Regularized Harmonic Surface Deformation

Computational Shape Modeling
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Presentation Contents

- Artifacts in Deformation
- Harmonic Surface Deformation - Intuition and Discretization
- Energy Regularization by smoothing - Intuition and Discretization
- Regularized Harmonic Surface Deformation - Discretization
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- Results

Artifacts in Deformation

Mostly Seen Artifacts

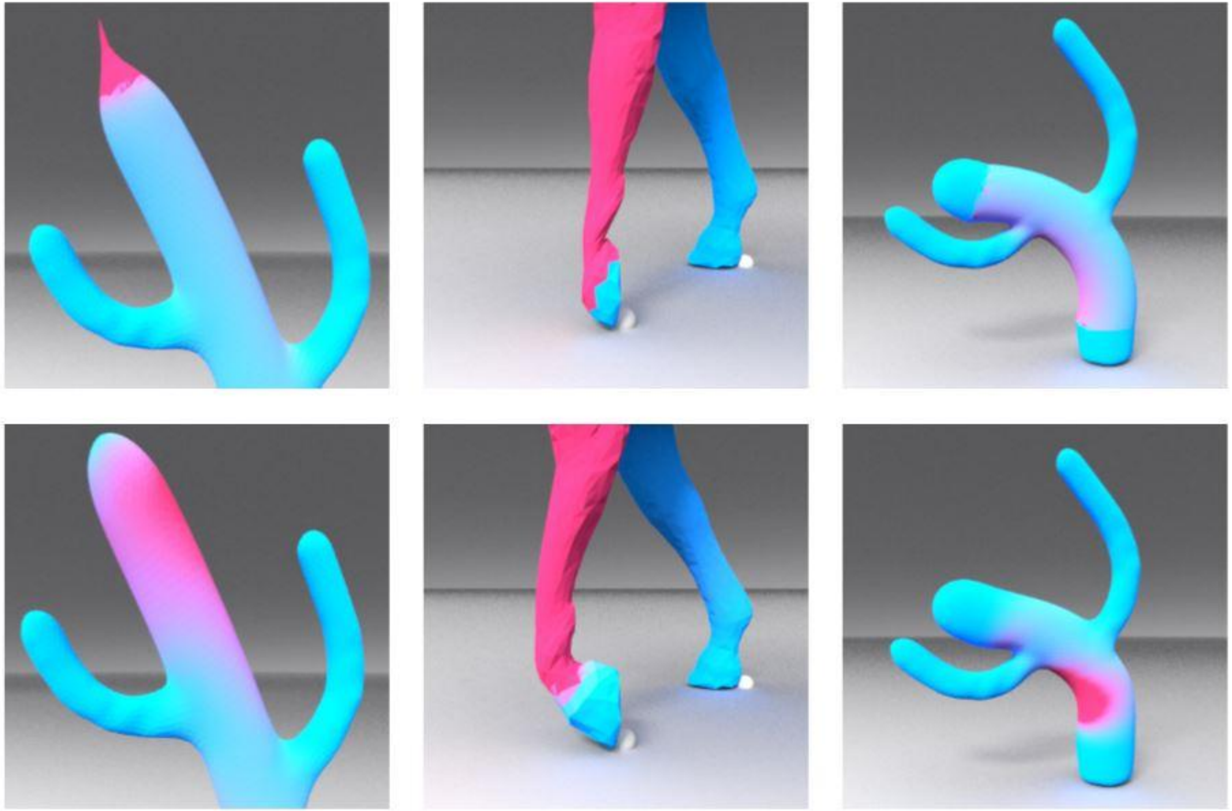
Distortion of Local Geometry

Topology preservation failure

Degenerate triangles

Intruding or protruding triangles or elements

Volume Loss



(a) (b) (c)

a) Protruding Elements

b) Volume Loss

c) Local Shape Distortion

Need of regularization?

