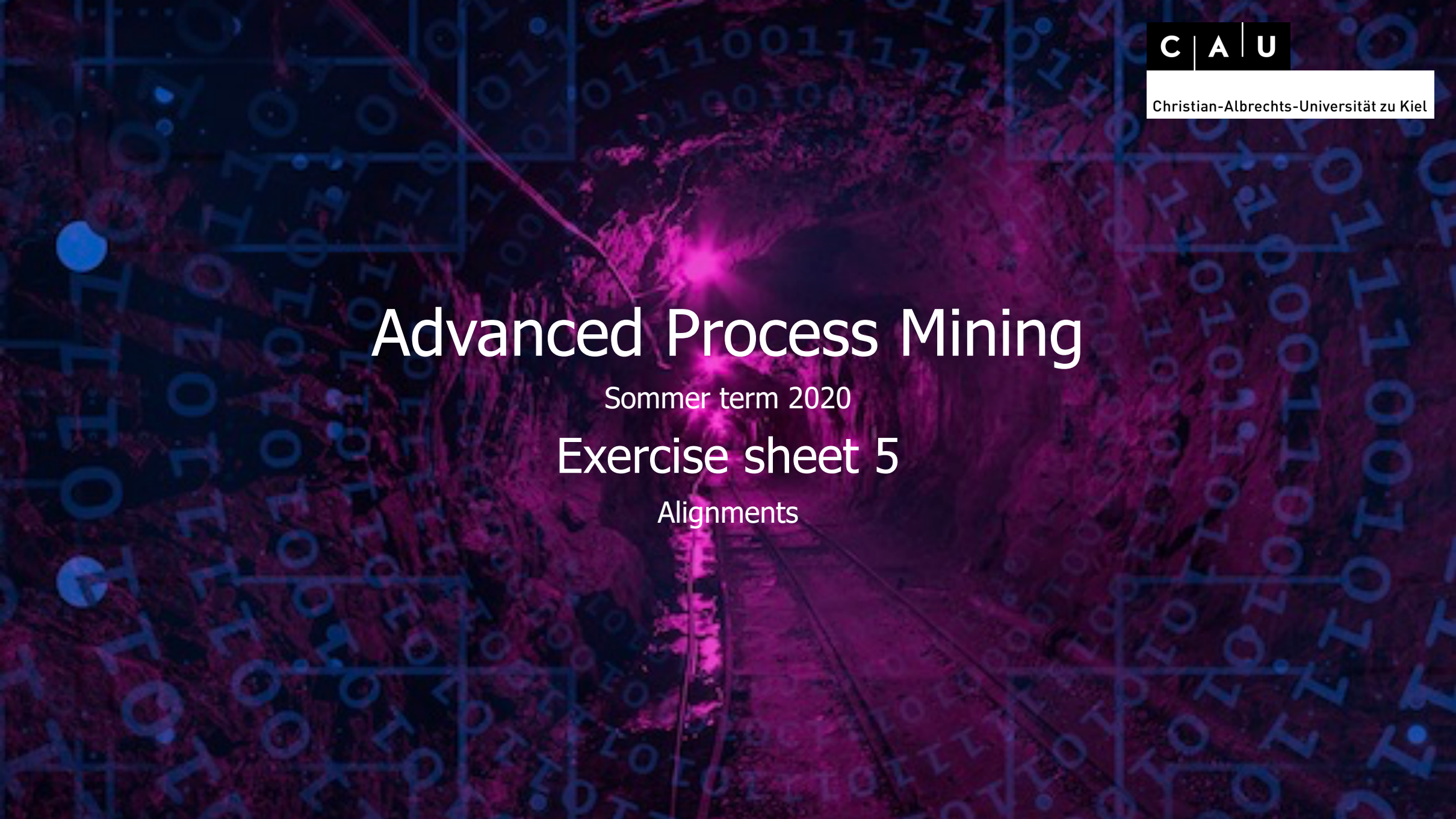


Christian-Albrechts-Universität zu Kiel

Advanced Process Mining

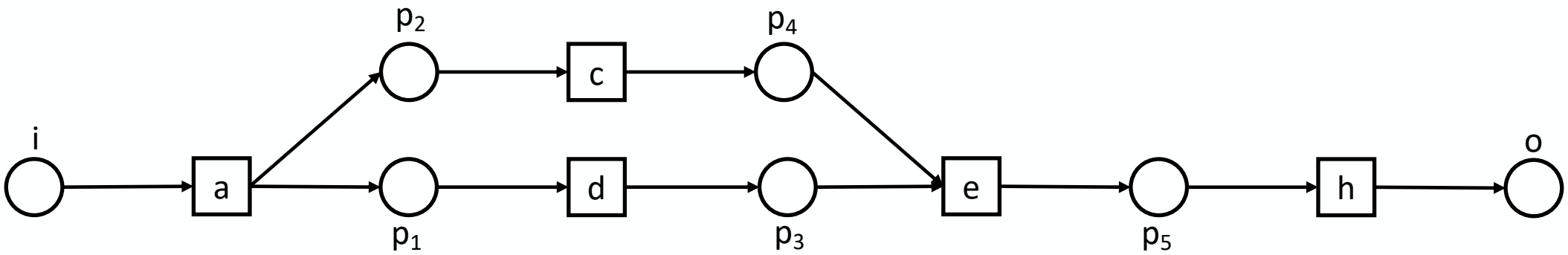
Sommer term 2020

Exercise sheet 5

Alignments

Alignments
Example

Christian-Albrechts-Universität zu Kiel



Trace σ =< a,d,b,e,h >

$\gamma_a =$

σ	a	≫	d	b	e	h
N	a	c	d	≫	e	h

← Trace σ

← Trace through process model

$\gamma_b =$

σ	a	d	≫	b	e	h
N	a	d	c	≫	e	h

$\gamma_c =$

σ	a	d	b	≫	e	h
N	a	d	≫	c	e	h

$\gamma_d =$

σ	a	d	b	e	h	≫	≫	≫	≫	≫
N	≫	≫	≫	≫	≫	a	c	d	e	h

Alignments
Example

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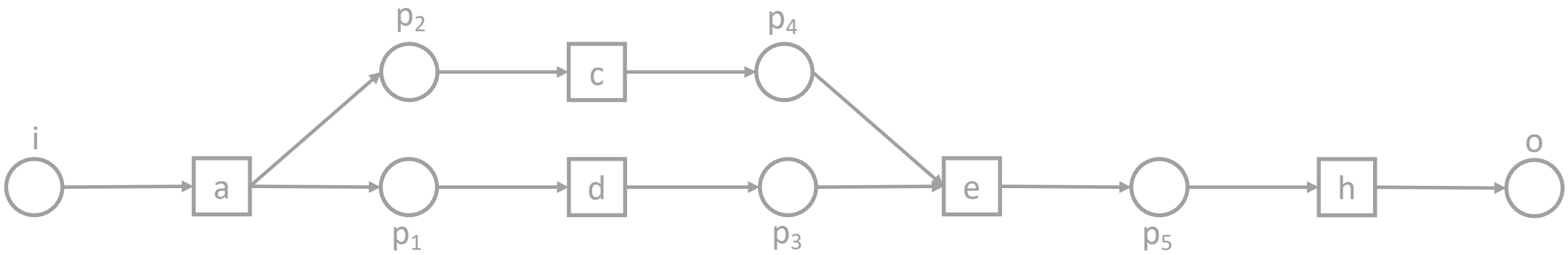

Trace σ =< a,d,b,e,h >

$\gamma_a =$

σ	a	$\gg$	d	b	e	h
N	a	c	d	$\gg$	e	h

$\delta_{\gamma_a} = 2$

$\gamma_b =$

σ	a	d	$\gg$	b	e	h
N	a	d	c	$\gg$	e	h

$\delta_{\gamma_b} = 2$

$\gamma_c =$

σ	a	d	b	$\gg$	e	h
N	a	d	$\gg$	c	e	h

$\delta_{\gamma_c} = 2$

$\gamma_d =$

σ	a	d	b	e	h	$\gg$	$\gg$	$\gg$	$\gg$	$\gg$
N	$\gg$	$\gg$	$\gg$	$\gg$	$\gg$	a	c	d	e	h

$\delta_{\gamma_d} = 10$

How to know which alignment is best?

