The Temperature Derivatives Market : From stochastic modeling to option pricing

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June 2025

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1 Motivations

Exotic derivatives are financial instruments that have more complex features than vanilla derivatives like plain call and put options. They often include features such as path-dependency, multiple underlying assets, non-standard payoff structures, or conditions based on external events.

In this assignment, I will study path-dependent options. These options derive their value not just from the final price of the underlying asset, but from the entire path or history of the asset's price over time. This makes them different from standard options like calls, which only depend on the price of the underlying asset at expiration.

I am not about to study the quite famous Asian option, but I will instead focus on Weather derivatives. These are based on weather outcomes like temperature, rainfall, or other meteorological data over a specific period. Their payoff depends on the path of the weather over that period (e.g., how the temperature fluctuates), making it path-dependent.

In short, these derivatives require models that can account for the full path or trajectory of an asset and not just its end value, making them more complex than standard options. I'm gonna dive deeper into this concept in this work in order to face the stakes raised by these interesting contracts : from the model construction to their pricing methods.

This project is a way for me to enhance my skills in : stochastic modeling, time series analysis, regression, model fitting and Python programming.

2 Summary

The approach to price weather derivatives follows a structured methodology.

- Import time series capturing temperature dynamics
- Model the temperature as a combination of trend, seasonality, and stochastic noise,
- Simulate future temperature paths using this model,
- Compute the payoff for each simulated path and average them to estimate the derivative's price.

3 Temperature Derivatives Market

3.1 Weather derivatives

As a first step, we give a quick introduction of Temperature Derivative Market before to dive deeper into this concept.

A weather derivative is a financial contract designed to protect businesses from the financial impact of adverse weather conditions, particularly temperature fluctuations. While nearly any quantifiable weather risk can be hedged using a weather derivative, provided there is a willing counterparty, certain industries have emerged as key users of such instruments :

- **Energy companies** use weather derivatives to hedge against revenue fluctuations due to mild weather, since consumer energy demand is highly sensitive to temperature variations.
- **Agricultural producers and transportation companies** rely on weather stability and use derivatives to mitigate the effects of temperature extremes or other unfavorable meteorological conditions.
- **Retail businesses** often experience seasonal sales patterns influenced by weather and use derivatives to manage such demand-side risks.
- **Leisure and tourism infrastructure** (e.g., hotels, amusement parks, and ski resorts) depend on predictable and favorable weather conditions for optimal operations.
- **Financial institutions** (e.g., investment banks, asset managers, insurers, and reinsurers) may include weather derivatives as part of broader portfolio risk management strategies.

3.2 Temperature derivatives

3.2.1 Definition

Temperature derivatives are the most widely used class of weather derivatives.
The majority of futures and options written on temperature indices are traded on the CME (Chicago Mercantile Exchange), while a significant portion of the market outside these standardized indices operates over-the-counter (OTC). Despite their availability, daily trading volumes remain relatively low, with multiple days often passing without a single transaction. Nonetheless, in 2023, driven by increased climate volatility and interest in weather risk management, the CME reported a growing open interest in these contracts.

The most common weather indices used are Heating Degree Days (HDD), Cooling Degree Days (CDD), and the Cumulative Average Temperature (CAT).

HDD measures how much and for how long the temperature stays below a certain baseline (typically 18°C or 65°F), indicating heating demand.
Conversely, CDD tracks how much the temperature exceeds that baseline, reflecting cooling needs.
CAT represents the sum of average daily temperatures over a given period.

These indices are tied to a predefined payoff function that determines compensation based on actual weather outcomes, allowing industries such as energy, agriculture, retail, and tourism to manage climate-related revenue risks effectively.

3.2.2 Mathematical definition

More formally, we describe those indices as follow.

The underlying variable when it comes to Temperature derivatives is the daily average temperature, defined as :

$$T_t = \frac{T_{\max,t} + T_{\min,t}}{2}$$

This value is aggregated over a period $[\tau_1, \tau_2]$ to compute indices such as Heating Degree Days (HDD), Cooling Degree Days (CDD), and Cumulative Average Temperature (CAT):

$$\mathrm{HDD}(\tau_1, \tau_2) = \int_{\tau_1}^{\tau_2} \max(c - T_u, 0)\, du$$

$$\mathrm{CDD}(\tau_1, \tau_2) = \int_{\tau_1}^{\tau_2} \max(T_u - c, 0)\, du$$

$$\mathrm{CAT}(\tau_1, \tau_2) = \int_{\tau_1}^{\tau_2} T_u\, du$$

where c is a reference baseline temperature, typically 18°C.

HDD is used to measure heating demand (usually from October to April), while CDD measures cooling demand (from April to October).
The CAT index replace the CDD in certain European cities.
A useful identity connects these three indices:

$$\max(T_u - c, 0) - \max(c - T_u, 0) = T_u - c$$

Integrating both sides gives the HDD–CDD parity:

$$\mathrm{CDD}(\tau_1, \tau_2) - \mathrm{HDD}(\tau_1, \tau_2) = \mathrm{CAT}(\tau_1, \tau_2) - c(\tau_2 - \tau_1)$$

This identity implies that CDD can be derived from HDD and CAT, reducing the dimensionality of models based on these indices.