Harmonic Surface Deformation

Intuition

Discretization

Harmonic Guidance for Surface Deformation - Intuition

Linear Method - Deformation by solving a linear system

Use of Gradient Manipulation - Preserves differential properties

Harmonic Fields are used to guide gradients

Harmonic field - **h** is defined by:

$$\Delta h = 0.$$

$$\begin{aligned}\mathbf{Lh} &= 0 \\ \mathbf{h} &= \begin{cases} 0 & \text{vertex is fixed} \\ 1 & \text{vertex is handle} \end{cases}\end{aligned}$$

Formulation

$$\begin{aligned}\mathbf{x}' &= \operatorname*{argmin}_{\mathbf{x}\in\mathbb{R}^{3|\mathcal{V}|}} E(\mathbf{x}) \\ E(\mathbf{x}) &= \int_\Omega \|\nabla\mathbf{x} - \mathbf{g}\|^2 \,\mathrm{d}\,\mathbf{x}\end{aligned}$$

Deformation Gradient guidance field is g.

Guidance field g is derived using harmonic fields.

Transformation Propagation process

Transformation propagation using geodesic or euclidean distances - Generally

Transformation Propagation using harmonic field

Transformations decomposed in translation, rotation, scaling, shearing matrices.

Harmonic field value used to interpolate between fixed and handle points for each matrix.

Solving the final transformation for each vertex (By multiplying individual matrices)

Transformation Propagation Process

For e.g. Harmonic field for rotation

For handle vertices h, Using Quaternions for rotation $q = (q_w; q_x, q_y, q_z)$.

$$\begin{aligned} \mathbf{L}\mathbf{q}_i &= \mathbf{0},\ i \in (w, x, y, z) \\ \mathbf{q}_i(h_j) &= q_i^{h_j} \end{aligned}$$

Deriving matrices from harmonic fields of each rotation, scaling, transformation

$$\mathbf{y} = \mathbf{G}\mathbf{x}$$
$$\mathbf{g}_i = \mathbf{y}_i\mathbf{R}_i^{\mathsf{T}}$$

Discretization

$$E(\mathbf{x}) = \sum_{t \in \mathcal{T}} A_t \, ||\mathbf{G}_t \, \mathbf{X}_t - \mathbf{Z}_t||_F^2$$

Defining a matrix norm as $||\mathbf{Y}||_{\mathbf{N}}^2 = \mathrm{Tr}\left(\mathbf{Y}^{\mathrm{T}} \mathbf{N} \, \mathbf{Y}\right)$

$$\begin{aligned} E(\mathbf{x}) &= ||\mathbf{G}\mathbf{x} - \mathbf{g}||_{\mathbf{A}}^2 \\ &= (\mathbf{G}\mathbf{x} - \mathbf{g})^{\mathsf{T}} \mathbf{A} (\mathbf{G}\mathbf{x} - \mathbf{g}) \end{aligned}$$

$$\frac{d}{dx} E(\mathbf{x}) = 2\mathbf{G}^{\mathsf{T}} \mathbf{A} \mathbf{G} \mathbf{x} - 2\mathbf{G}^{\mathsf{T}} \mathbf{A} \mathbf{g} \stackrel{!}{=} 0$$

$$\mathbf{G}^{\mathsf{T}} \mathbf{A} \mathbf{G} \mathbf{x} = \mathbf{G}^{\mathsf{T}} \mathbf{A} \mathbf{g}$$

Energy Regularization

Intuition

Discretization

Tikhonov Regularization

$$E_{\mathcal{R}}(\mathbf{x}) = (1-\beta)\underbrace{\int_{\Omega} \|\mathbf{A}\mathbf{x}-\mathbf{b}\|^2 \,\mathrm{d}\,\mathbf{x}}_{E_{\mathcal{D}} \text{ data term}} + \beta \underbrace{\int_{\Omega} \|\mathbf{\Gamma}\mathbf{x}\|^2 \,\mathrm{d}\,\mathbf{x}}_{E_{\mathcal{S}} \text{ smoothness term}}$$

Choices for smoothness term:-

norm(x)

norm(grad(x))

norm(Lx)

Regularization Proposed by Esturo[4]

Smoothness term Tikhonov - Not problem specific , Data term independent

Large local energy variations around artifacts

By penalizing variations , Proposed formulation will be:

$$E_{\mathcal{D}}(\mathbf{x}) = \int_{\Omega} \|\mathbf{e}_{\mathbf{f}}(\mathbf{x})\|^2 \, \mathrm{d}\,\mathbf{x}$$

