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ELEN3081
Experiment 4
Dec 3, 2021

Simulating the Circuit

In building the initial circuit, seen in *Figure 1*, and reading the voltage for the output at vx, we can run a transient simulation to see the dampening in the circuit.

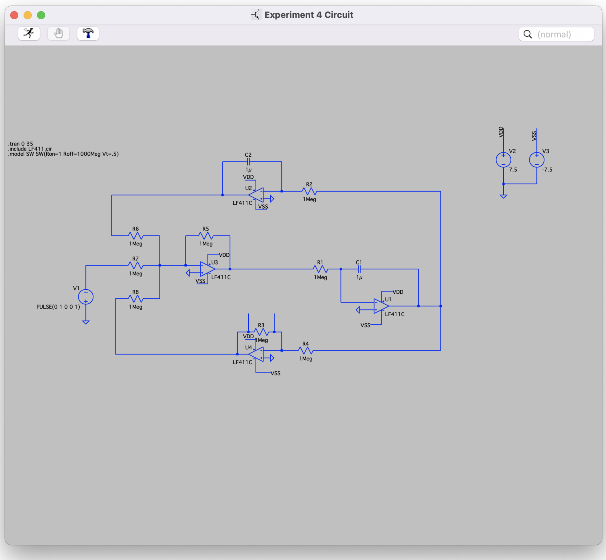

Figure 1: The initial circuit, modeling a constant-coefficient second order differential equation.

When run, this circuit has a response point x(t) that is seen in *Figure 2*. This response is either critically or overdamped, although at this point we are not sure which is the case—we'll explore this later, however. We can then use the equations for ω, ζ, and Q. Plugging these into the second order system we can see an ideal response in *Figure 3*. Here ω = 500Hz, ζ = 1, and Q = 0.5.

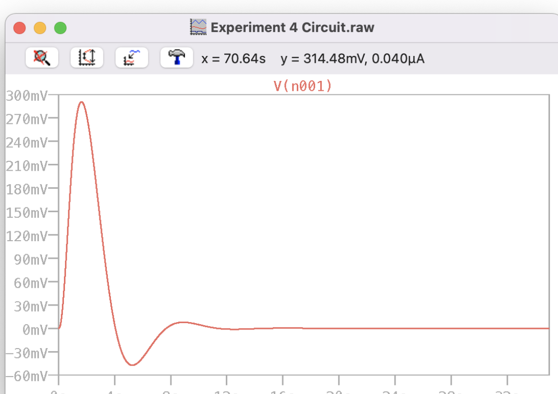

Figure 2: The response at x(t) for the initial circuit, seen in Figure 1.

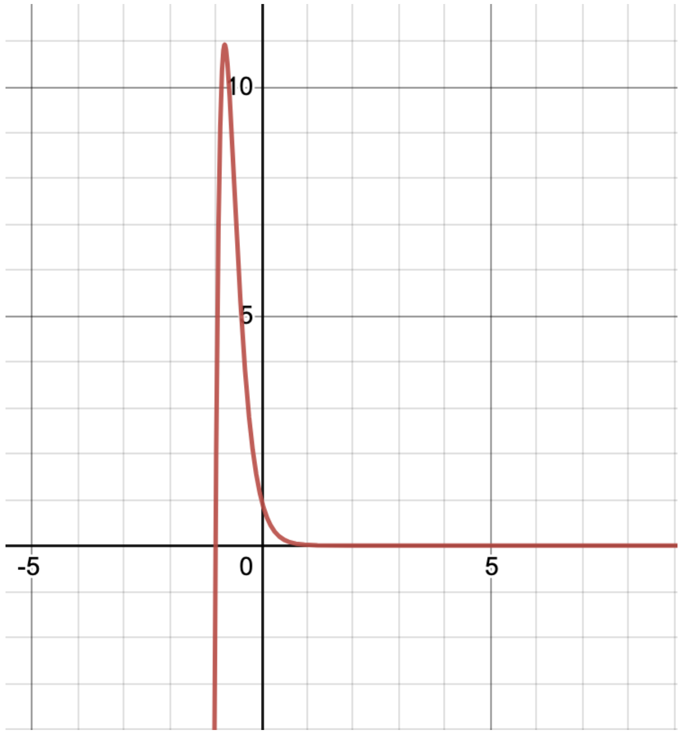

Figure 3: The ideal response of this circuit.