Apply a cost function:

Move in log	$+1$
Move in model	$+1$
Equal move in both	± 0
Different move in both	$+\infty$

Alignments
Exercise 1a

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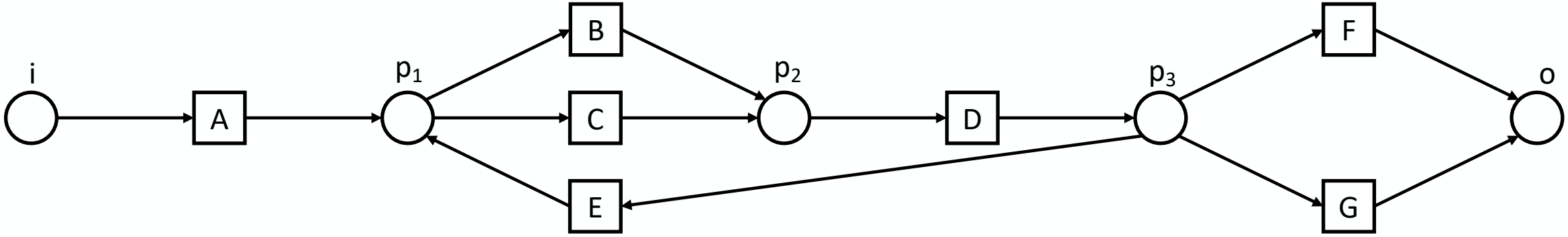
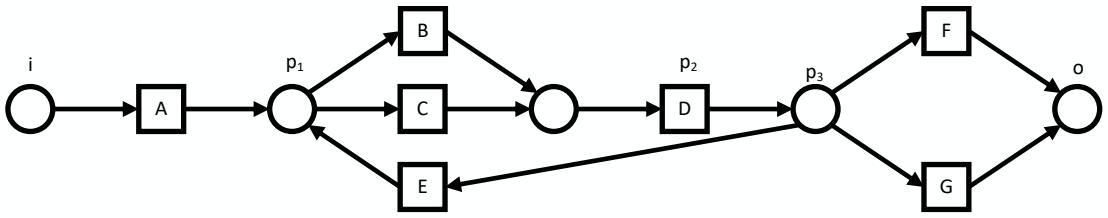


Trace
ACDF
ADCEG
AFG
FDCA
C

Apply alignment-based conformance checking and apply the cost function in order to find an optimal alignment

Alignments
Exercise 1a



Apply alignment-based conformance checking and calculate the cost function for the given traces.

Trace	δ
ACDF	
ADCEG	
AFG	
FDCA	
C	

$\gamma_1 =$

σ_1	A	C	D	F
N	A	C	D	F

$\gamma_2 =$

σ_2	A	D	C	≫	E	G
N	A	≫	C	D	≫	G

$\gamma_3 =$

σ_3	A	≫	≫	F	G
N	A	C	D	F	≫

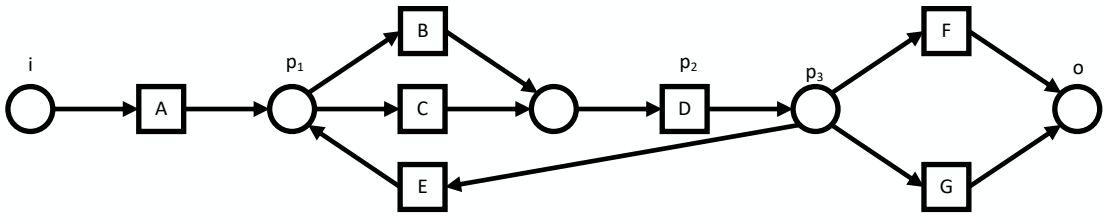
$\gamma_4 =$

σ_4	≫	≫	≫	F	D	C	A
N	A	C	D	F	≫	≫	≫

$\gamma_5 =$

σ_5	≫	C	≫	≫
N	A	C	D	F

Alignments
Exercise 1a



Apply alignment-based conformance checking and calculate the cost function for the given traces.

Trace	δ
ACDF	0
ADCEG	3
AFG	3
FDCA	6
C	3

$\gamma_1 =$

σ_1	A	C	D	F
N	A	C	D	F

$\gamma_2 =$

σ_2	A	D	C	≫	E	G
N	A	≫	C	D	≫	G

$\gamma_3 =$

σ_3	A	≫	≫	F	G
N	A	C	D	F	≫

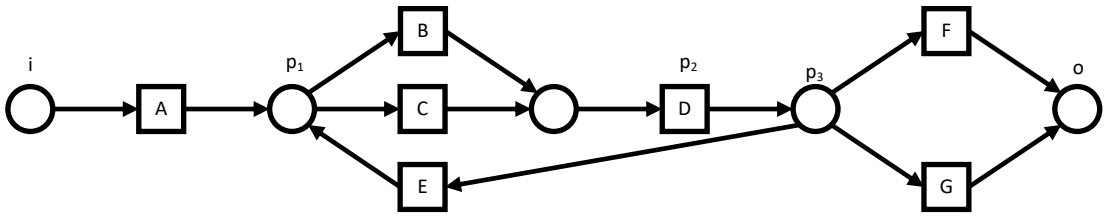
$\gamma_4 =$

σ_4	≫	≫	≫	F	D	C	A
N	A	C	D	F	≫	≫	≫

$\gamma_5 =$

σ_5	≫	C	≫	≫
N	A	C	D	F

Alignments
Exercise 1b



Determine the fitness of the process model above and the observed traces by considering the alignments.

#	Trace	δ
200	ACDF	0
120	ADCEG	3
100	AFG	3
80	FDCA	6
10	C	3

$\gamma_1 =$

σ_1	A	C	D	F
N	A	C	D	F

$\gamma_2 =$

σ_2	A	D	C	$\gg$	E	G
N	A	$\gg$	C	D	$\gg$	G

$\gamma_3 =$

σ_3	A	$\gg$	$\gg$	F	G
N	A	C	D	F	$\gg$

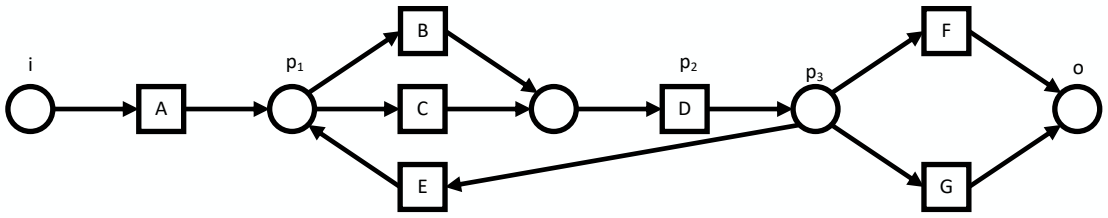
$\gamma_4 =$

σ_4	$\gg$	$\gg$	$\gg$	F	D	C	A
N	A	C	D	F	$\gg$	$\gg$	$\gg$

$\gamma_5 =$

σ_5	$\gg$	C	$\gg$	$\gg$
N	A	C	D	F

Alignments
Exercise 1b



$\gamma_1 =$

σ	A	C	D	F
N	A	C	D	F

$\delta = 0$

Determine the fitness of the process model above and the observed traces by considering the alignments.