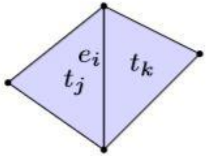
$$E_{\mathcal{S}}(\mathbf{x}) = \int_{\Omega} \|\nabla \mathbf{e}_{\mathbf{f}}(\mathbf{x})\|_F^2 \, \mathrm{d}\,\mathbf{x}$$

Discretization - 2D meshes

Denote smoothness term as linear operator multiplied by energy function.

$$E_{\mathcal{S}}(\mathbf{x}) = \|\mathbf{D}_n(\mathbf{E}\mathbf{x} - \mathbf{g})\|^2_{\mathbf{B}_n}$$

$$D_{it} = \begin{cases} 1 & t \text{ is left neighbor of } i \\ -1 & t \text{ is right neighbor of } i \\ 0 & \text{else} \end{cases}$$



$$D_{ij} = 1, D_{ik} = -1.$$

Discretization

$$\begin{aligned}
E_{\mathcal{S}}(\mathbf{x}) &= \|\mathbf{D}_n(\mathbf{E}\mathbf{x}-\mathbf{g})\|^2_{\mathbf{B}_n} \\
&= (\mathbf{D}_n(\mathbf{E}\mathbf{x}-\mathbf{g}))^{\top}\mathbf{B}_n(\mathbf{D}_n(\mathbf{E}\mathbf{x}-\mathbf{g})) \\
&= (\mathbf{E}\mathbf{x}-\mathbf{g})^{\top}\underbrace{\mathbf{D}_n^{\top}\mathbf{B}_n\mathbf{D}_n}(\mathbf{E}\mathbf{x}-\mathbf{g}) \\
&= \|(\mathbf{E}\mathbf{x}-\mathbf{g})\|^2_{\mathbf{D}_n^{\top}\mathbf{B}_n\mathbf{D}_n}
\end{aligned}$$

$$\begin{aligned}
E_{\mathcal{R}}(\mathbf{x}) &= (1-\beta)E_{\mathcal{D}}(\mathbf{x}) + \beta\, E_{\mathcal{S}}(\mathbf{x}) \\
&= (1-\beta)\|\mathbf{E}\mathbf{x}-\mathbf{g}\|^2_{\mathbf{A}} + \mathbf{E}\mathbf{x}-\mathbf{g}\|^2_{\mathbf{D}_n^{\top}\mathbf{B}_n\mathbf{D}_n} \\
&= \|\mathbf{E}\mathbf{x}-\mathbf{g}\|^2_{\mathbf{W}_\beta}
\end{aligned}$$

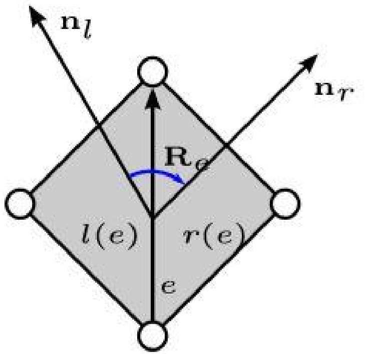
$$\mathbf{W}_\beta = (1-\beta)\mathbf{A} + \beta\,\mathbf{D}_n^{\top}\mathbf{B}_n\mathbf{D}_n$$

Regularized Harmonic Surface Deformation

Discretization

For 3D meshes, the D operator should be changed.

$$D^R_{et} = \begin{cases} \mathbf{R}_e & \text{if } l(e) = t \\ -\mathbf{I}_3 & \text{if } r(e) = t \end{cases} \qquad \mathbf{n}_r = \mathbf{R}_e \mathbf{n}_l$$



Apply Regularization

Harmonic Deformation:

$$\begin{aligned} \mathbf{e_f} &= \nabla\mathbf{x} - \mathbf{g} \\ E(\mathbf{x}) &= \int_\Omega \|\mathbf{e_f}(\mathbf{x})\|^2 \,\mathrm{d}\,\mathbf{x} \end{aligned}$$

