)^8(1 + 8p)$. [2 marks]
 - (b) It is decided to modify the sampling scheme so that if only one defective item is found in the sample of nine, a second sample of nine is taken and the batch rejected if this contains any defective items. With this exception, the original scheme is continued. Find an expression in terms of p for the probability that a batch is accepted. [3 marks].
 - (c) Suppose again the sampling scheme is modified. Now a random sample of 100 items is taken and examined. If 10 or more of the 100 items are defective then the batch is rejected; otherwise it is accepted. Use an approximation method to find the probability that a batch is accepted given $p = 0.08$. [3 marks]
2. A certain cellular device is advertised as being able to last for 7.0 to 9.0 hours of screen-on time. In experiments carried out to test this claim, the time in hours, X , was measured on a random sample of 250 occasions, and the data obtained is summarized by $\sum(x - 7.6) = 68.3$ and $\sum(x - 7.6)^2 = 17855$. The population mean and variance of X are denoted by μ and σ^2 respectively.
 - (a) Find, correct to one decimal place, an unbiased estimate of σ^2 . [2 marks]
 - (b) Test the hypothesis, using the critical region method, that $\mu = 8.0$ against the alternative hypothesis that $\mu < 8.0$, using a 5% significance level. Make sure you write down any assumptions associated with this test. [5 marks]
 - (c) Calculate a 95% confidence interval for μ . [2 marks]
3. Researchers want to know the effects of smoking on heart rates at rest in young adults. Resting pulse rates were measured for a random sample of smokers and nonsmokers and the observations are given in the dataset `smokingdata.txt`.

Is there evidence of a difference in mean pulse rate between smokers and non-smokers? Test at a 1% significance level. State the necessary conditions and plot the appropriate graphical display(s) to check one or more of these conditions. (You may use R for calculating means and standard deviations, however please write out calculations for hypothesis testing procedures). [6 marks]
4. Insurance companies track life expectancy information to help in determining the cost of life insurance policies. Last year, the life expectancy of its policyholders was 77 years. They want to know if their clients this year have a longer life expectancy, on average, so the company randomly samples some recent policies to check if the mean life expectancy of its policyholders has increased. The insurance company will only change their premium structure if there is evidence that people who buy their policies are living longer than before.

A random sample of 15 records had a mean of 78.6 years and standard deviation of 4.48 years. The data was roughly symmetric and unimodal.

- (a) Does this sample indicate that the insurance company should change its premiums because life expectancy has increased? Test an appropriate hypothesis at a 1% level and state your recommendation for the company. Write down any assumptions associated with this test. [5 marks]
- (b) Based on the conclusion you made in part (a), what type of error is the company at risk of making? Briefly explain in the context of the question. [2 marks]