What is the worst valid alignment with the shortest path from initial state to final state?

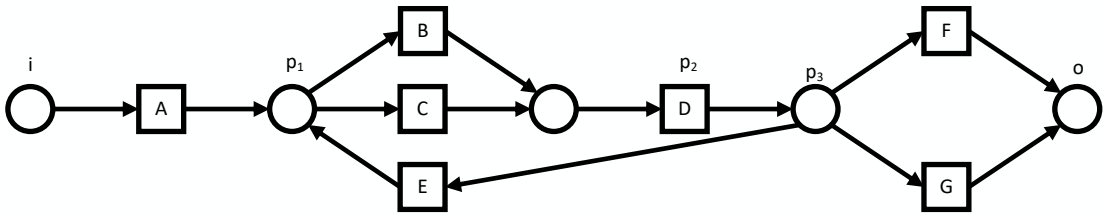
σ_1	A	C	D	F	$\gg$	$\gg$	$\gg$	$\gg$
N	$\gg$	$\gg$	$\gg$	$\gg$	A	C	D	F

What is the cost?

$\delta = 8$

$$fitness(\sigma, N) = 1 - \frac{\delta(\lambda^N_{opt}(\sigma))}{\delta(\lambda^N_{worst}(\sigma)} = 1 - \frac{0}{8} = 1$$

Alignments
Exercise 1b



$\gamma_1 =$

σ	A	C	D	F
N	A	C	D	F

$\delta = 0$

Determine the fitness of the process model above and the observed traces by considering the alignments.

What is the worst valid alignment with the shortest path from initial state to final state?

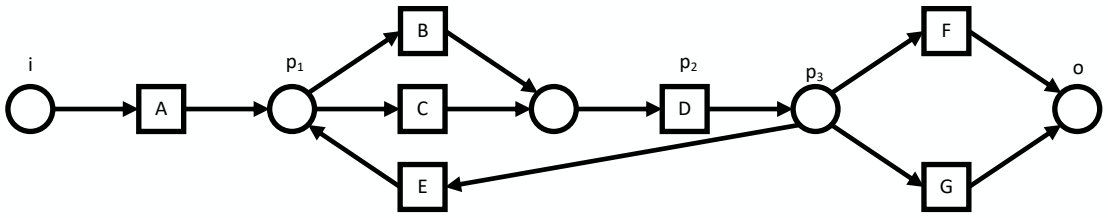
σ_1								
N								

What is the cost?

$\delta =$

$$fitness(\sigma, N) = 1 - \frac{\delta(\lambda^N_{opt}(\sigma))}{\delta(\lambda^N_{worst}(\sigma)}$$

Alignments
Exercise 1b



$\gamma_2 =$

σ_2	A	D	C	$\gg$	E	G
N	A	$\gg$	C	D	$\gg$	G

$\delta = 3$

Determine the fitness of the process model above and the observed traces by considering the alignments.

What is the worst valid alignment with the shortest path from initial state to final state?

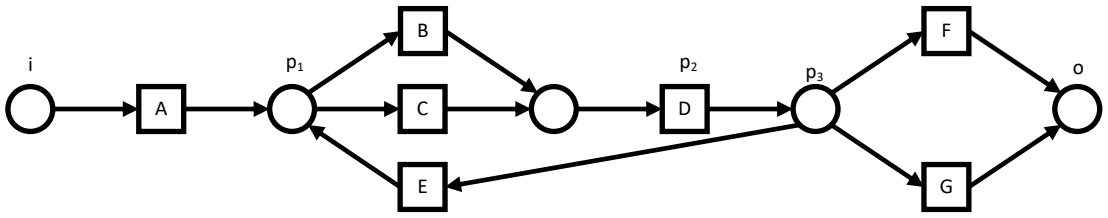
σ_2								
N								

What is the cost?

$\delta =$

$$fitness(\sigma, N) = 1 - \frac{\delta(\lambda^N_{opt}(\sigma))}{\delta(\lambda^N_{worst}(\sigma)}$$

Alignments
Exercise 1b



$\gamma_2 =$

σ_2	A	D	C	$\gg$	E	G
N	A	$\gg$	C	D	$\gg$	G

$\delta = 3$

Determine the fitness of the process model above and the observed traces by considering the alignments.

What is the worst valid alignment with the shortest path from initial state to final state?

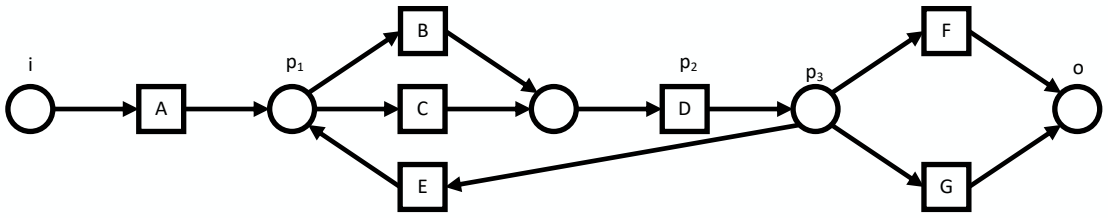
σ_2	A	D	C	E	G	$\gg$	$\gg$	$\gg$	$\gg$
N	$\gg$	$\gg$	$\gg$	$\gg$	$\gg$	A	C	D	G

What is the cost?

$\delta = 9$

$$fitness(\sigma, N) = 1 - \frac{\delta(\lambda^N_{opt}(\sigma))}{\delta(\lambda^N_{worst}(\sigma)} = 1 - \frac{3}{9} = 0.6667$$

Alignments
Exercise 1b



$\gamma_3 =$

σ_3	A	$\gg$	$\gg$	F	G
N	A	C	D	F	$\gg$

$\delta = 3$

Determine the fitness of the process model above and the observed traces by considering the alignments.

What is the worst valid alignment with the shortest path from initial state to final state?

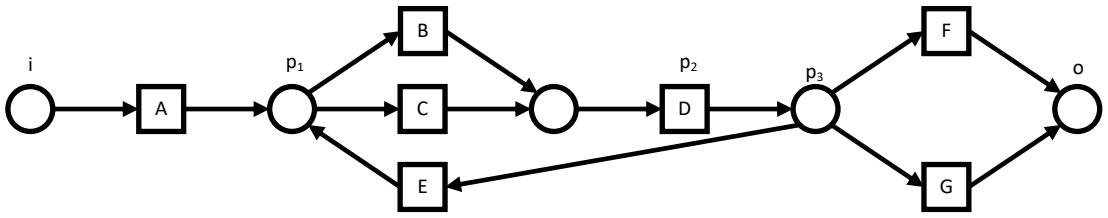
σ_3							
N							

What is the cost?