After Regularization:

$$E(\mathbf{x}) = \|\mathbf{G}\mathbf{x} - \mathbf{g}\|^2_{\mathbf{W}_\beta}$$

Implementation Details

Implementation

From previous discrete equation, on solving:

$$\mathbf{G}^{\mathsf{T}}\mathbf{W}_{\beta}\mathbf{G}\mathbf{x}' = \mathbf{G}^{\mathsf{T}}\mathbf{W}_{\beta}\mathbf{g}$$

Deriving corresponding metrices from mesh,

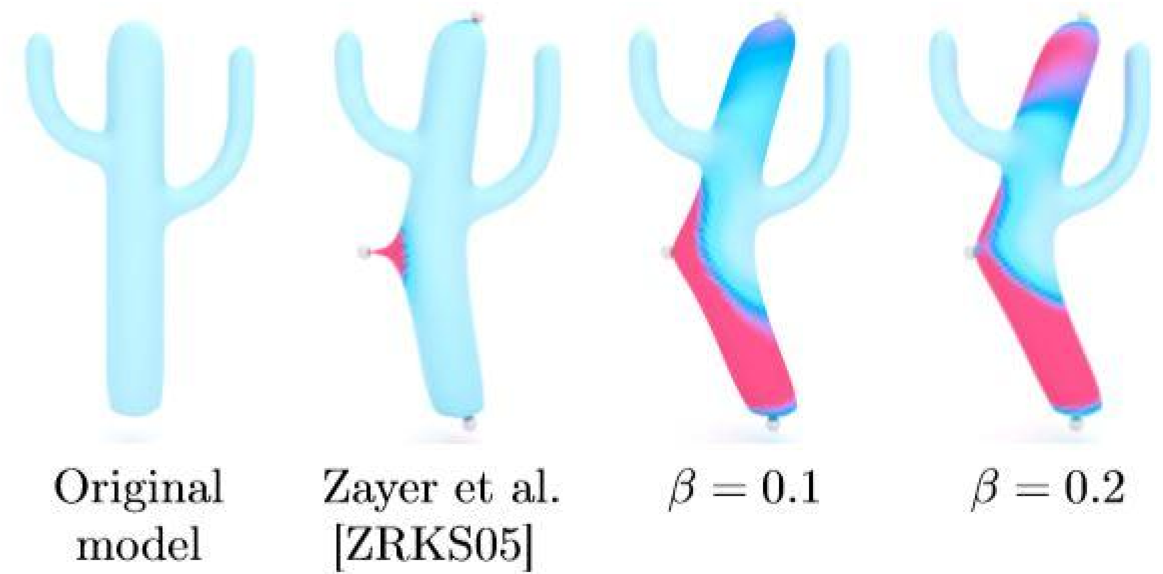
Finding harmonic gradient guidance g

And solving abouve linear system

