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Quantum HW #8
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1.) $\Psi(x,y,z) = \frac{1}{4\sqrt{\pi}} \frac{2z^2 - x^2 - y^2}{r^2} + \sqrt{\frac{3}{\pi}} \frac{xz}{r^2}$

$\Rightarrow$ Rewrite in terms of spherical harmonics

$$\Psi = \sqrt{\frac{1}{5}}\, Y_{2,0} + \sqrt{\frac{2}{5}}\left(Y_{2,-1} - Y_{2,1}\right)$$

(a) $\underline{l = 2}$ for all possible states.

(b)
$P(m=0) = \underline{\frac{1}{5}}$, $\underline{L_z = 0}$

$P(m=1) = P(m=-1) = \underline{\frac{2}{5}}$, $\underline{L_z = \pm\hbar}$ respectively

ok!

2.) $\Psi_0 = |l=2, m=0\rangle$

$$D(0,\beta,0) = \sqrt{\frac{4\pi}{(2l+1)}}\, Y_l^{m\,*}(\theta,\phi)\Big|_{\theta=\beta,\ \phi=0} \qquad (3.6.52)$$

$P(m=0) = \left|\langle 2,0|\sqrt{\frac{4\pi}{5}}\, Y_2^0\,|2,0\rangle\right|^2 = \underline{\frac{1}{4}\left(3\cos^2(\beta) - 1\right)^2}$

$P(m=\pm1) = \left|\langle 2,\pm1|\sqrt{\frac{4}{5}}\, Y_2^{\pm1}\,|2,0\rangle\right|^2 = \left|\pm\sqrt{\frac{3}{2}}\sin\beta\cos\beta\right|^2 = \underline{\frac{3}{2}\sin^2(\beta)\cos^2(\beta)}$ ✓

$P(m=\pm2) = \left|\langle 2,\pm2|\sqrt{\frac{4}{5}}\, Y_2^{\pm2}\,|2,0\rangle\right|^2 = \left|\sqrt{\frac{3}{8}}\sin^2(\beta)\right|^2 = \underline{\frac{3}{8}\sin^4(\beta)}$

✓

$\Rightarrow$

$|j_1, j_2; m_1, m_2\rangle$ $\frac{2}{\sqrt{6}} = \frac{2\sqrt{6}}{6} =$

Order convention

3.)

(a) Trivially, we can see that the $j=2$, $m=\pm 2$ states correspond to $1|1,1;1,1\rangle$ and $1|1,1;-1,-1\rangle$ respectively. 1, 2

The intermediate $j=2$ states can be found by applying ladder operators.

$$J_-|J=2, m=2\rangle = \sqrt{(j+m)(j-m+1)}\,|J=2, m=1\rangle \qquad J_- = J_{1-} + J_{2-}$$

$$\sqrt{(2+2)(1)}\,|j=2, m=1\rangle = \sqrt{(j_1+m_1)(j_1-m_1+1)}\,|1,1;0,1\rangle + \sqrt{(j_2+m_2)(j_2-m_2+1)}\,|1,1;1,0\rangle$$

$$\Rightarrow\ 2|j=2, m=1\rangle = \sqrt{2}\,|1,1;0,1\rangle + \sqrt{2}\,|1,1;1,0\rangle$$

$$|j=2, m=1\rangle = \frac{\sqrt{2}}{2}|1,1;0,1\rangle + \frac{\sqrt{2}}{2}|1,1;1,0\rangle \quad 3$$

Continuing on...

$$J_-|j=2, m=1\rangle = \sqrt{2\cdot 3}\,|J=2, m=0\rangle$$

$$= \frac{\sqrt{2}}{2}\left(\sqrt{2}\,|1,1;0,0\rangle + \sqrt{2}\,|1,1;-1,1\rangle + \sqrt{2}\,|1,1;1,-1\rangle + \sqrt{2}\,|1,1;0,0\rangle\right)$$

$$= 2|1,1;0,0\rangle + |1,1;-1,1\rangle + |1,1;1,-1\rangle$$

$$\Rightarrow\ |j=2, m=0\rangle = \frac{2}{\sqrt{6}}|1,1;0,0\rangle + \frac{1}{\sqrt{6}}|1,1;-1,1\rangle + \frac{1}{\sqrt{6}}|1,1;1,-1\rangle \quad 4$$

$$|j=2, m=-1\rangle = \frac{\sqrt{2}}{2}|1,1;0,-1\rangle + \frac{\sqrt{2}}{2}|1,1;-1,0\rangle \quad 5$$

For $|j=1, m=1\rangle$, exactly one of $m_1, m_2 = 1$ and exactly one of $m_1, m_2 = 0$. Since there is no reason to prefer one over the other, I will start with

got the sign from a table...

$$|j=1, m=1\rangle = \frac{2}{\sqrt{2}}|1,0\rangle - \frac{2}{\sqrt{2}}|0,1\rangle \quad 6 \quad (|m_1, m_2\rangle \text{ shorthand convention})$$

$$J_-|j=1, m=1\rangle = \sqrt{2}\,|j=1, m=0\rangle = \frac{2}{\sqrt{2}}\left[\cancel{\sqrt{2}|0,0\rangle - \sqrt{2}|0,0\rangle} + \sqrt{2}|1,-1\rangle - \sqrt{2}|-1,1\rangle\right]$$

$$\Rightarrow\ |j=1, m=0\rangle = \frac{2}{\sqrt{2}}\left[|1,-1\rangle - |-1,1\rangle\right] \quad 7$$

Cont. ⟹

From here it is easy to see

$$|j=1, m=-1\rangle = \frac{2}{\sqrt{2}}|0,-1\rangle + \frac{2}{\sqrt{2}}|-1,0\rangle$$ 8 ✓

Lastly, for $|j=0, m=0\rangle$, we have a linear combination of $|1,-1\rangle$, $|-1,1\rangle$, and $|0,0\rangle$, again with no preference

again, sign came from a table...

So $$|j=0, m=0\rangle = \frac{\sqrt{3}}{3}\left[|1,-1\rangle + |-1,1\rangle - |0,0\rangle\right]$$ 9 ✓

(b) ρ^0 has $I=1$, $I_z=0$

Each π^0 also has $I=1$, $I_z=0$.

For the ρ^0 to decay into $\pi^0\pi^0$, the only possible final state (conserving isospin) is $|j_1=1, j_2=1, m_1=0, m_2=0\rangle$

Spin analog to isospin

For the $|j=1, m=0\rangle$ decay, this transition has a coefficient of zero (by #7) so the decay does not occur!

This makes sense now because strong / EM interactions conserve parity, while this decay process does not.

ok!

15/15