Evaluating QPU Performance with the Ising Model

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4 Apr 2019

Thanks to Dr. Sanders, Seyed Shakib Vedaie, and Archismita Dalal for their support of this project.

Aim

To use state of the art classical computational solving techniques to evaluate Ising model instances, evaluate the performance, and compare against D-Wave's 2000Q.

Claim

There exists at least one Ising model optimization problem which is evaluated with a smaller time-to-solution metric on D-Wave's 2000Q than with an optimized, classical simulated annealer.

Optimization Problems

Optimization

An optimization problem seeks to extremize a given function.

Ising Problem

$$H = -\sum_{i=1}^{N} B_i s_i - \sum_{i}^{N} \sum_{j \in \text{nn(i)}} J_{ij} s_i s_j$$

Which vector $\vec{S}$ (with each $s_i \in \{-1, +1\}$) minimizes the Ising Hamiltonian H, given some input sets:

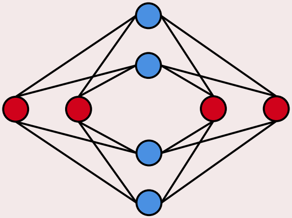
$$\vec{B} = \{B_1, B_2, \ldots, B_N\}; \qquad \vec{J} = \begin{pmatrix} J_{1,1} & J_{1,2} & \cdots & J_{1,N} \\ J_{2,1} & J_{2,2} & \cdots & J_{2,N} \\ \vdots & \vdots & \ddots & \vdots \\ J_{N,1} & J_{N,2} & \cdots & J_{N,N} \end{pmatrix}$$

The Chimera Unit Cell

Bipartite Cell

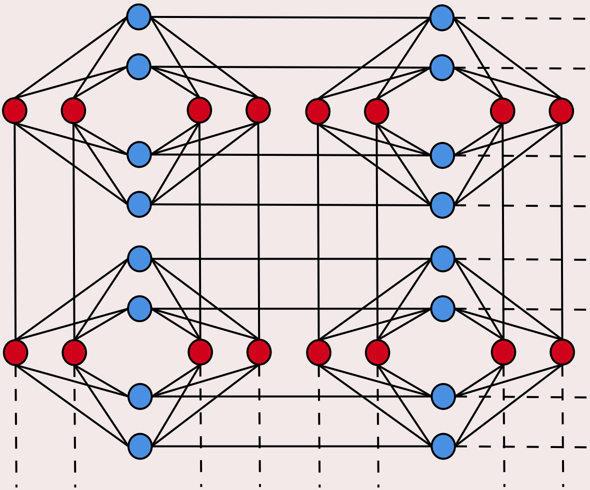
A graph with a set of vertices V and a set of non-zero edges E is **bipartite** if its vertices can be partitioned into two disjoint subsets A and B such that every edge $e_i \in E$ connects one vertex from set A to one vertex in set B.

One Chimera Unit Cell



The Chimera Graph

Several Interconnected Chimera Unit Cells



From Ising Model to Chimera Graph

Embedding Ising Problems onto the Chimera Graph

Features of the Ising problem need to be embedded onto the Chimera graph, solved, and then interpreted.

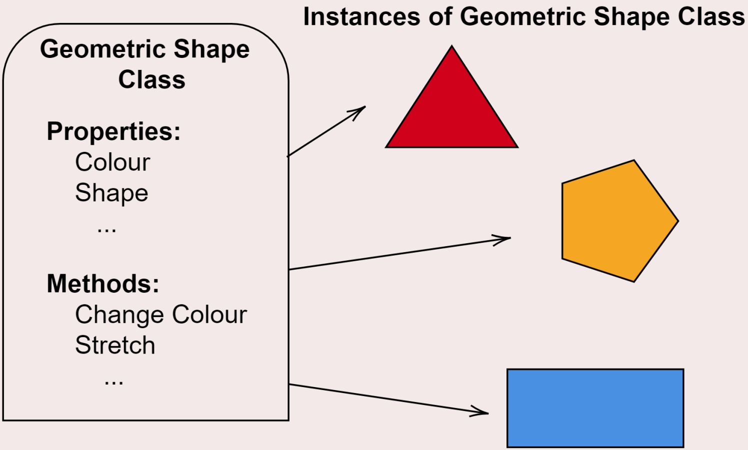
- The set of vertices (qubits) V are binary and therefore should correspond to the set of spins $\vec{S}$.
- The weight associated with each vertex should correspond to the magnetic field strength $\vec{B}$ at the location each spin.
- The set of edges E correspond to the set of interspin coupling strengths J.

