

Integrator with Reset

In building a predesigned integrator circuit with the LF411 it could be determined that the output voltage of the integrator would be $v_o = -\frac{1}{R_1 C_1} \int_0^t v_1 dt$ when a switch, connected in parallel with the capacitor, was open. When closed, this was found to be $v_o = 0$. Hand calculations can be found in *Figure 1*.

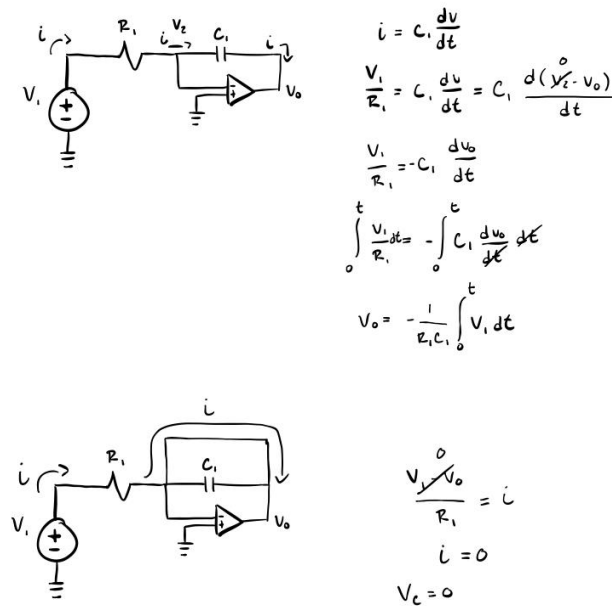


Figure 1: Hand Calculations for the Output Voltage when the Switch is Open and Closed

This can be verified in LTSpice using values of resistance of $1M\Omega$ and capacitance of $5\mu F$. The schematic built to verify this calculation can be seen in *Figure 2*, with the resulting graph in *Figure 3*. These show that the circuit integrates as expected. Then, we can start the integration with an offset voltage by adding a voltage source of the desired voltage before the switch. This pre-charges the capacitor. The new output, with a 2V offset can be seen in *Figure 4*, and the circuit with an additional voltage source can be seen in *Figure 5*.

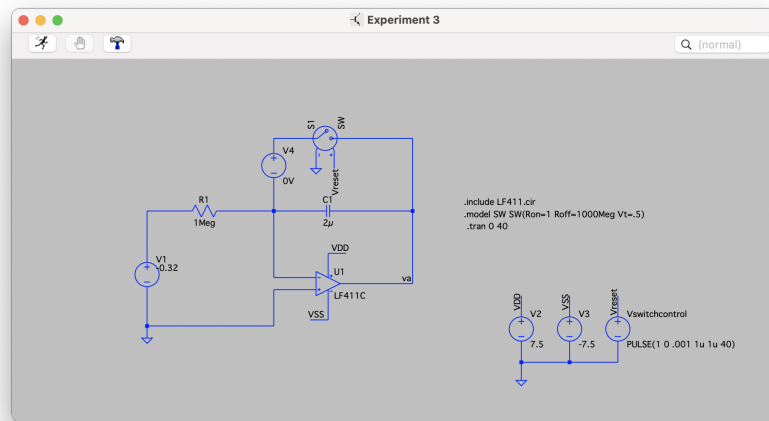


Figure 2: The First Stage of the Integrator.