Further, we can short R3 which results in the graph seen in *Figure 4*. This graph is consistently underdamped, as expected since oscillations continue throughout the analysis. To see some convergence, however, we'll need to have some small resistance across this stage. This can be achieved through the addition of a relatively small—in this case 200kΩ—resistor being placed where the short for R3 occurred. The resulting graph, seen in *Figure 5*, demonstrates the consistently underdamped case while still showing convergence, unlike that of *Figure 4*.

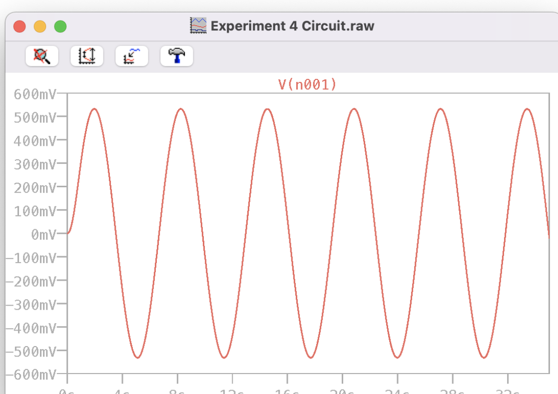

Figure 4: The consistently(and non-converging) underdamped case.

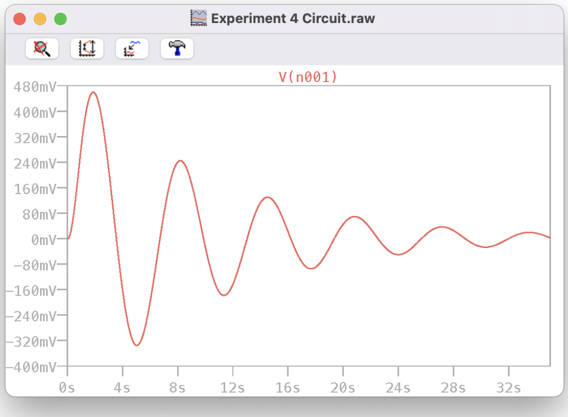

Figure 5: The converging consistently underdamped case.

We can calculate cases, like that of critical damping, through analyzing the components and their connections to the critical frequency, quality factor, and damping factor of the circuit. For example, the following calculations demonstrate a method used to find that critical damping occurs when R3=1MΩ.

$$\frac{R3R5}{R4R8} = K_3 \qquad \omega_0 = \sqrt{K_1/\tau_1\tau_2} \qquad Q = 1/2 \qquad \zeta = \sqrt{K_1/\tau_1\tau_2}$$

$$\zeta = 1,\ \alpha = \omega_0 1 = \sqrt{(1M\Omega/1M\Omega)/((1M\Omega * 1\mu F)^2} * (1M\Omega * 1\mu F)/((R3 * 1M\Omega)/(1M\Omega * 1M\Omega))$$

$$R3 = 1M\Omega$$

This would suggest that for 10x the critical damping, R3 must equal 10MΩ. Likewise, for 0.1x the critical damping, R3 must equal 100kΩ.

The critically damped case, when R3=1MΩ was already analyzed, as it was the initial resistor value for this circuit. The figure for x(t) critically damped can be seen in *Figure 2*.

The overdamped case, with R3 at 10MΩ, can be seen in *Figure 6* and the underdamped case, with R3 at 100kΩ, can be seen in *Figure 7*.

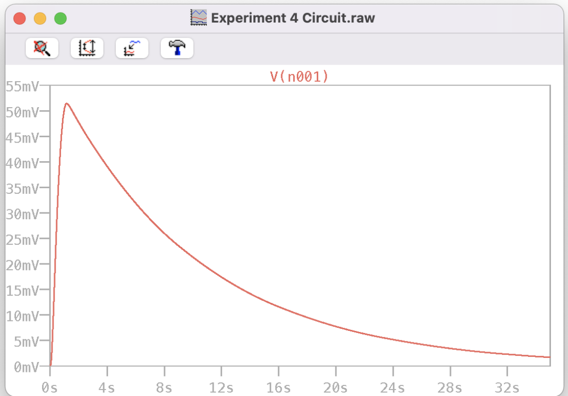

Figure 6: An overdamped case with R3 at 10MΩ.