Hardware Limitations

Only problems which can be embedded onto the Chimera graph represent feasible problems.

Instance Definition

Object-Oriented Programming Analogy



Example Problem

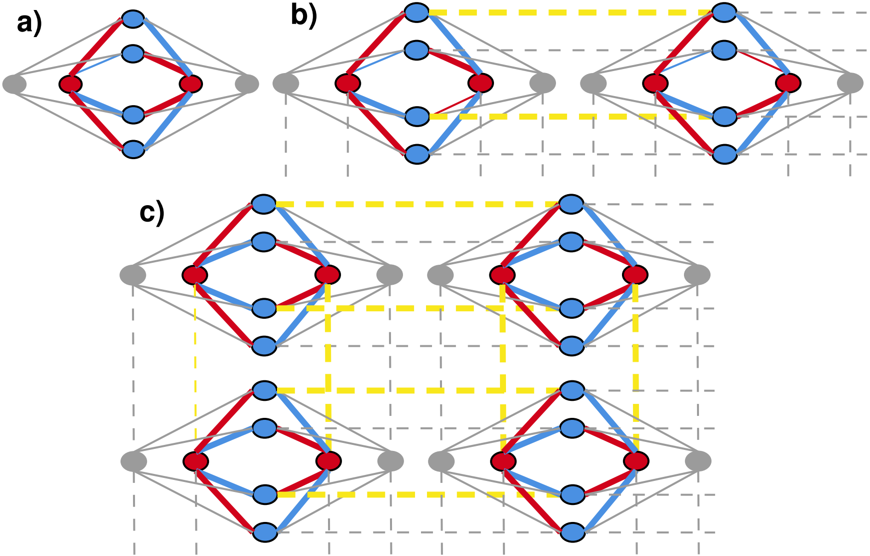
Embedding this Problem

Consider the following problem:

$$\vec{B} = \{2, 2, 2, 2, 0, -2, -2, 0\};$$

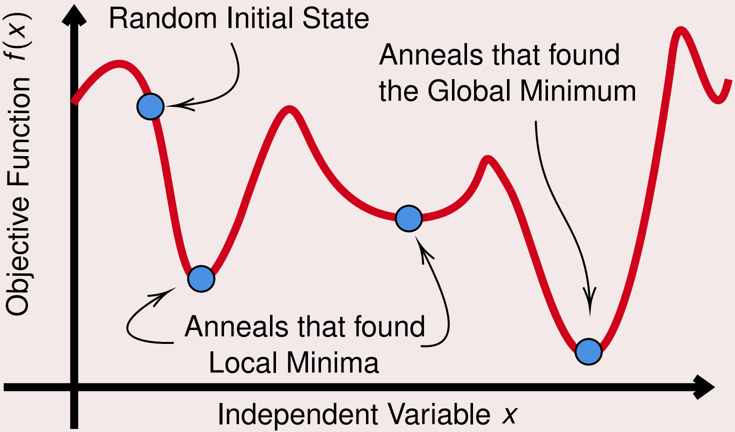
$$\vec{J} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & -2 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 & -2 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -2 & 2 & 2 & -2 & 0 & 0 & 0 & 0 \\ 2 & -2 & -2 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Example Problem



Simulated Annealing

Annealers Don't Always find the Global Minimum



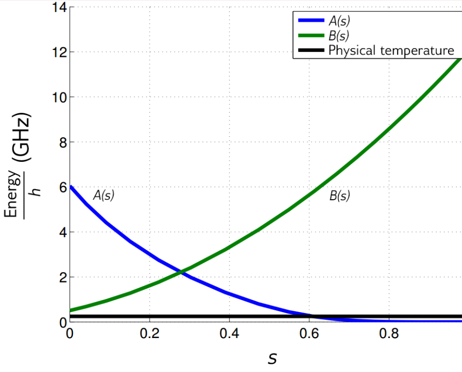
D-Wave's Quantum Processing Unit (QPU)

D-Wave's 2000Q

The QPU inside of the 2000Q is described by the Hamiltonian:

$$\mathrm{H}_{\mathrm{Ising}} = \underbrace{-\frac{A(s)}{2}\sum_i \hat{\sigma}_x^{(i)}}_{\text{initial Hamiltonian}} \underbrace{+ \frac{B(s)}{2}\left(\sum_i h_i \hat{\sigma}_z^{(i)} + \sum_{i>j} J_{i,j}\hat{\sigma}_z^{(i)}\hat{\sigma}_z^{(j)}\right)}_{\text{final Hamiltonian}}$$

Annealing Schedule



Performance Metric

Success probability P_s

The probability of finding the global minimum in a single annealing run of time t_A.