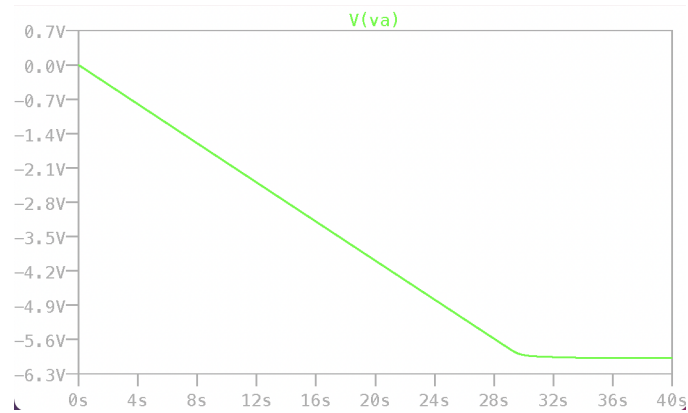


Figure 3: Output of the Integrator in Figure 2

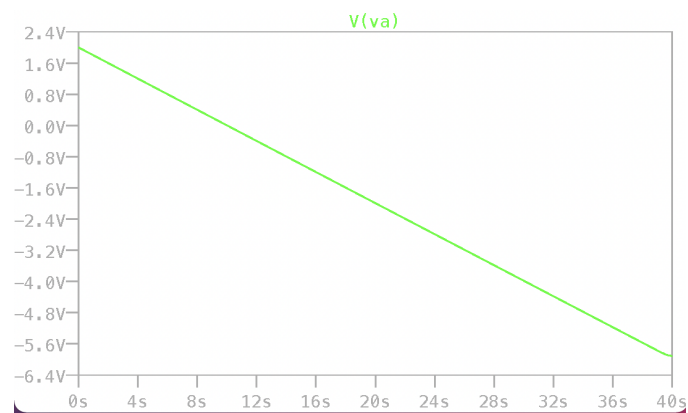


Figure 4: Output of Integrator with Offset

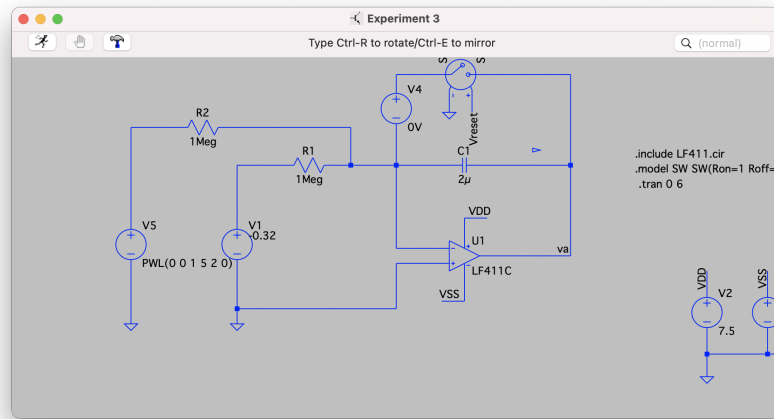


Figure 5: The Modified Integrator with an Offset on the Switch

The Ballistics Calculation: Creating the Electrical Analog

Using the integrator circuit, we can model ballistics calculations—this allows us to model the velocity and displacement of a rocket. With the basic integrator circuit, we can define voltages and polarity to represent magnitude and direction. For this, we can define 1V as $100 \frac{ft}{s^2}$ in the downward direction.

Running this in LTSpice, we can see that with a $1\text{M}\Omega$ resistor and a $2\mu\text{F}$ capacitor that we have a negatively increasing velocity, and that at $t = 1$ second that there is a velocity of $10 \frac{ft}{s}$. The graph for the velocity can be seen in Figure 6. The schematic used was that in Figure 5 with the second voltage source set to 0V .

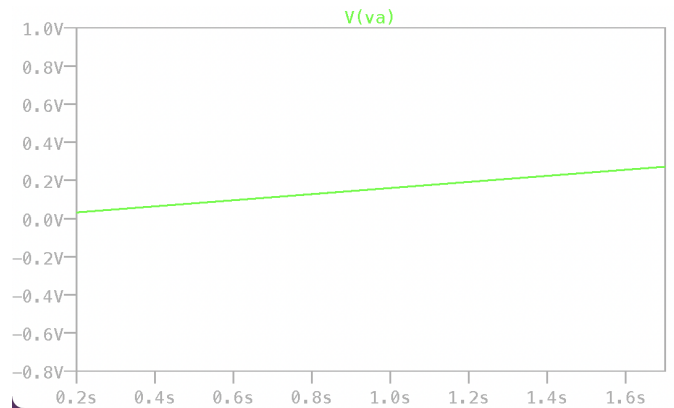


Figure 6: The Graph of the Velocity seen from the Integrator

By modifying the integrator circuit that we used before by adding a voltage source to represent acceleration due to gravity, we are able to calculate velocity. The circuit used for this can be seen in Figure 5. By running this simulation, we can see that there is an expected output that models an upward acceleration with a gravity component. This graph can be seen in Figure 7.