4 Stochastic Modeling of Temperature Dynamics

4.1 Theoretically

In order to simulate realistic temperature trajectories for the purpose of pricing weather derivatives and managing climate-related risks, we adopt a stochastic modeling approach that captures both seasonal patterns and random variability in temperature data, as in real life.

The temperature process T_t is initially modeled using a mean-reverting stochastic differential equation of the form :

$$dT_t = \kappa(\bar{\mu}(t) - T_t)\,dt + \sigma(t)\,dW_t,$$

where $\bar{\mu}(t)$ is a deterministic seasonal trend, κ is the speed of mean reversion, $\sigma(t)$ is the volatility (assumed positive), and W_t is a standard Brownian motion.
This model reflects the natural tendency of temperatures to revert toward a seasonally varying mean.

The seasonal trend $\bar{\mu}(t)$ incorporates both long-term drift and periodic behavior, and is typically expressed as :

$$\boxed{\bar{\mu}(t) = \alpha + \beta t + \gamma \sin(\omega t + \phi)}$$

with $\omega = \frac{2\pi}{365}$ to capture the annual cycle.

Remark : As we will see, its parameters can be estimated from historical data using least-squares regression. After fitting $\bar{\mu}(t)$, residuals can be further modeled using an autoregressive (AR) process to reflect short-term dependencies.

However, the basic mean-reverting model does not guarantee that the expected temperature exactly follows the seasonal trend, i.e.,

$$\mathbb{E}[T_t] \neq \bar{\mu}(t)$$

To address the issue, we modify the model by adding the derivative of the trend function :

$$\boxed{dT_t = \left(\frac{d\bar{\mu}(t)}{dt} + \kappa(\bar{\mu}(t) - T_t)\right) dt + \sigma(t)\,dW_t}$$

We could use Itô's formula, but it turns out that we can now solve this SDE using the traditional integrating factor method.

Multiplying by $e^{\int_0^t \kappa du}$ we obtain, where the left-hand side of the above expression is just the differential of a product :

$$e^{\int_0^t \kappa du} d\bar{\mu}(u) - e^{\int_0^t \kappa du} \kappa(\bar{\mu}(u) - T_u)du + e^{\int_0^t \kappa du} dT_u = e^{\int_0^t \kappa du} \sigma_t dW_u$$

So,

$$d\left[e^{\int_0^t \kappa du}(\bar{\mu}(u) - T_u)\right] = e^{\int_0^t \kappa du} \sigma_t dW_u$$

If we consider Itô process $Z_t = e^{\int_0^t \kappa du}(\bar{\mu}(u) - T_u)$, then its dynamics are as the equation above :

$$dZ_t = d\left[e^{\int_0^t \kappa du}(\bar{\mu}(u) - T_u)\right] = e^{\int_0^t \kappa du} \sigma_t dW_u$$

$$Z_t = Z_0 - \int_0^t e^{\int_0^s \kappa du} \sigma_s dW_s$$

Now substituting $Z_t = e^{\int_0^t \kappa du}(\bar{\mu}(u) - T_u)$ into the equation above with $\bar{\mu}_0 = T_0$ lead to :

$$e^{\int_0^t \kappa du}(\bar{\mu}(t) - T_t) = e^{\int_0^t \kappa du}(\bar{\mu}_0 - T_0) - \int_0^t e^{\int_0^s \kappa du} \sigma_s dW_s$$

Finally, rearranging we get :

$$\boxed{T_t = \bar{\mu}(t) + e^{-\int_0^t \kappa du} \int_0^t e^{\int_0^s \kappa du} \sigma_s dW_s}$$

This implies immediately that :

$$\boxed{\mathbb{E}[T_t] = \bar{\mu}(t)}$$

This correction ensures that the mean of the stochastic process exactly tracks the deterministic seasonal trend.

In the following, we calibrate this model using historical temperature data by first estimating the seasonal trend $\bar{\mu}(t)$, and subsequently modeling the residuals using an autoregressive (AR) process to capture short-term dependencies.

4.2 Data Retrieval

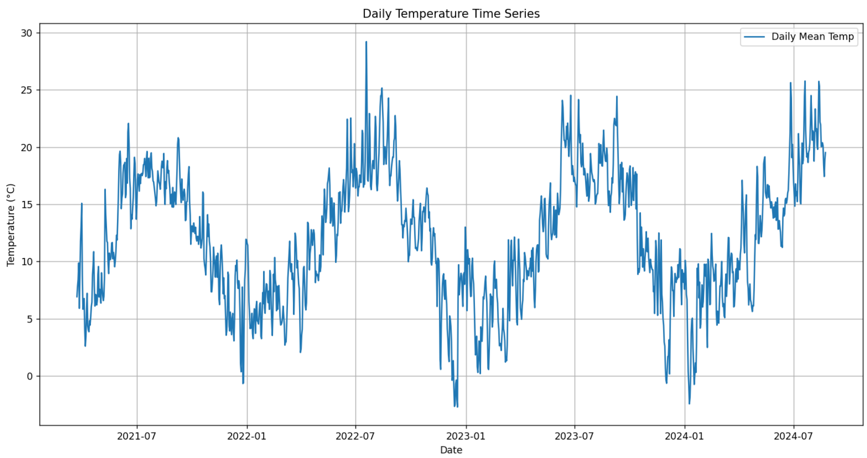
We first collect historical daily and hourly 2-meter temperature data for **Amsterdam** using the *Open-Meteo API*, ensuring reliable and high-resolution weather observations. These time series will allow us to calibrate our model for temperature dynamics.