$\delta =$

$$fitness(\sigma, N) = 1 - \frac{\delta(\lambda^N_{opt}(\sigma))}{\delta(\lambda^N_{worst}(\sigma)}$$

Alignments
Exercise 1b



$\gamma_3 =$

σ_3	A	$\gg$	$\gg$	F	G
N	A	C	D	F	$\gg$

$\delta = 3$

Determine the fitness of the process model above and the observed traces by considering the alignments.

What is the worst valid alignment with the shortest path from initial state to final state?

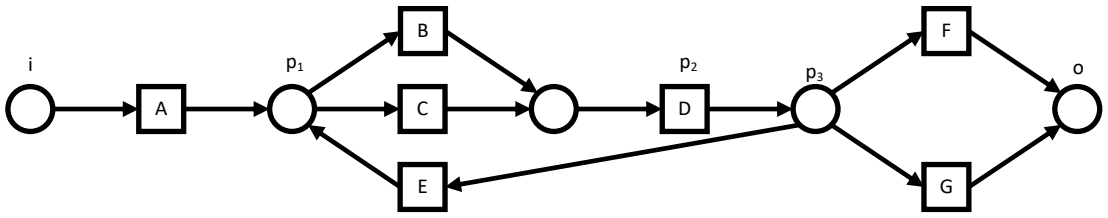
σ_3	A	F	G	$\gg$	$\gg$	$\gg$	$\gg$
N	$\gg$	$\gg$	$\gg$	A	C	D	F

What is the cost?

$\delta = 7$

$$fitness(\sigma, N) = 1 - \frac{\delta(\lambda^N_{opt}(\sigma))}{\delta(\lambda^N_{worst}(\sigma)} = 1 - \frac{3}{7} = 0.5714$$

Alignments
Exercise 1b



$\gamma_4 =$

σ_4	≫	≫	≫	F	D	C	A
N	A	C	D	F	≫	≫	≫

$\delta = 6$

Determine the fitness of the process model above and the observed traces by considering the alignments.

What is the worst valid alignment with the shortest path from initial state to final state?

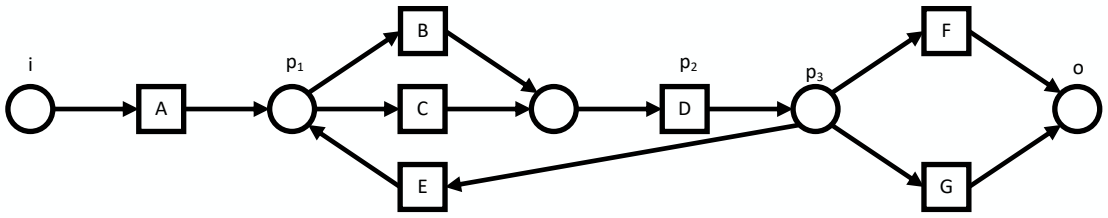
σ_4	F	D	C	A	≫	≫	≫	≫
N	≫	≫	≫	≫	A	C	D	F

What is the cost?

$\delta = 8$

$$fitness(\sigma, N) = 1 - \frac{\delta(\lambda^N_{opt}(\sigma))}{\delta(\lambda^N_{worst}(\sigma)} = 1 - \frac{6}{8} = 0.25$$

Alignments
Exercise 1b



$\gamma_4 =$

σ_4	$\gg$	$\gg$	$\gg$	F	D	C	A
N	A	C	D	F	$\gg$	$\gg$	$\gg$

$\delta = 6$

Determine the fitness of the process model above and the observed traces by considering the alignments.

What is the worst valid alignment with the shortest path from initial state to final state?

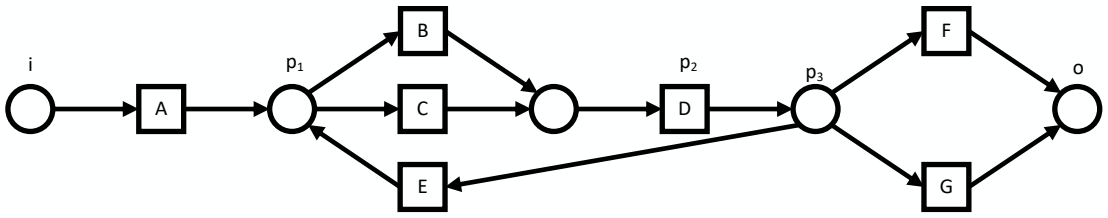
σ_4							
N							

What is the cost?

$\delta =$

$$fitness(\sigma, N) = 1 - \frac{\delta(\lambda^N_{opt}(\sigma))}{\delta(\lambda^N_{worst}(\sigma)}$$

Alignments
Exercise 1b



$\gamma_5 =$

σ_5	$\gg$	C	$\gg$	$\gg$
N	A	C	D	F

$\delta = 3$

Determine the fitness of the process model above and the observed traces by considering the alignments.

What is the worst valid alignment with the shortest path from initial state to final state?

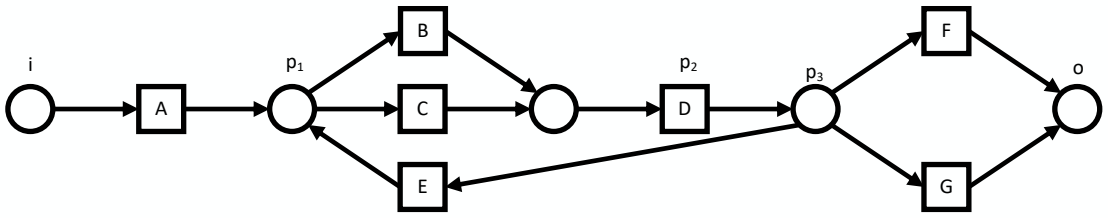
σ_5					
N					

What is the cost?

$\delta =$

$$fitness(\sigma, N) = 1 - \frac{\delta(\lambda^N_{opt}(\sigma))}{\delta(\lambda^N_{worst}(\sigma)}$$

Alignments
Exercise 1b



$\gamma_5 =$

σ_5	$\gg$	C	$\gg$	$\gg$
N	A	C	D	F

$\delta = 3$

Determine the fitness of the process model above and the observed traces by considering the alignments.

What is the worst valid alignment with the shortest path from initial state to final state?

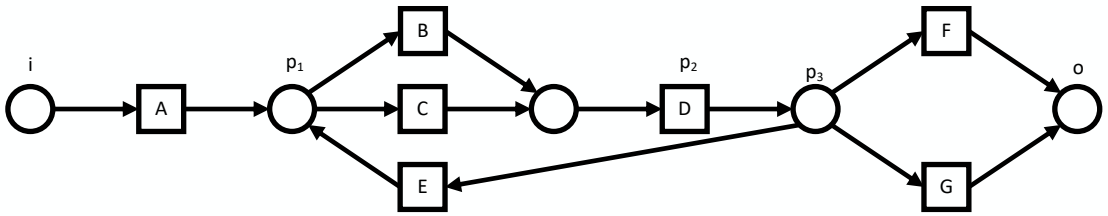
σ_5	C	$\gg$	$\gg$	$\gg$	$\gg$
N	$\gg$	A	C	D	F

What is the cost?