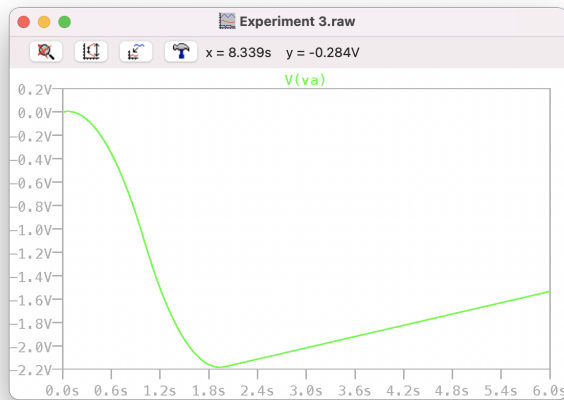


Figure 7: The Addition of Gravity to the Integrator

Completing the Design of the Analog Computer

We can further apply the integrator circuit to model not only the velocity of the rocket, but also the displacement. This can be done by building a second stage for the integrator. The design for this circuit can be seen in *Figure 8*. We can define 1V as $1000 \frac{ft}{s^2}$ in the positive direction, which allows us to have a maximum distance of 6,000 ft in either direction. We'll use components of the same value as before, so we'll have a resistor of $1M\Omega$ and a capacitor of $2\mu F$. Then, we can test with a fixed velocity $100 \frac{ft}{s}$. We'll see a final displacement of -500 feet. This can be seen in *Figure 9*.

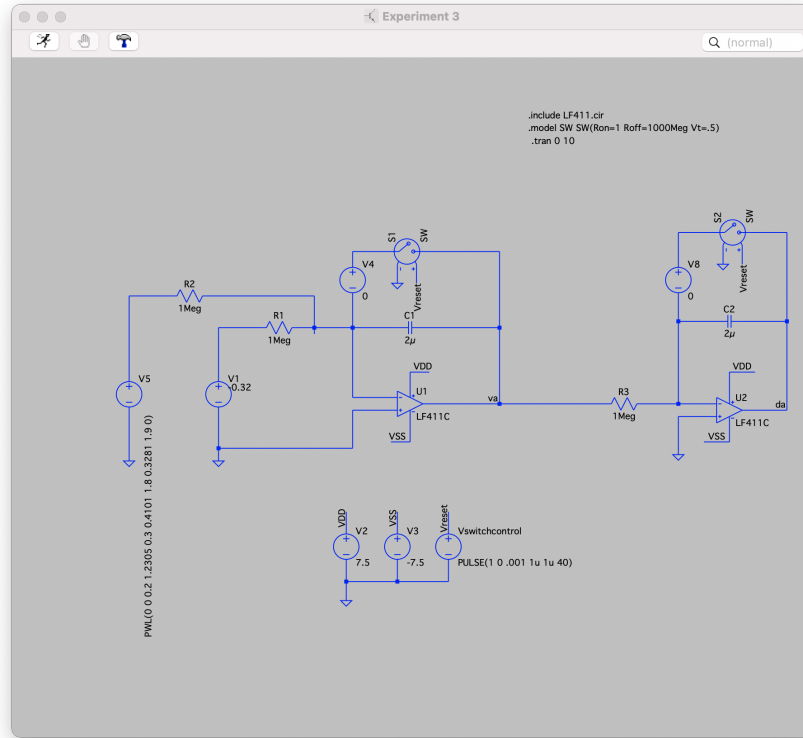


Figure 8: The Integrator Circuit with Second Stage

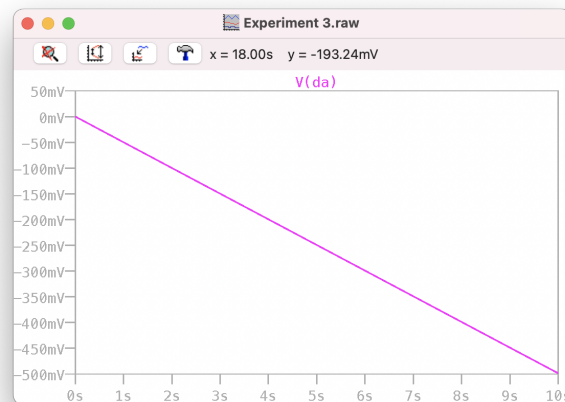


Figure 9: The Output of the Second Integrator

Then, we can connect both stages and see that both the velocity and displacement can be connected together. Their outputs, when measured, display both components—albeit scaled. In *Figure 10* we can see this. We can read these outputs like we did before to see that the velocity at 10 seconds is $160 \frac{ft}{s}$ and the displacement is 5600 feet. We can also model a projectile fired from the ground using this model—with a $-0.25V$ source connected in addition to the first stage—allowing for an initial velocity of $250 \frac{ft}{s}$ up. This

can then be simulated, as seen in *Figure 11*. The projectile's maximum height is 6,000 feet and the maximum velocity is $1,200 \frac{ft}{s}$. After 5 seconds, the physical simulation fails because we hit the limits of the simulation.