4.3 Constructing a Stochastic Model

The objective of this section is to construct a model for temperature dynamics that consists of both a deterministic component, capturing the trend and seasonality, and a stochastic component to account for the residual variation.

4.3.1 Mean temperature time series

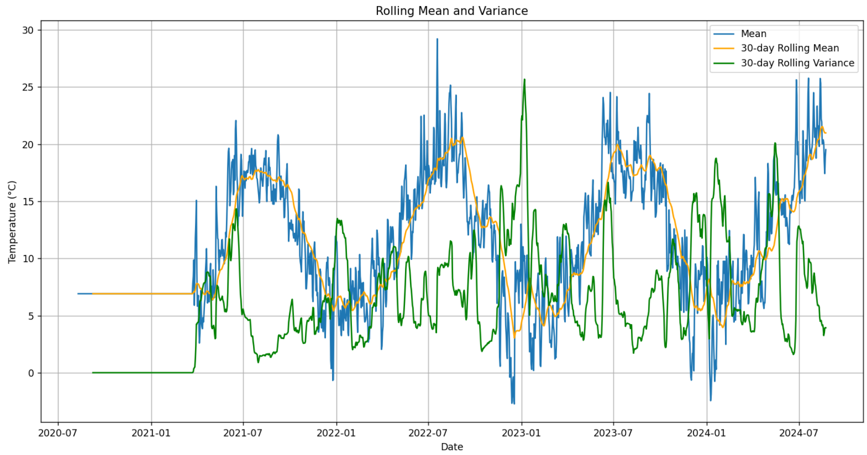
We begin with the plot of the mean temperature time series.



Interpretation : As a first look, we remark a periodic evolution of the temperature through the years, with a maximum and a minimum respectively reached in July and January, which perfectly aligns with what we experience. The plot seems obviously composed of a sinusoidal deterministic trend on which is added a noise.

4.3.2 Rolling mean/variance

To better understand the short-term fluctuations and long-term trends, a rolling mean and rolling variance were calculated using a 30-day window. Here is the graphical result :

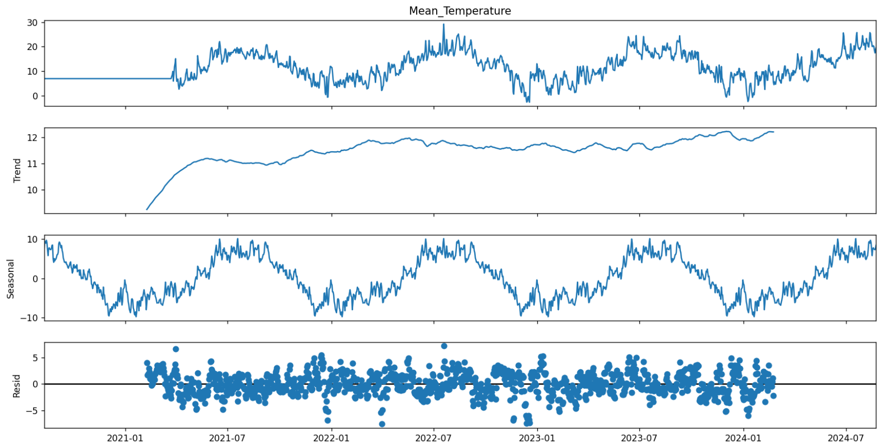


Interpretation : The plot of the daily mean temperature along with its 30-day rolling mean and rolling variance reveals key patterns in the temperature dynamics.
The rolling mean closely follows the original series, smoothing out short-term fluctuations and highlighting seasonal trends.
While the rolling variance shows that the temperature variability is not constant throughout the year.

These observations suggest the presence of strong seasonality and time-varying volatility in the temperature data, which justifies the use of a model containing both a deterministic trend and a stochastic component.

4.3.3 Decomposition

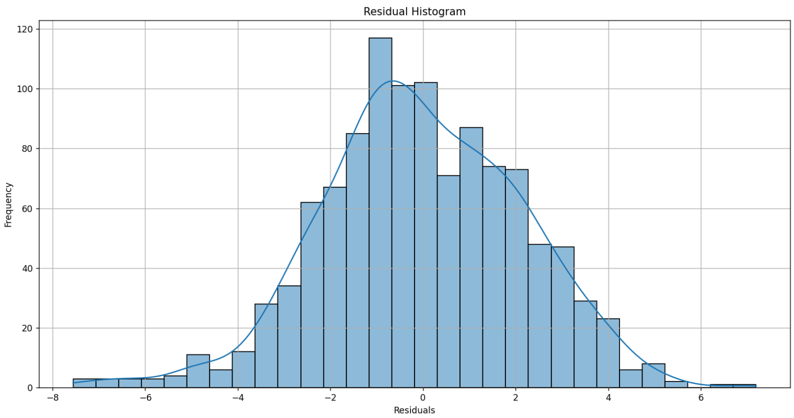
Finally, the time series was decomposed using the seasonal decomposition method, which separated the data into trend, seasonality, and residual components.