$\delta = 5$

$$fitness(\sigma, N) = 1 - \frac{\delta(\lambda^N_{opt}(\sigma))}{\delta(\lambda^N_{worst}(\sigma)} = 1 - \frac{3}{5} = 0.8$$

Alignments
Exercise 1b



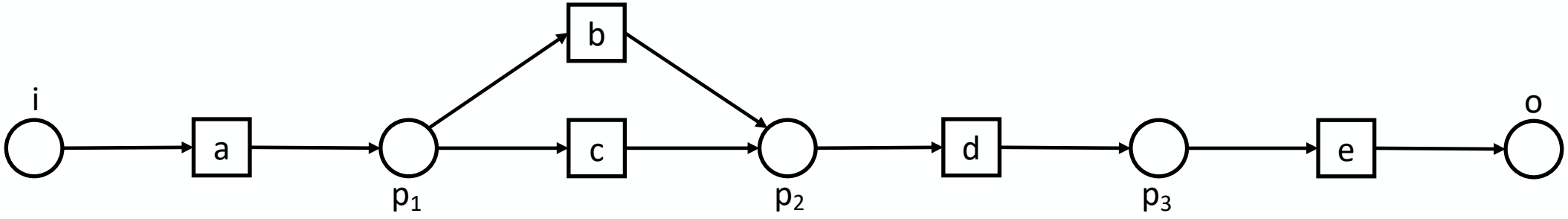
Determine the fitness of the process model above and the observed traces by considering the alignments.

#	Trace	$\delta(\lambda^N_{opt}(\sigma))$	$\delta(\lambda^N_{worst}(\sigma)$	$fitness(\sigma, N)$
200	ACDF	0	8	1
120	ADCEG	3	9	$^6/_9$
100	AFG	3	7	$^4/_7$
80	FDCA	6	8	$^1/_4$
10	C	3	5	$^2/_5$

$$fitness(L,N) = 1 - \frac{\Sigma_{\sigma \in L} L(\sigma) \times \delta\left(\lambda^N_{opt}(\sigma)\right)}{\Sigma_{\sigma \in L} L(\sigma) \times \delta\left(\lambda^N_{worst}(\sigma)\right)} = 1 - \frac{0 + 360 + 300 + 480 + 30}{1600 + 1080 + 700 + 640 + 50} = 0.713$$

Petri Net Construction
Exercise 2a

If possible draw a Petri net that has exactly two optimal alignments with replay fitness of $^{6}/_{7}$, given the trace $\sigma_i = < a, d, e >$



best:

σ_i	a	≫	d	e
N	a	b	d	e

σ_i	a	≫	d	e
N	a	c	d	e

worst:

σ_i	a	d	e	≫	≫	≫	≫
N	≫	≫	≫	a	c	d	e

Petri Net Construction
Exercise 2b

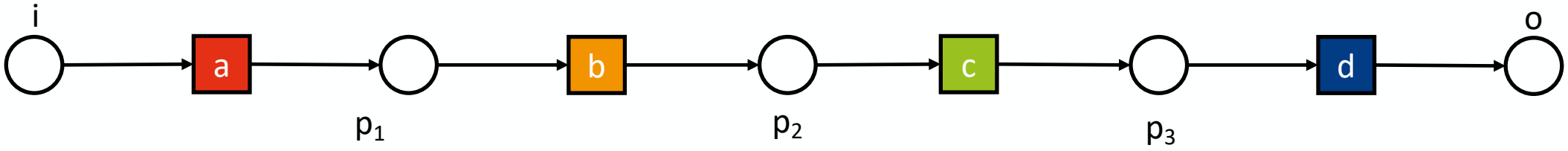
If possible draw a Petri net that has exactly two optimal alignments with replay fitness of 1, given the trace $\sigma_{ii} =< a, b, c, d >$.

Not possible.

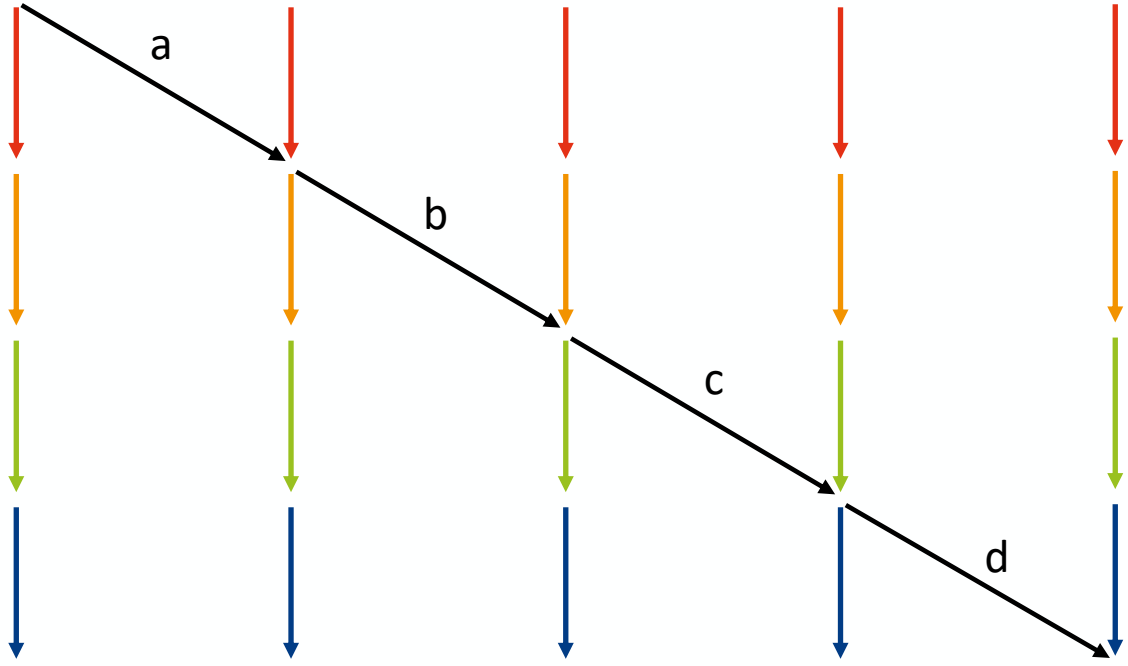
In order for $fitness(\sigma_{ii}, N) = 1$, the expression $\delta(\lambda^N_{opt}(\sigma_{ii})$ has to be zero.
This will only be the case if the alignment fits perfectly with synchronous moves only.
Exactly one alignment can fulfil this requirement.