Figure 10: Both Stages Connected



Figure 11: Projectile Motion Model using the Integrator

Modeling a Rocket Flight

Finally, we can model a real rocket flight using a C6 engine by plotting the PWL voltage to model the acceleration of the rocket. *Figure 8* shows the circuit for this simulation. In *Figure 12* we can see acceleration, velocity, and displacement of the rocket. The maximum height of the rocket is 2,400 ft, the maximum velocity is $240 \frac{ft}{s}$, and the duration of the flight is 4 seconds.

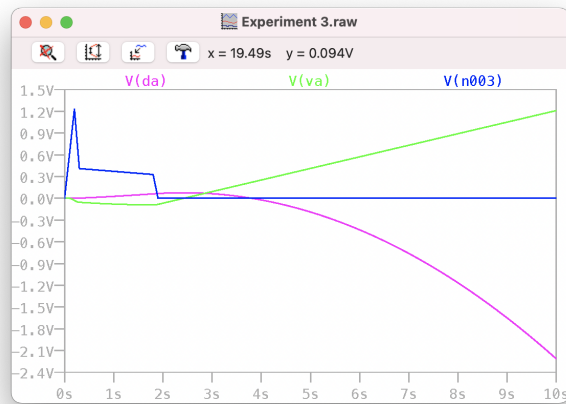


Figure 12: Modeling a C6 Engine with the Integrator Circuit

Build It

Finally, we can build the actual circuit to do these calculations on an oscilloscope. The circuit was built in *Figure 13*, and was tested on the oscilloscope to see that there was an expected output—where the switch pulse reset the output, which integrated the input. The output can be seen in *Figure 14*.

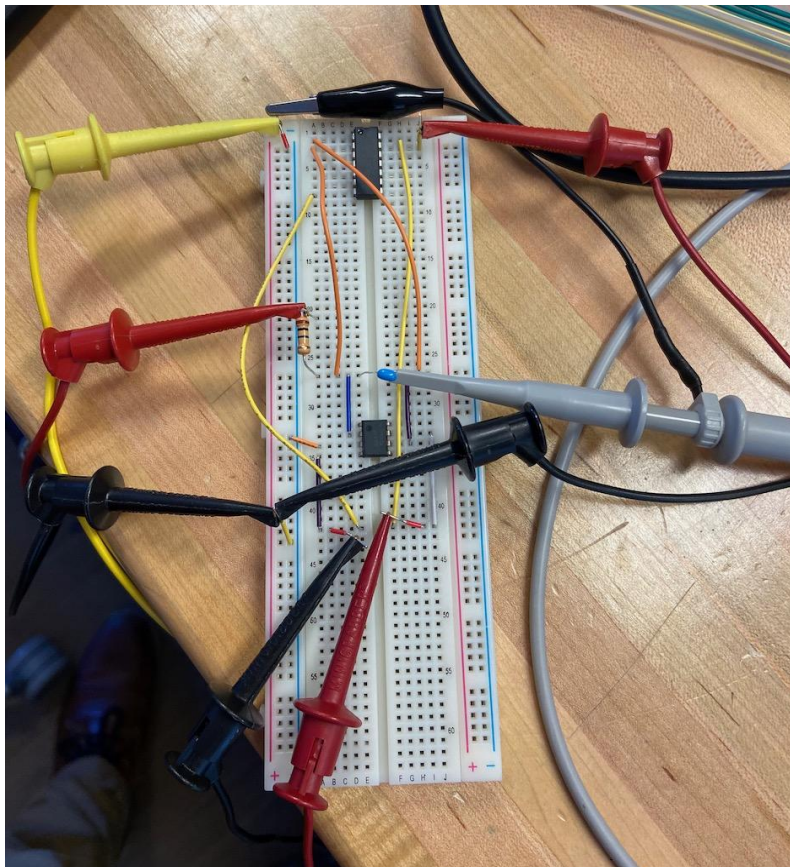


Figure 13: The Physical Integrator Circuit



Figure 14: Output of the Integrator as seen on the Oscilloscope.