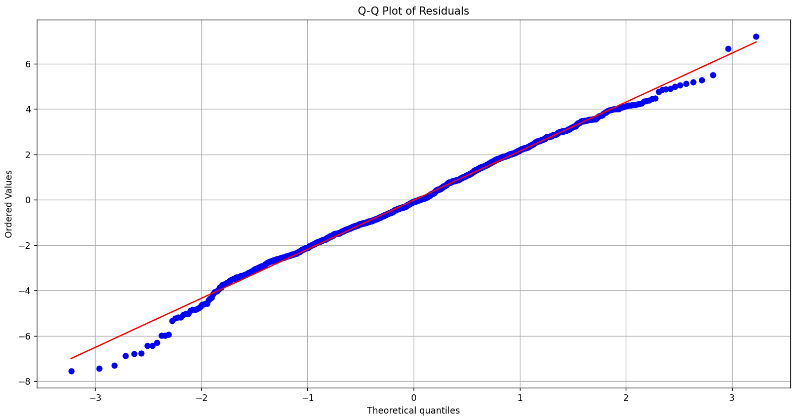


Interpretation : The seasonal decomposition plot of the mean temperature shows a clear yearly seasonality pattern, a relatively smooth upward trend over time, and residuals that appear to be centered around zero. This suggests that the additive model adequately captures the main structure of the temperature dynamics.

4.3.4 Residuals Analysis

Now a residuals analysis is conducted to evaluate the stochastic properties of the residuals. This includes visualizing the residuals using histograms and Q-Q plots, which leads to :





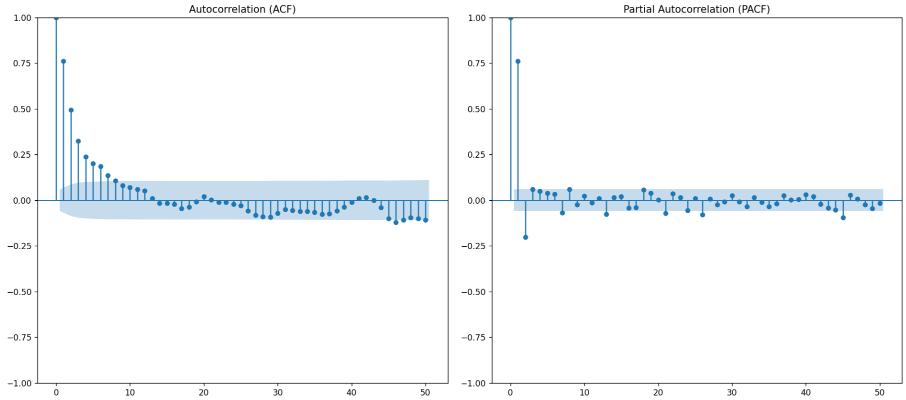
Interpretation : The residual histogram highlights a bell-shaped curve looking like a Gaussian distribution, indicating that the variations not captured by the modelnare random and symmetrically distributed around zero. This suggests that the model effectively captures the deterministic structure of the temperature data, including both its long-term trend and seasonal patterns.

The "normally" distributed residuals support the assumption that the remaining variations can be modeled as stochastic noise.

Furthermore, the Q-Q plot of the residuals aligns closely with the 45 degrees reference line, indicating that the residuals follow a normal distribution. This confirms that the deviations are approximately Gaussian.

4.3.5 Autocorrelation

Finally we compute the autocorrelation (ACF) and partial autocorrelation (PACF) plots, and we apply a statistical tests such as the Augmented Dickey-Fuller test to assess the stationarity of the residuals.



Interpretation : As we can see, the ACF and PACF values drop close to zero after a few lags. This in fact suggests that the residuals are stationary. Indeed, this behavior indicates that there is no significant autocorrelation remaining in the data, meaning that the residuals do not exhibit any predictable patterns over time. The model has likely captured all the time-dependent structures in the data, and the residuals are now essentially random, with no further dependency or trend.

4.3.6 Augmented Dickey-Fuller Test

Here we give the results from the ADF test :

- **ADF Statistic :** -6.8741
- **p-value :** 1.49×10^{-9}
- **Critical Value (1%) :** -3.4364
- **Critical Value (5%) :** -2.8642
- **Critical Value (10%) :** -2.5682

The ADF test yields a test statistic of -6.8741, which is significantly lower than all the critical values at the 1%, 5%, and 10% significance levels. Furthermore, the p-value is extremely small, far below the conventional threshold of 0.05.
These results highlight that we can reject the null hypothesis of a unit root, providing strong evidence that the residuals are stationary.

4.4 Fitting the model under the real measure $\mathbb{P}$

4.4.1 Deterministic Model Fitting

The model is gonna be calibrated under the physical measure by estimating a deterministic trend-seasonality component.
Consider the following expression for T :

$$T(t) = a + bt + \alpha \sin(\omega t + \theta)$$

Remark : Here we model T according to our previous $\bar{\mu}(t)$.

Using historical daily temperature data, we estimate the parameters a, b, α, and θ with the `curve_fit` function from the `scipy.optimize` library, which minimizes the squared differences between the observed temperatures and the model.