σ_{ii}	a	b	c	d
N	a	b	c	d

Alignment Search Space
Example



Trace σ =< a,b,c,d >

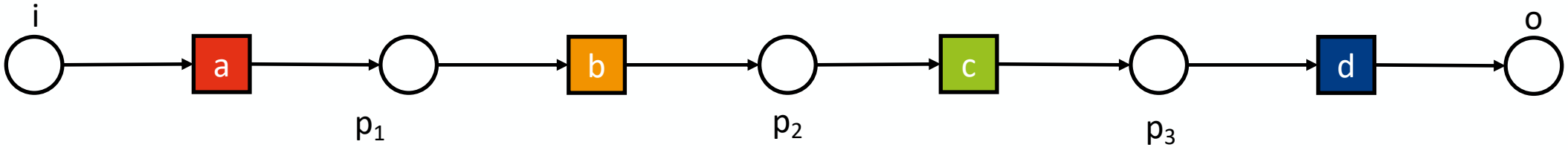


σ	a	b	c	d
N	a	b	c	d

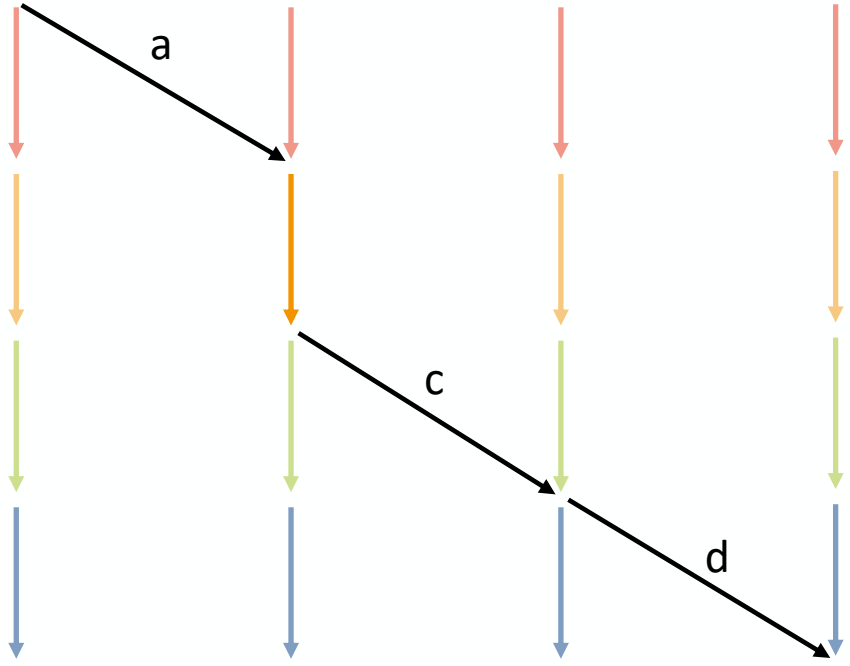
Objective: Find shortest path from top left (initial state before first log move) to bottom right (final state of last log move).

Diagonal arcs result in a synchronous move

Alignment Search Space
Example



Trace σ =< a,c,d >

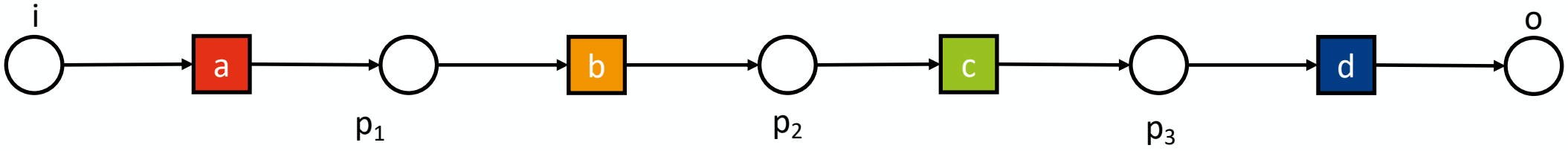


σ	a	≫	c	d
N	a	b	c	d

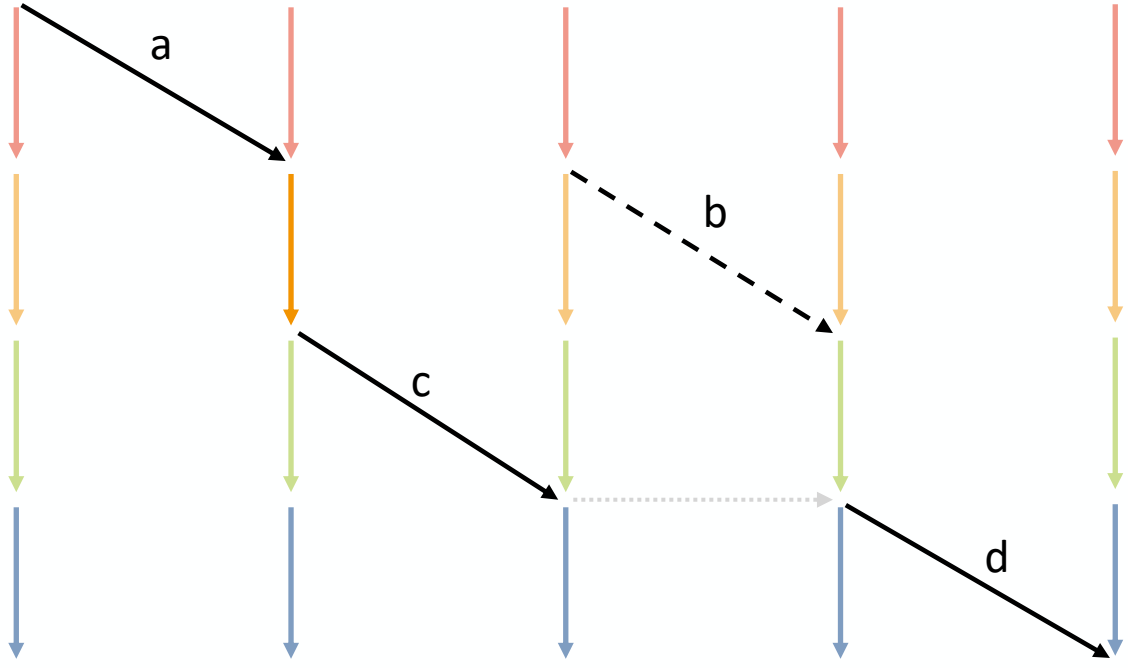
Activity *b* not in log.

Vertical arcs result in a skip move in the log

Alignment Search Space
Example



Trace σ =< a,c,b,d >



σ	a	≫	c	b	d
N	a	b	c	≫	d

Activity *b* and *c* are switched.