Time-to-Solution

The time-to-solution (TTS) required to achieve a success probability of P_s at least once with a desired probability P is found using:

$$\mathrm{TTS} = t_A \frac{\log(1-P)}{\log(1-P_s)}$$

Problem Definition

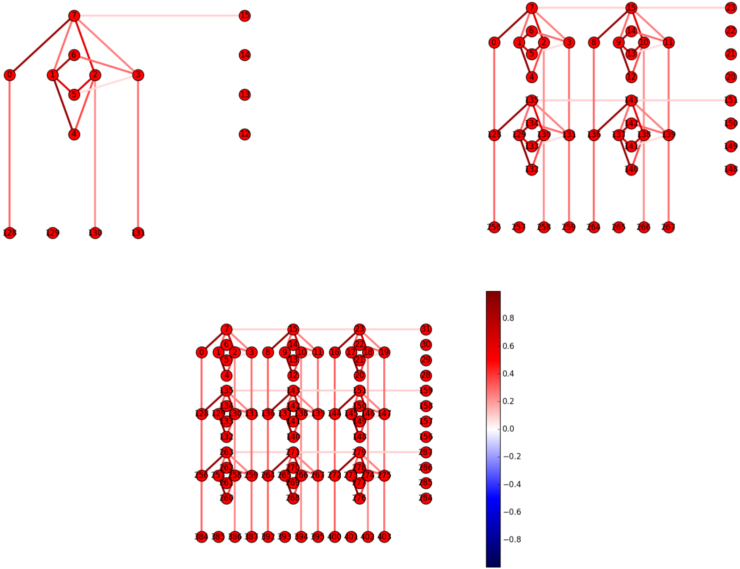
Problem Definition

Consider the problem described by:

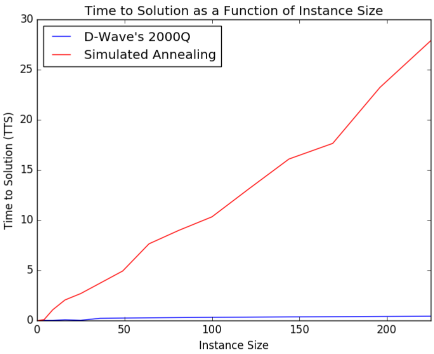
$$\vec{B} = \{0.5143, 0.9702, 0.7975, 0.5248, 0.5397, 0.1735, 0.6969, 0.6760\}$$

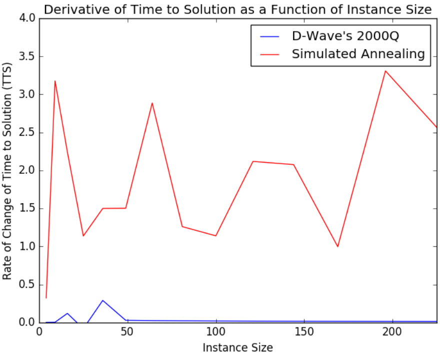
$$\vec{J} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.909 \\ 0 & 0 & 0 & 0 & 0.917 & 0.623 & 0.832 & 0.292 \\ 0 & 0 & 0 & 0 & 0.361 & 0.650 & 0 & 0.603 \\ 0 & 0 & 0 & 0 & 0 & 0.063 & 0.291 & 0.255 \\ 0 & 0.917 & 0.361 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.623 & 0.650 & 0.063 & 0 & 0 & 0 & 0 \\ 0 & 0.832 & 0 & 0.291 & 0 & 0 & 0 & 0 \\ 0.909 & 0.292 & 0.603 & 0.255 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Chimera Graph Representation



Solver Performance





Summary

- Minimizing the Ising model's Hamiltonian constitutes an optimization problem.
- The Ising model Hamiltonian can be embedded onto the Chimera graph, which represents the 2000Q's hardware structure.
- The structure of the Chimera graph restricts the problems that can be solved by the 2000Q.
- Chimera problems can be instantiated with an integer multiple size of the smallest instance and grown by embedding the problem again in an adjacent unit of the Chimera graph.
- Both simulated annealing and quantum annealing can be used to solve optimization problems.
- There exists at least one problem which can be solved with a smaller time to solution metric on the 2000Q than with an optimized simulated annealer.