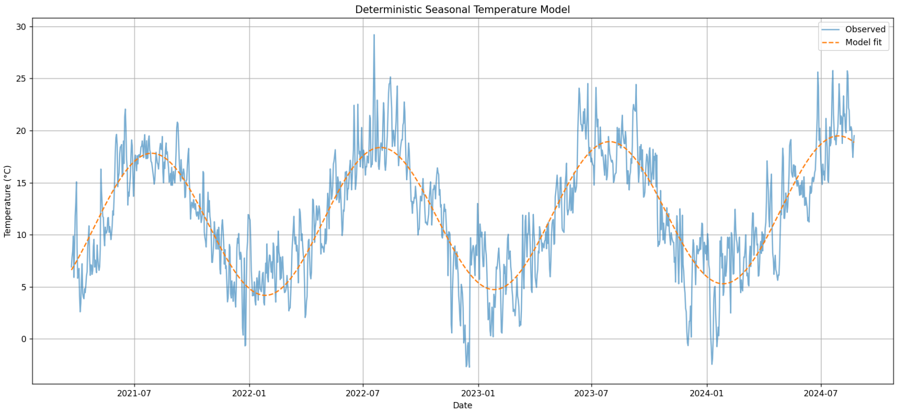
After implementation on Python, we get :

- $a = 10.6639$
- $b = 0.0015$
- $\alpha = 6.9630$
- $\theta = -6.9051$

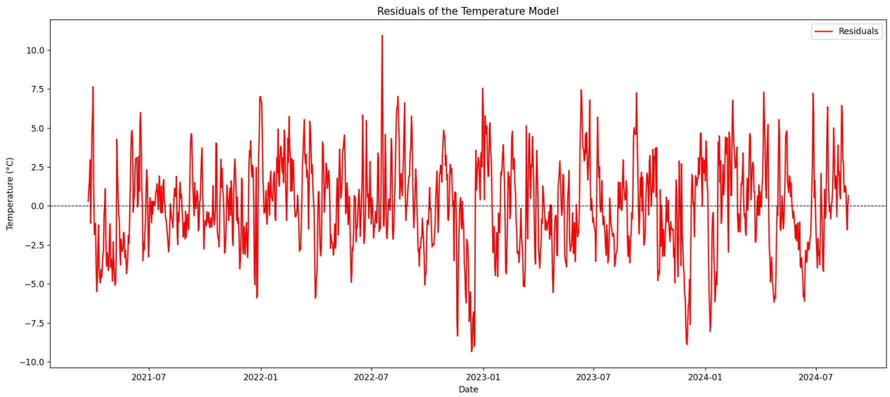
where,

- a represents the estimated average annual temperature for Amsterdam. Such a value of a aligns with the typical average temperature observed in Amsterdam, reflecting a realistic baseline temperature.
- b indicates a slight upward trend in the temperature over time. The positive value suggests a gradual increase in annual temperature, although this trend is modest. This could be related to long-term climatic changes or other factors affecting the region.
- α represents the amplitude of the seasonal temperature variation. A value of α equal to 6.96°C indicates a significant seasonal temperature variation, with maximum temperatures in summer reaching about 17°C and minimum temperatures in winter around 4°C, which is typical for Amsterdam ; even if the temperature reaches more extrem values occasionally during summer and winter.
- A value of θ close to -2π suggests that the peak temperature occurs just before or after the typical maximum, which is intuitive.

Now we give the graphs of comparison between the model with the previous determined parameters and the real temperature evolution.



Furthermore we calculate the residuals, defined as the differences between the actual observed temperatures and the model predictions. The following graph gives us the residual fluctuations :



Interpretation : Although the deterministic model captures the general seasonal trend and long-term linear evolution of temperature, the residuals reveal significant day-to-day fluctuations. Sometimes even reaching up to 10 degrees Celsius.
These residuals represent the portion of the temperature dynamics that is not explained by the seasonal model and are due to short-term meteorological variability, such as passing weather fronts, cold spells, or heatwaves. Such fluctuations confirm that the deterministic model alone is insufficient to fully capture the observed temperature behavior.
Therefore, to model these deviations more accurately, it is necessary to introduce a stochastic component. This leads to the formulation of the stochastic

differential equation we highligh before, which incorporates random fluctuations around the seasonal mean, thereby providing a more realistic representation of temperature dynamics for both simulation and forecasting purposes.

4.4.2 Residuals

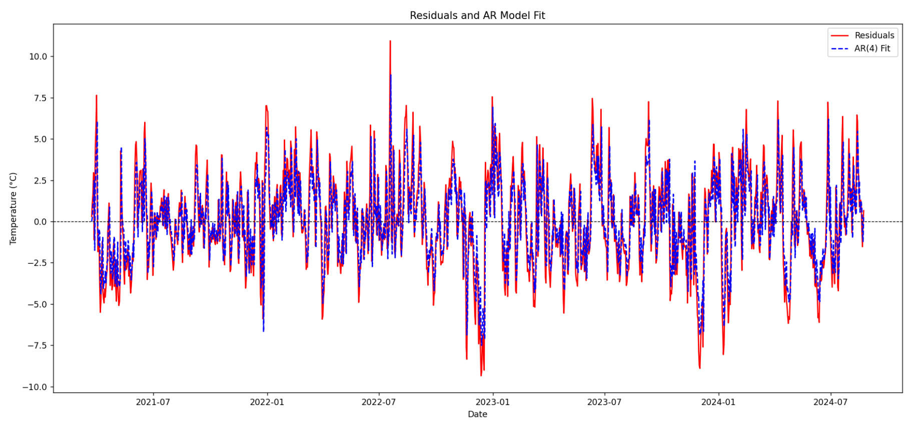
Now we focus on the residuals using an autoregressive (AR) process to capture short-term dependencies.
The autoregressive (AR) model is a type of model where the current value of the series is regressed on its previous values, capturing temporal dependencies in the residuals. In fact, the AR model of order p uses the past p observations (here p represents the number of "period" we look back to) to predict the current value. In our case, the period is given in years, so p represents the number of years we look back to.