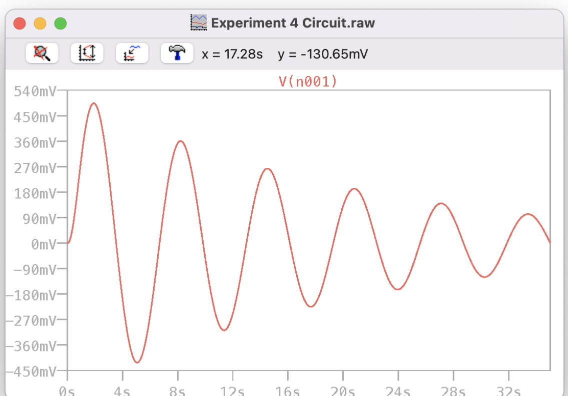

Figure 7: An underdamped case with R3 at 100kΩ.

Increasing the natural frequency of the circuit by a factor of 100, we can calculate that R3 must now be equal to 100kΩ to achieve critical damping. The following calculation shows this.

$$When\ R5\ =\ 5M\Omega,\ R1\ =\ 50k\Omega,\ then\ \omega_0\ =\ \sqrt{R5/R6/(R1C1\ *\ R2C2)},\ \ \omega_0\ =\ 100\ \omega\ \ where\ \omega\ \ is\ the\ initial\ frequen$$

This is as a result of R5 taking a value of 5MΩ and R1 taking a value of 50kΩ. This now also means that R3 is overdamped with a value of 1MΩ and is underdamped with a value of 10kΩ. The critically damped case can be seen in *Figure 8*, while the underdamped and overdamped cases can be seen in *Figure 9* and *Figure 10*, respectively.

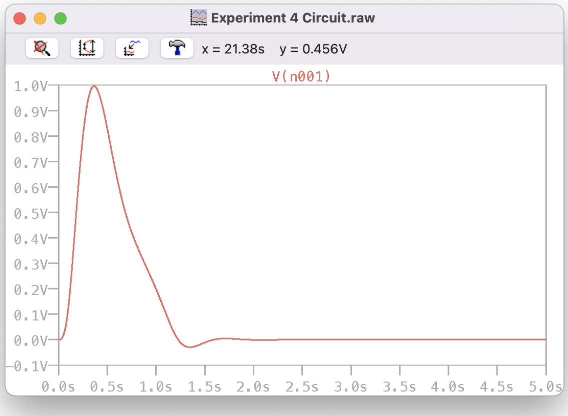

Figure 8: The critically damped case with 100ω.

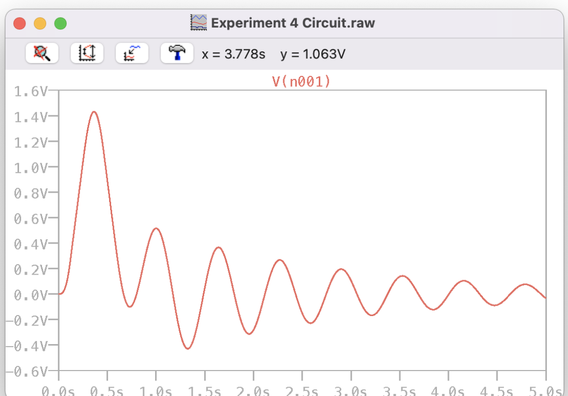

Figure 9: The underdamped case with 100ω.

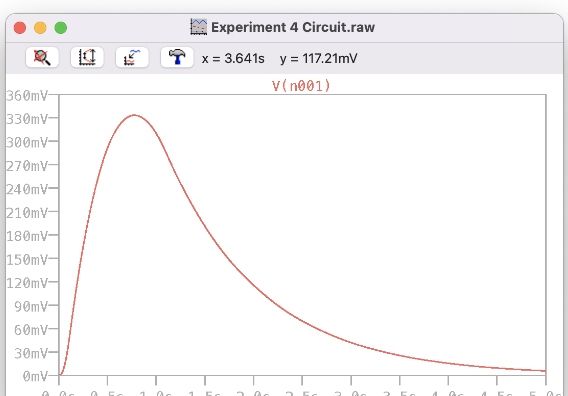

Figure 10: The overdamped case with 100ω.

While keeping the natural frequency at a multiple of 100, we can find the frequency response at the point x(t). The response is that of *Figure 11*. The simulated response can be seen in *Figure 12*.

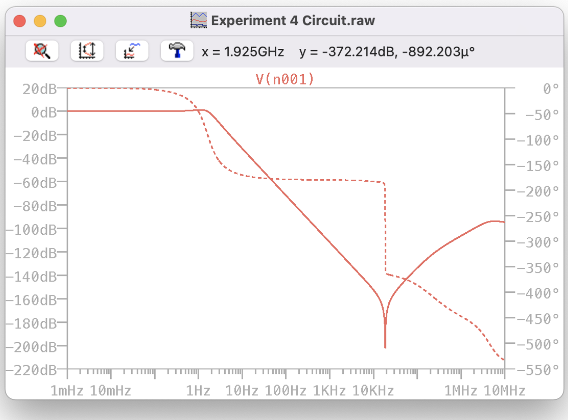

Figure 11: The frequency response of the circuit at x(t).

