Kiel University
Christian-Albrechts-Universität zu Kiel

Advanced Process Mining

Sommer term 2020

Exercise sheet 1

Recap

Soundness

When is a Petri net sound?
What criteria have to be met?

Safe

Option to complete

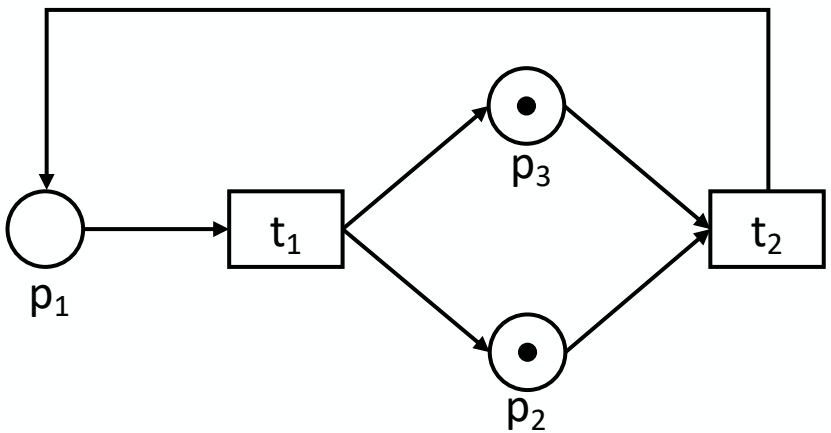
Proper completion

No dead transitions
(initial marking)

Soundness
Safe

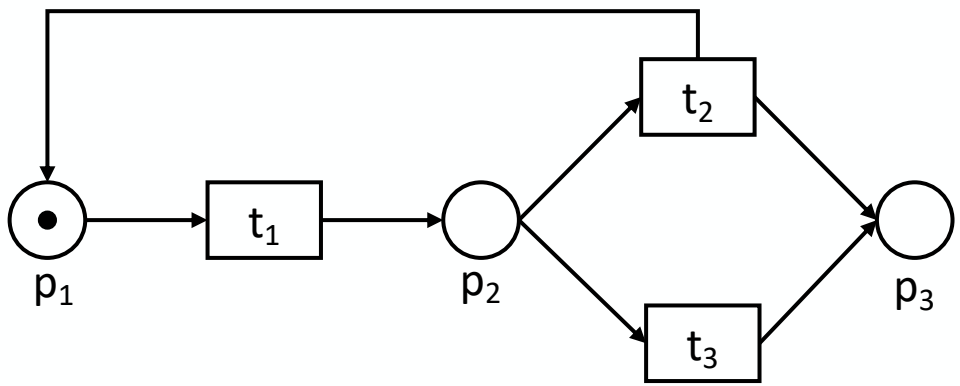
- Synonym: 1-bounded
- A place is safe (or 1-bounded) if for all reachable markings the tokens in that place never exceed one
- A Petri net is safe (or 1-bounded) if all its places are safe

Which one of these Petri-Nets is safe:



Safe

No place exceeds the allowed token limit of one



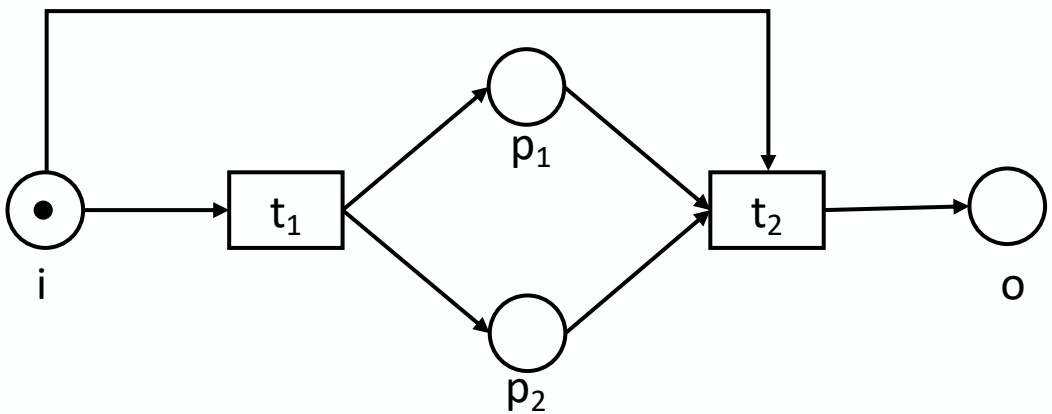
Not Safe

There are possible markings where p_3 will exceed the token limit of one

Soundness
Option to complete

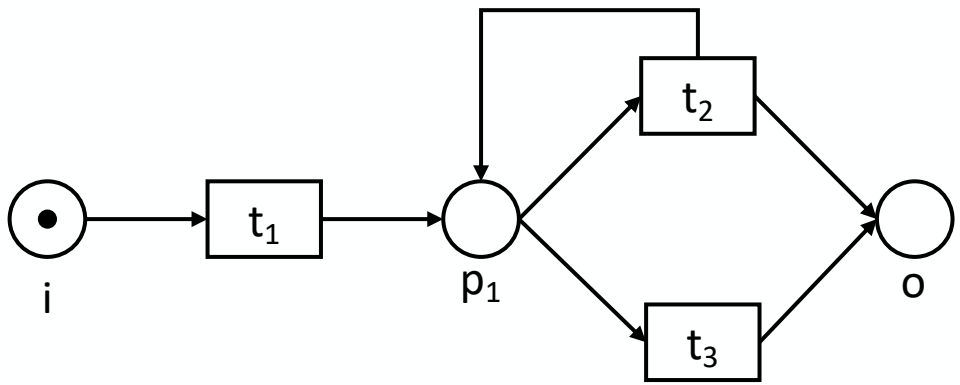
- It must always be possible to mark the sink place