Mathematically it is written as :

$$\hat{T}_i = \gamma \hat{T}_{i-1} + Z_i,$$

where $\hat{T}_t = T_t - \bar{\mu}(t)$ is the residual, γ is the autocorrelation coefficient, and $Z_i \sim \mathcal{N}(0, \sigma_i^2)$ is the white noise.
As soon as $\gamma = 0$, there is no autocorrelation and residuals are purely random noise.

To determine the optimal order of the AR model, we used the Akaike Information Criterion (AIC). AIC allows to select the model that best balances fit accuracy and model complexity. Lower AIC values indicate better fitting models. In our case, the optimal value of p is 4.
Here is the graph resulting of the regression :



Interpretation : The result of the AR model fitting is a smoothed version of the initial residuals' one, where extreme fluctuations are reduced, and the overall trend of the data is better captured.
Particularly, this implies that the remaining noise (not taken into account by the model) has temporal dependence, which is now accounted for by the AR process.
Moreover, in our case, where the model is based on a 4-year period, it it makes sense for the optimal value of p to be 4.
Indeed each year in the model corresponds to a distinct cycle, and using 4 years allows the model to capture the dependencies and residual patterns across the total period. Thus this value enable to take into account the fluctuations from the previous 4 years, which is particularly relevant knowing the model can effectively consider long-term temperature dynamics and seasonal variations across multiple years.

4.4.3 Mean-reversion parameter estimation

Now the aim is to use the stochastic modeling evoked before. We estimate the mean-reversion parameter κ by discretizing the stochastic differential equation for temperature dynamics using the Euler method.

Recall that we have the following SDE for the temperature dynamics :

$$dT_t = \left(\frac{d\bar{\mu}(t)}{dt} + \kappa \left(\bar{\mu}(t) - T_t \right) \right) dt + \sigma_t dW_t$$

First, consider the Euler discretization of our SDE over the interval $[i-1, i]$:

$$T_i - T_{i-1} = \bar{\mu}_i - \bar{\mu}_{i-1} + \kappa \left(\bar{\mu}_{i-1} - T_{i-1} \right) + \sigma_i z_i$$

Rewriting, we get :

$$T_i - \bar{\mu}_i = T_{i-1} + \bar{\mu}_i - \bar{\mu}_{i-1} - \kappa \left(T_{i-1} - \bar{\mu}_{i-1} \right) + \sigma_i z_i$$

where $z_i \sim \mathcal{N}(0, 1)$.

Now let us define the de-trended and deseasonalized temperature as :

$$\hat{T}_t = T_t - \bar{\mu}(t)$$

The discretized form of the model becomes :

$$\hat{T}_i = \hat{T}_{i-1} - \kappa \hat{T}_{i-1} + \sigma_i z_i,$$

which is equivalent to an autoregressive process of order 1 (AR(1)) :

$$\hat{T}_i = \gamma \hat{T}_{i-1} + \varepsilon_i,$$

where $\gamma = 1 - \kappa$, and $\varepsilon_i = \sigma_i z_i$ with $z_i \sim \mathcal{N}(0, 1)$.

By fitting an AR(1) model to the temperature series $\hat{T}_i$, we can estimate γ, and thus compute the mean-reversion speed as $\kappa = 1 - \gamma$.

The Python implementation gives us :

$$\boxed{\kappa = 0.0531 = 5.31\%}$$

Interpretation : κ controls the speed at which temperature returns to the mean after deviation.
This value means that only about 5.31% of the deviation is corrected each day, which implies slow but steady mean reversion.
This seems realistic for daily temperature data, which typically does not revert to the seasonal average abruptly, but indeed gradually over several days.

4.4.4 Calibrating volatility

What we have seen so far is that empirically the magnitude of the residuals varies over the course of the year, indicating that the volatility itself has a seasonal pattern.
To model the stochastic fluctuations in temperature around its seasonal mean, we introduce a time-dependent volatility function $\sigma(t)$ in the stochastic differential equation.
Since such pattern is periodic, a natural and mathematically efficient way to model $\sigma(t)$ is to use a Fourier series expansion.
We also include a linear trend $V + Ut$ to account for potential long-term variations in annual volatility, such as the fact that colder months tend to have higher variance than warmer ones.
Moreover, the introduction of the constant term V (which has to be relevantly chosen) ensures the positivity of $\sigma(t)$.