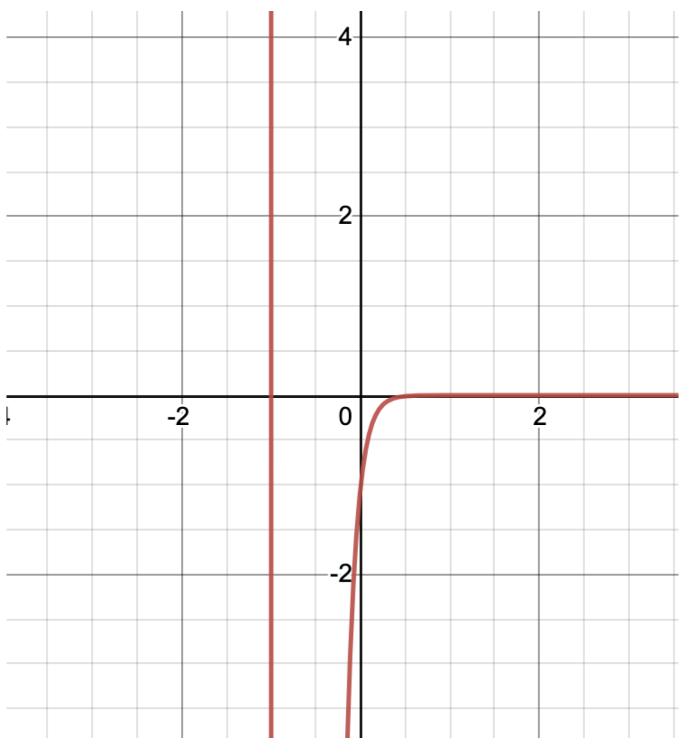

Figure 12: The simulated response of the circuit at x(t).

We can then make changes, such as setting the values of R3 and K3 to achieve a Q of 10. We can do this by recognizing the new value of K3 becomes $K3 = 10k\Omega * 5M\Omega/1M\Omega * 1M\Omega = 0.05$. When we change these values to achieve this quality factor we see the frequency responses changes slightly, producing the output seen in *Figure 13*. We can see that the cutoff frequencies are much more gradual, especially at low frequencies, suggesting that there is a larger bandwidth when the quality factor is increased.

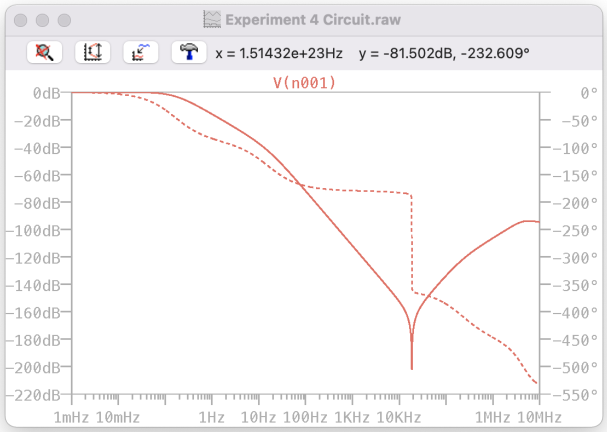

Figure 13: The frequency response of the circuit when Q=10.

Simulating the frequency response at U1, we can see the output in *Figure 14*, indicating that this is a low pass filter.

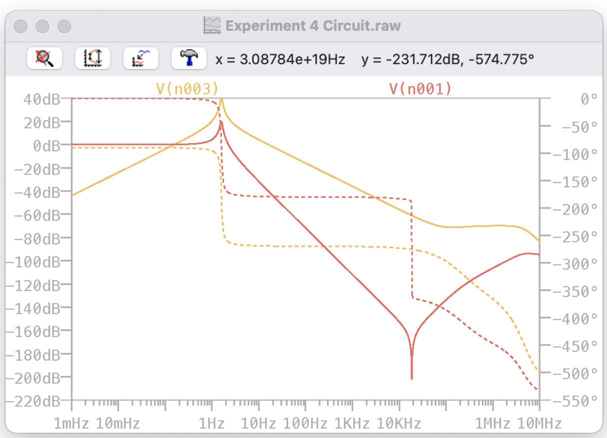

Figure 14: The frequency response of U1 shown in orange.