Horizontal arcs result in a skip move in the model

Alignment Search Space

Exercise 3a

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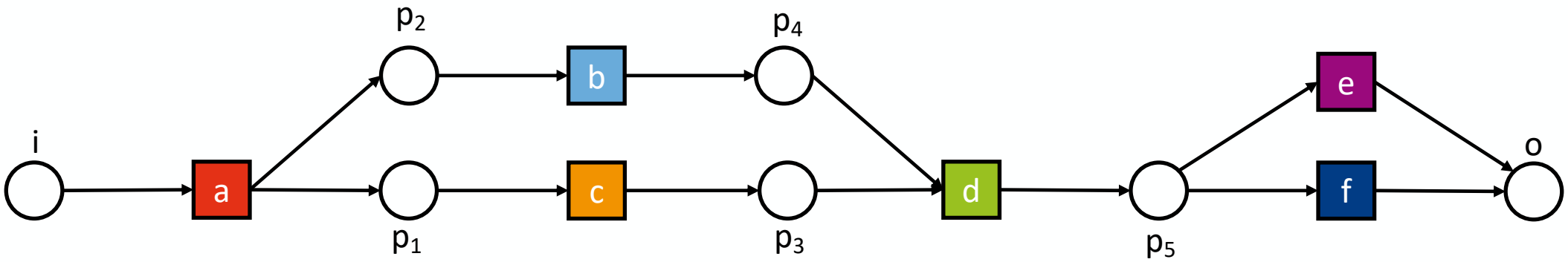
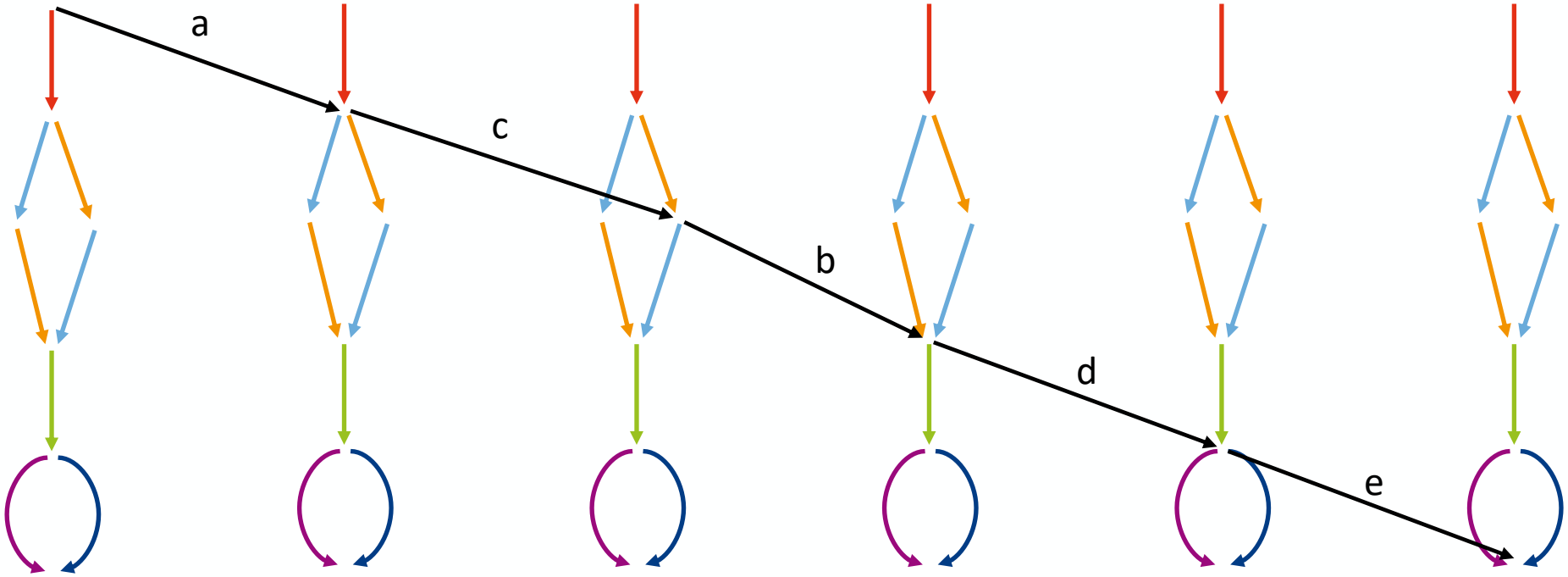


Trace σ =< a,c,b,d,e >



Alignment Search Space

Exercise 3b

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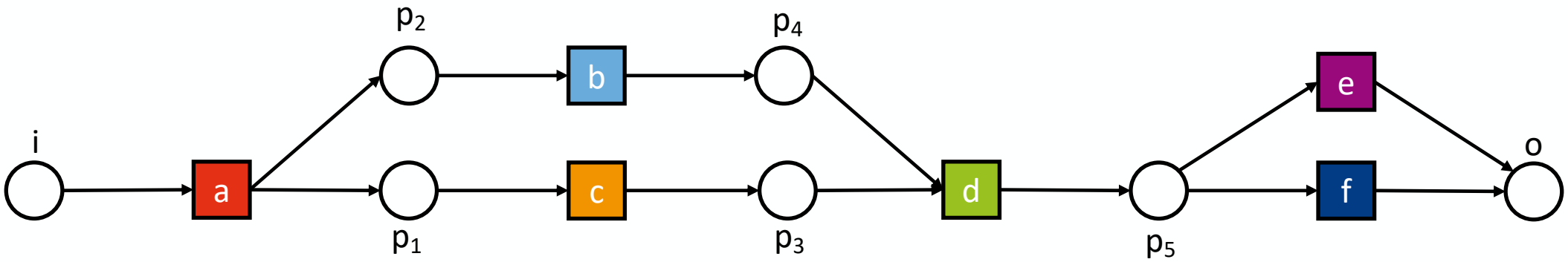
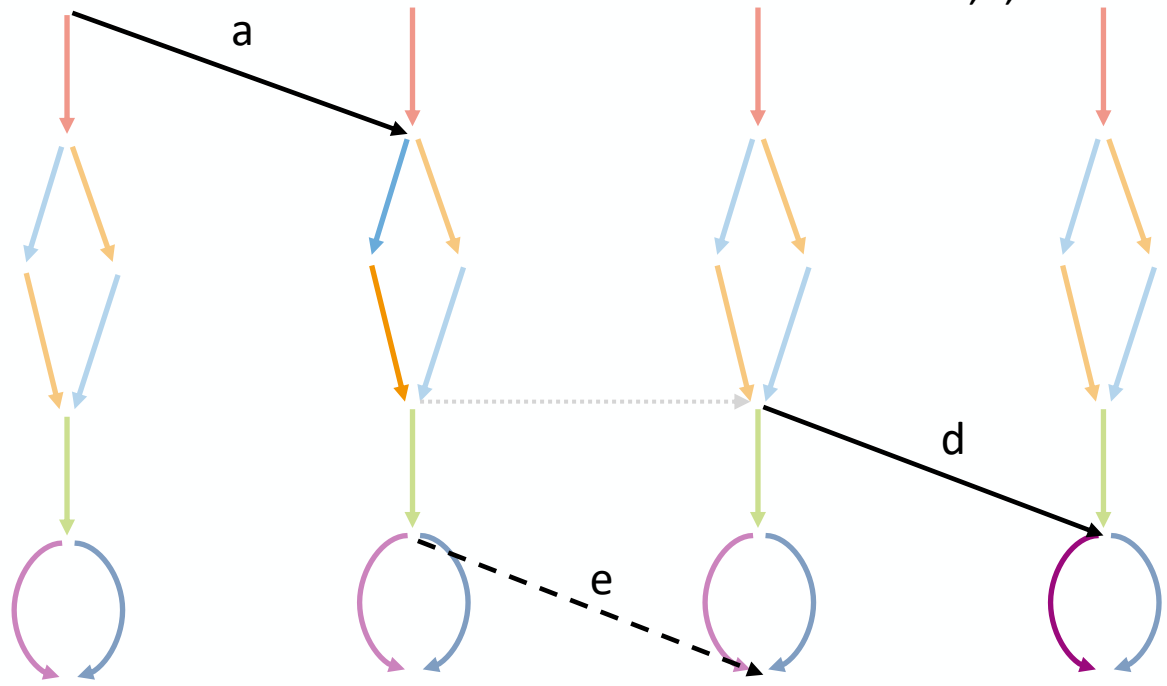