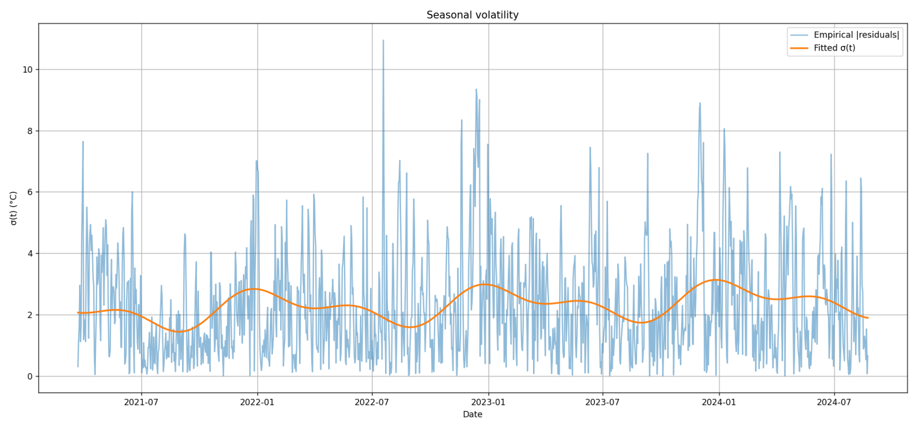
To estimate the seasonal volatility function $\sigma(t)$, we begin by computing the residuals from the deterministic seasonal model, and interpret their absolute values as empirical proxies for volatility. Given the observed periodic behavior of these residuals, we fit a function of the form

$$\sigma(t) = V + Ut + \sum_{i=1}^{I} c_i \sin(i\omega t) + \sum_{j=1}^{J} d_j \cos(j\omega t),$$

where $\omega = \frac{2\pi}{365.25}$.

We estimate the parameters V, U, c_i, d_j using a least-squares fit on the daily absolute residuals.

Here we throw light on the graphical result of the simulation :



Remark : We deliberately restrict the number of sine and cosine terms in the Fourier expansion ($I = J = 2$) to avoid overfitting.
Indeed, since the seasonal variation in volatility is mainly driven by annual and semiannual cycles, the first harmonics (with frequencies ω and 2ω) are sufficient to capture the dominant seasonal effects observed in temperature residuals.
Including higher-frequency terms would just increase model complexity without substantial gains in accuracy.

The estimated parameters for the seasonal volatility model $\sigma(t)$ are :

$$\boxed{V = 2.0641, \quad U = 0.0004, \quad c_1 = -0.3676, \quad c_2 = 0.1053, \quad d_1 = 0.2908, \quad d_2 = -0.2903}$$

Interpretation :

- V represents the constant baseline level of volatility, capturing the overall level of variation in temperature residuals across the entire year.
- U corresponds to the linear trend in volatility, capturing gradual changes in volatility over time. Given the small magnitude, it indicates that volatility changes very slightly over the year, with a negligible upward trend.
- c_1 and c_2 capture the periodic oscillations in volatility. The negative value of c_1 suggests that volatility is lower during certain parts of the year (likely in the summer), while the positive value of c_2 indicates higher volatility at other times (likely in the winter). Together, these coefficients model the annual cycle of volatility.
- d_1 and d_2 model additional periodic fluctuations in volatility. The similar magnitudes of d_1 and d_2, but with opposite signs, suggest that the volatility pattern is symmetric, with peaks and valleys occurring at regular intervals throughout the year, according to the graph.

5 Temperature Options pricing

Finally, based on the fitted model, we can perform Monte Carlo simulations to generate temperature paths and compute the expected payoff of weather derivatives, such as HDD and CDD options.

5.1 Euler Scheme

In this context we use an Euler discretization scheme, to implement path simulations.
We want to model the evolution of the temperature over time, considering both a deterministic seasonal trend and stochastic residuals modeled by an autoregressive (AR) process. The temperature evolution is thus given by the following equation :

$$\boxed{T_{t+\Delta t} = T_t + \kappa(\mu(t) - T_t + \epsilon_t)\Delta t + \sigma(t) dW_t}$$

Where :

- T_t is the temperature at time t
- $\mu(t)$ is the deterministic seasonal trend at time t
- κ is the mean-reversion rate, controlling how the temperature reverts to the trend
- ϵ_t is the residual term, modeled using the AR process
- $\sigma(t)$ is the volatility of the temperature at time t
- dW_t is the Wiener process increment

The update formula consists of two components :

1. The deterministic part, the mean-reversion term, $\kappa(\mu(t) - T_t + \epsilon_t)\Delta t$, which reflects the predictable seasonal pattern and forces the temperature to revert toward the seasonal trend, with some deviation caused by the residuals ϵ_t.
2. The stochastic part $\sigma(t)dW_t$, which introduces randomness into the temperature based on the volatility function $\sigma(t)$ and a Wiener process increment dW_t, simulating the noise from the AR process.

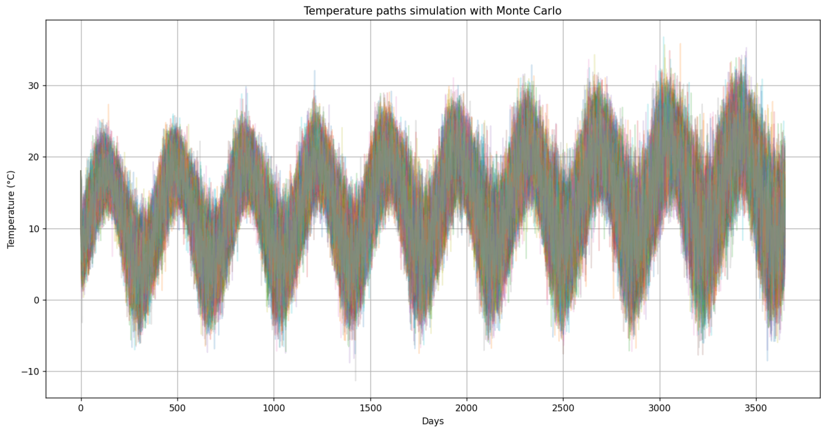
Remark : Here we have :

$$dW_t = W_{t+\Delta t} - W_t \sim \mathcal{N}(0, \Delta t)$$

so we generate paths using $dW_t = \sqrt{\Delta t} \cdot Z$ for $Z \sim \mathcal{N}(0, 1)$.