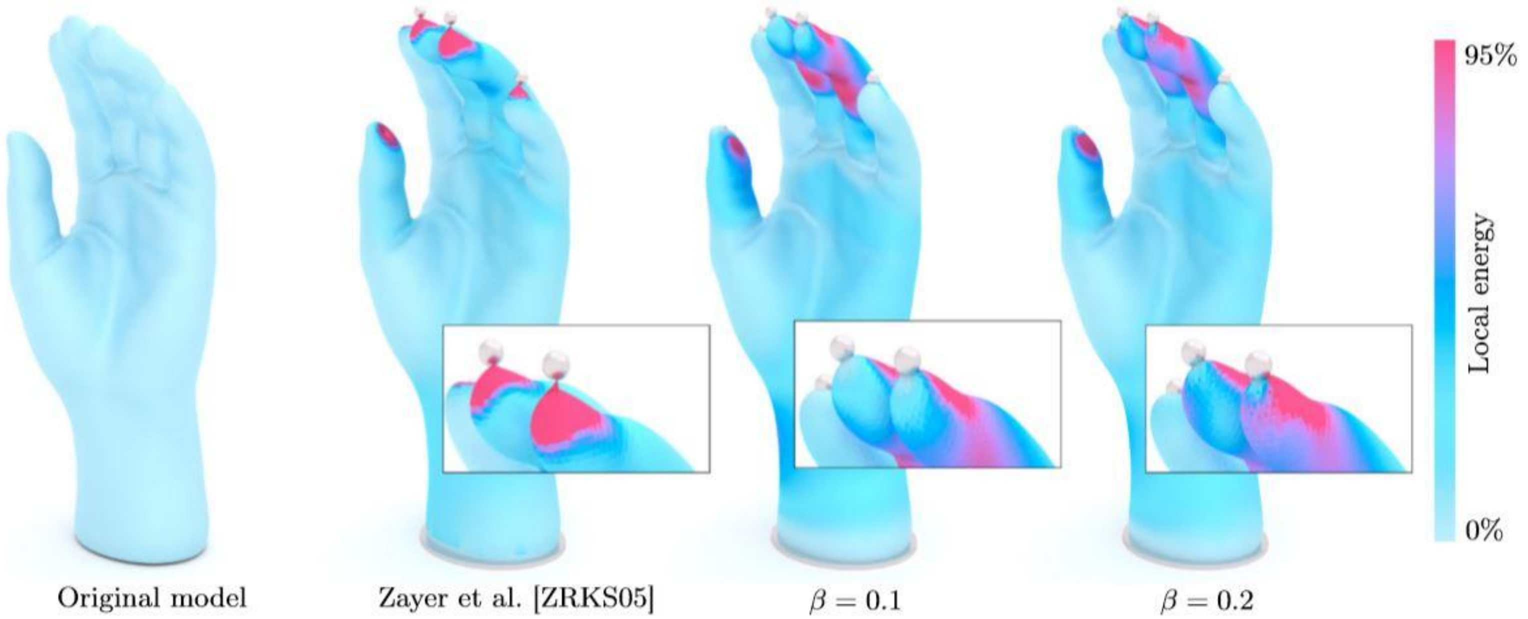
$$\mathbf{W}_{\beta} = (1-\beta)\mathbf{A} + \beta\,\mathbf{D}_n^{\mathsf{T}}\mathbf{B}_n\mathbf{D}_n$$

Results

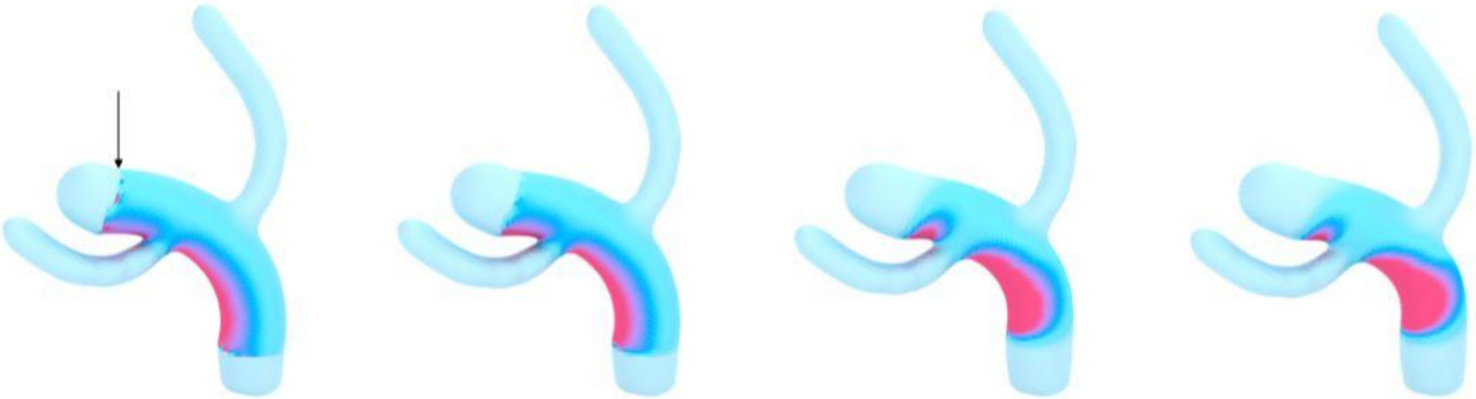
Results



Results


95%
Local energy
0%
Original model
Zayer et al. [ZRKS05]
$\beta = 0.1$
$\beta = 0.2$

Results



Zayer et al. [ZRKS05] $\beta = 0.003$ $\beta = 0.1$ $\beta = 0.2$

References

[1] HarmonicGuidanceforSurfaceDeformation

[2] Smoothed Quadratic Energies on Meshes , Esturo