Trace σ =< a,e,d >



σ	a	≫	≫	e	d	≫
N	a	b	c	≫	d	e

Alignment Search Space

Exercise 3b

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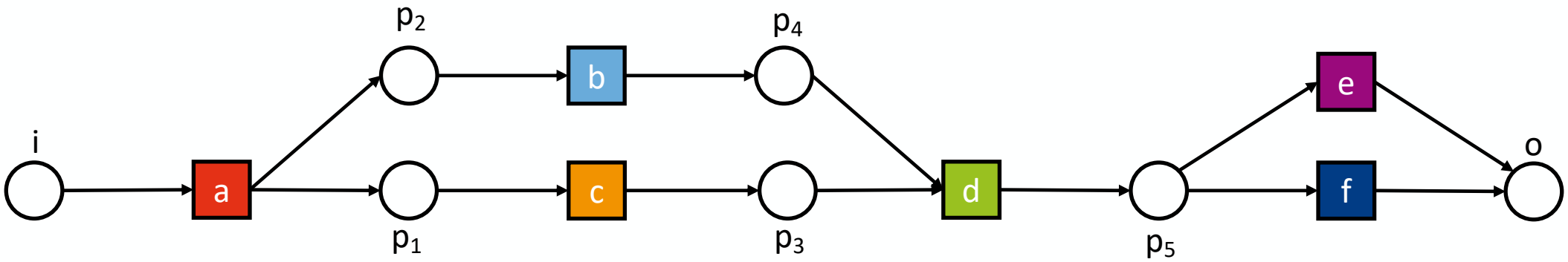
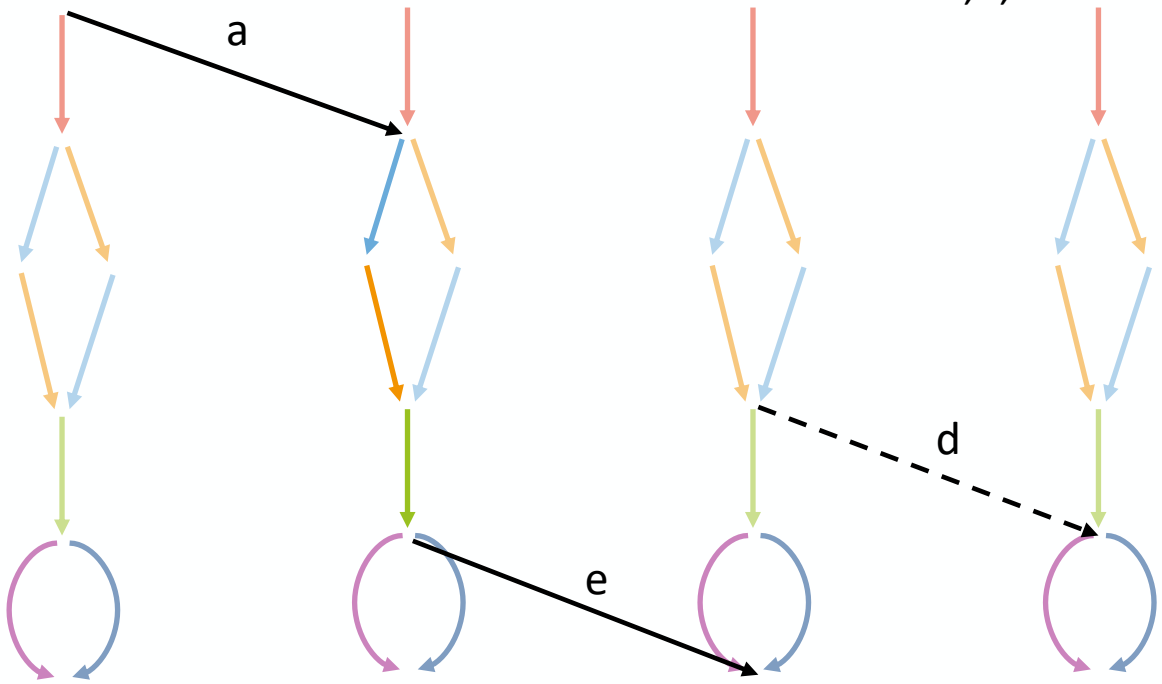


Trace σ =< a,e,d >



σ	a	≫	≫	≫	e	d
N	a	b	c	d	e	≫

Alignment Search Space

Exercise 3c

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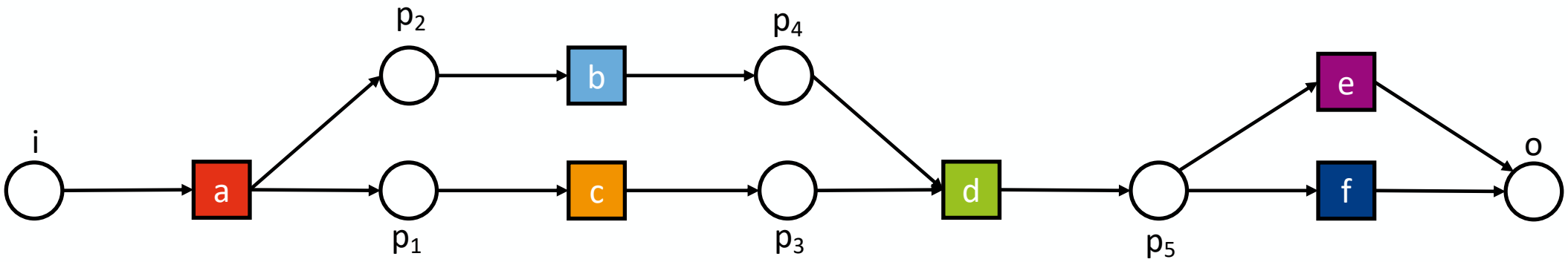
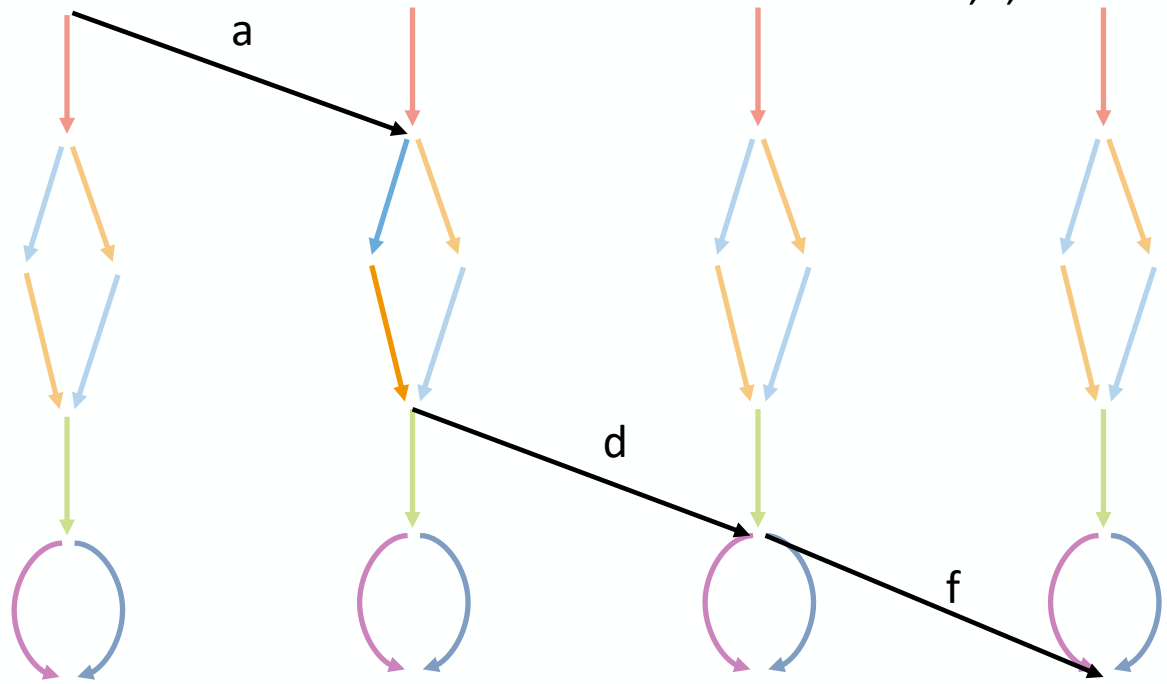


Trace σ =< a,d,f >



σ	a	≫	≫	d	f
N	a	b	c	d	f