The simulation is performed in discrete time steps with the temperature updated iteratively at each step and the residuals ϵ_t are generated from the fitted AR model.

The resulting simulation produces a time series of temperature values.
We moreover use the values determined before to model $\mu(t)$ and $\sigma(t)$.
For $M = 100$ simulated paths over 10 years, we get the following graph :



Interpretation : The simulation incorporates both a deterministic trend-seasonality model and a stochastic component including mean-reverting residuals and time-varying volatility. The deterministic part is calibrated to capture the annual temperature cycle in Amsterdam, producing clear sinusoidal patterns aligned with seasonal transitions.
The changes in the amplitude of noise throughout the year is consistent with empirical weather behavior, where transitional seasons tend to have more volatile temperatures than mid-summer or mid-winter.

5.2 Heating Degree Day (HDD) Option Pricing

According to the previous paths simulation we can go further and provide the pricing of Temperature derivatives.

As a beginning, we focus on a **call** option with a classical payoff function $max(0, DD - K)$, where DD represents the cumulative degree days over the period of interest.

In this simulation, we consider a HDD call option over a 3-months ($T = 90$ days) winter period (January 10 to April 10, 2021) for the city of Amsterdam. The notional amount per HDD unit is set to $N = 20$, and the strike level is fixed at $K = 200$. These values were chosen based on typical contractual specifications in the CME weather derivatives market, where nominal values often range between \$20 and \$100 per degree-day unit, and strike levels are usually aligned with climatological expectations for the region, based on publicly available meteorological data (Copernicus website).

The option price is computed using the Monte-Carlo estimator :

$$C_{\text{HDD}} = e^{-rT} \frac{1}{M} \sum_{i=1}^{M} N \max(H_T^{(i)} - K, 0)$$

where $H_T^{(i)}$ is the total HDD accumulated on the i-th simulated path, and r is the risk-free rate, assumed constant at 1%. M denotes the number of generated paths.

Result : The resulting simulated option price is approximately of $\boxed{C_{HDD} = \mathbf{\$19600}}$. This result is consistent with expectations.

Indeed :

$$\mathbb{E}[\text{Payoff}] = N \max(\mathbb{E}[H_T] - K, 0) = 20(\mathbb{E}[H_T] - 200) \approx \$19600 \implies \mathbb{E}[H_T] \approx 1180$$

Interpretation : The result for the period from January 10 to April 10, 2021, seems to be relevant. Indeed, the estimated option price of approximately \$19600 aligns well with the expectations based on climatological averages. The average cumulative HDDs of around 1180 over 90 days, implying an *average daily temperature* of approximately **5°C**, indicate that the weather during this period was indeed much colder than 18°C, as expected during the winter months in Amsterdam.

Conclusion : This temperature level is consistent with typical winter conditions in the city. This highlights the model accuracy.

5.3 Cooling Degree Day (CDD) Option Pricing Results

Now we focus on a put option in a different season.

In this simulation, we consider a CDD put option over a 3-month period ($T = 90$ days), spanning from September 10 to December 10, 2020, for Amsterdam. The notional amount per Cooling Degree Day (CDD) is set to $N = 20$, and the strike level is now set at $K = 80$. The selected period corresponds to the autumn season, where cumulative temperatures tend to be low.
The option price is computed using the Monte-Carlo estimator :

$$P_{\text{CDD}} = e^{-rT} \frac{1}{M} \sum_{i=1}^{M} N \max(K - H_T^{(i)}, 0)$$

where $H_T^{(i)}$ is the total CDD along the i-th simulated path, and $r = 1\%$ is the constant risk-free rate. M denotes the number of simulation paths.

Result : The simulated option price is approximately $\boxed{P_{CDD} = \mathbf{\$1480}}$.

Given the payoff structure:

$$\mathbb{E}[\text{Payoff}] = N \max(K - \mathbb{E}[\text{CDD}], 0),$$

we can infer the expected cumulative Cooling Degree Days (CDD) during this period :

$$1480 = 20(80 - \mathbb{E}[\text{CDD}]) \Rightarrow \mathbb{E}[\text{CDD}] = 6.$$

Interpretation : This expected value aligns with climatological data for Amsterdam, where average daily temperatures drop below the 18°C threshold throughout September to November. As a result, CDD accumulation becomes minimal, with only a few days (here 6) potentially contributing to the index in September.

Conclusion : The low expected CDD value and resulting option price are consistent with typical weather data.

6 Conclusion

Our pricing results suggest that the protocol is likely to be correct.
This pricing model is thus accurate, providing good insights into how temperature-based contracts can be valued in real-world scenarios.
Weither it is the consistence of the regressions aiming to calibrate the temperature dynamics model or the consistency of the simulated option prices, they validate the methodology, showcasing its potential application for future weather derivative contracts valuation.

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7 References

I referred to the following paper as support on certain points through this work : https://digitalcommons.lsu.edu/cgi/viewcontent.cgi?article=1026&context=josa

For Temperature time series documentation : https://github.com/open-meteo/open-meteo

For meteorological data : https://www.copernicus.eu/fr