We can verify the relationship between the bandwidth, center frequency, and Q by looking at the equation $Q = \omega_r/B$. Here, we know that the quality factor is 10 and that $\omega_r = 1Hz$. From this, we can conclude that the bandwidth is 0.1Hz. There is an additional frequency response that can be seen at the output of U3. This stage of the circuit is proportional to the second derivative of x, which has a frequency response seen in *Figure 15*.

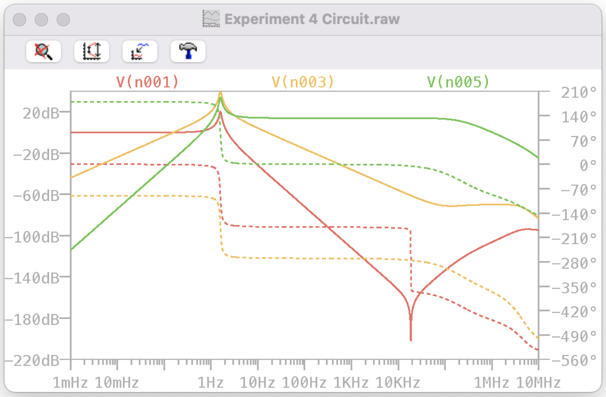

Figure 15: The frequency response of U3 shown in green.

Building the Circuit

The circuit seen in *Figure 1* can be constructed using physical components. When tested, we expect this circuit to have responses much like that seen in the simulation, barring errors due to parasitic resistance, physical limitations, etc. The physical circuit can be seen in *Figure 16*.

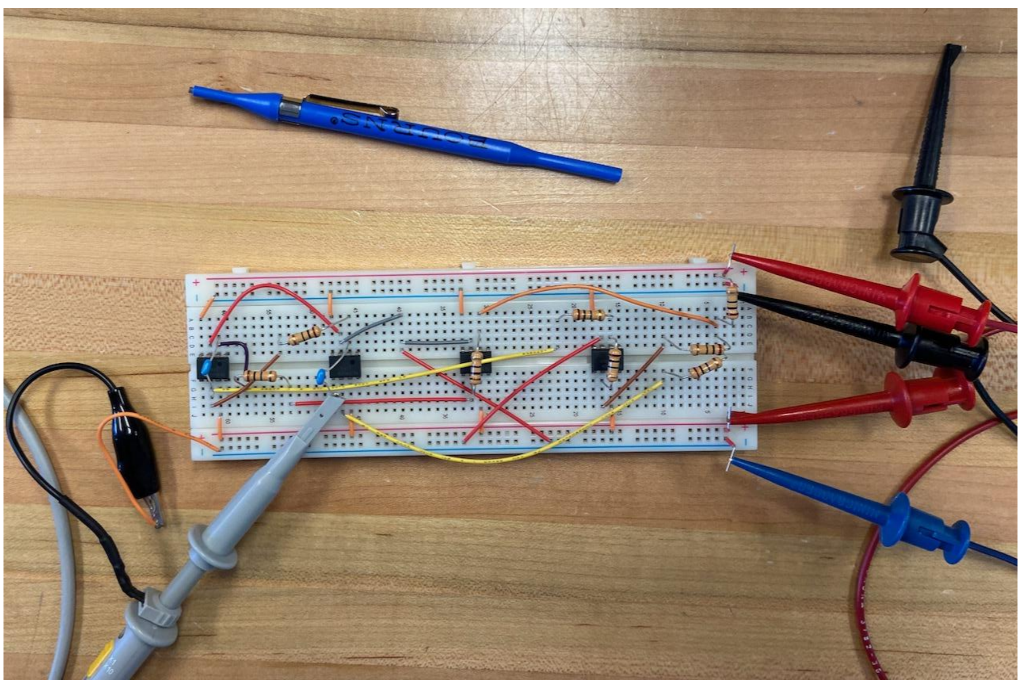

Figure 16: The physical circuit.

Much like seen before, we can test the circuit for different cases, revealing the behavior of the circuit. The first of which is the response at x(t). The physical response, seen in *Figure 17* does infact look similar to the modeled response—the biggest difference being that the physical circuit must have an input that is a square wave.

Figure 17: The physical response of the circuit at x(t).

We can further test this circuit for cases such as shorting R3, which creates an underdamped case, seen in *Figure 18*.

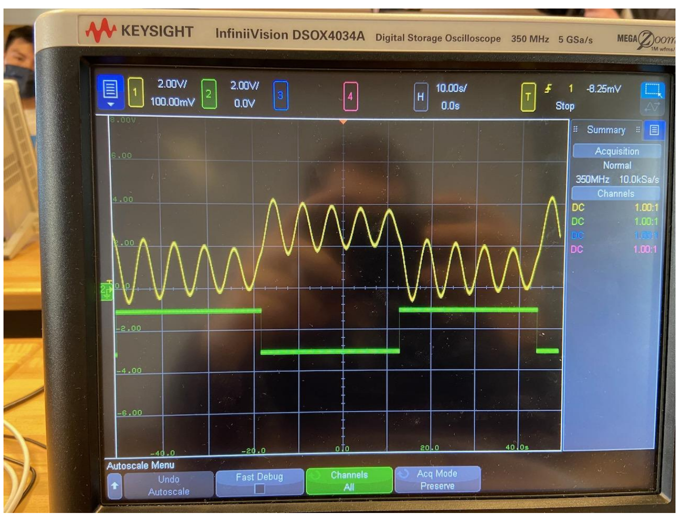

Figure 18: The response when R3 is shorted.

Noting that the critical damping occurs when R3 has a value of 1MΩ, we know that the response will be that of *Figure 17*, however, we can test the response at 10x and 0.1x the value of R3 for critical damping. The response for the overdamped case is seen in *Figure 19* and the response for the underdamped case is seen in *Figure 20*.

Figure 19: The overdamped case of the circuit, with R3=10MΩ.

Figure 20: The underdamped case of the circuit, with R3=100kΩ.

As seen above, we can increase the value of ω to 100 by changing the values of resistors R5 to 5MΩ and R1 to 50kΩ. This results in a circuit that is critically damped when R3 is 100kΩ. We can test the critically damped case, *Figure 21*, the overdamped case, *Figure 22*, and the underdamped case, *Figure 23*, which are all seen below.

Figure 21: The modified circuit, critically damped.

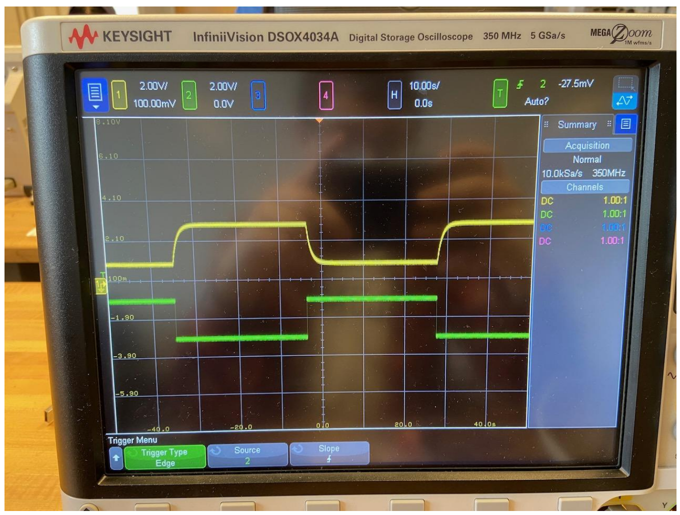

Figure 22: The modified circuit, overdamped.

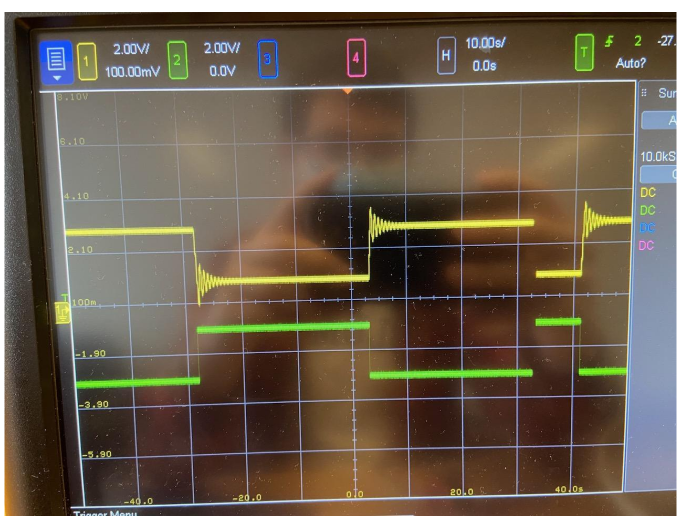

Figure 23: The modified circuit, underdamped.