Which one of these Petri-Nets fulfils this requirement:



Violated

The sink place o will never be reached.



Fulfilled

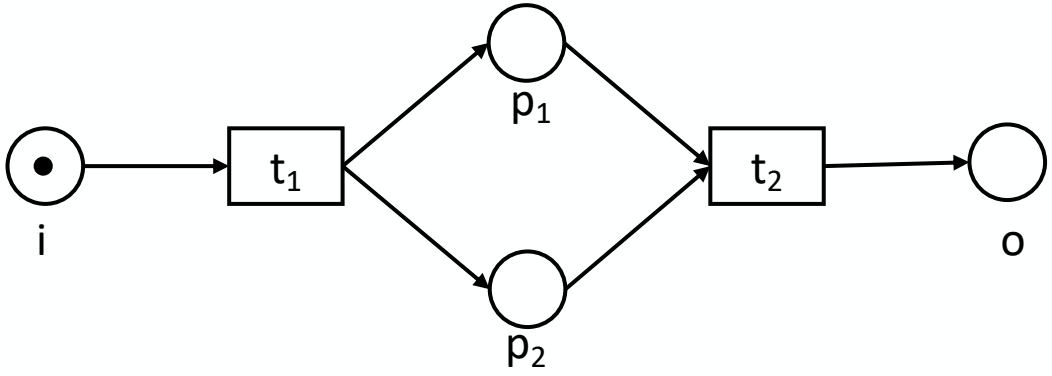
The sink place o can be reached for all possible markings.

Soundness
Proper completion

Kiel University
Christian-Albrechts-Universität zu Kiel

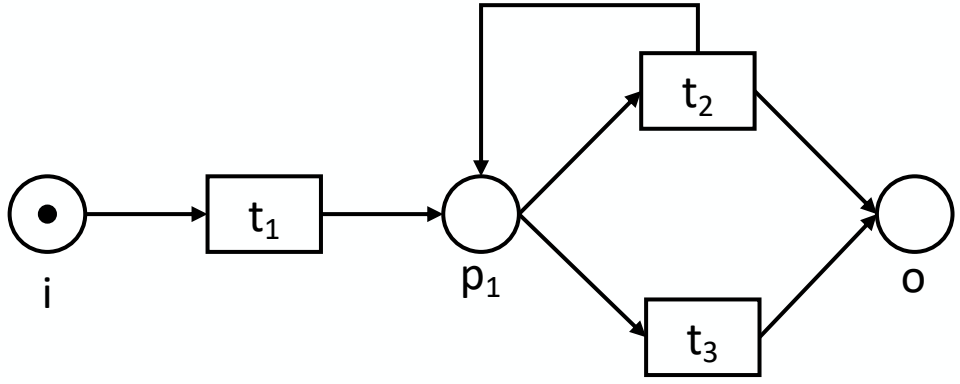
- If the sink place is marked, all other places should be empty

Which one of these Petri-Nets fulfils this requirement:



Fulfilled

Once the sink place o is reached, all other places are empty.



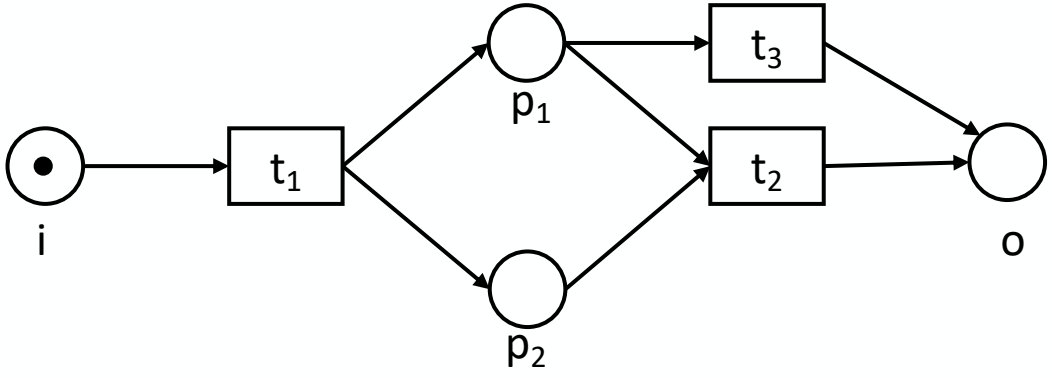
Violated

Once activating transition t_2 the sink place is marked, but place p_1 is not empty.

Soundness
No dead transitions

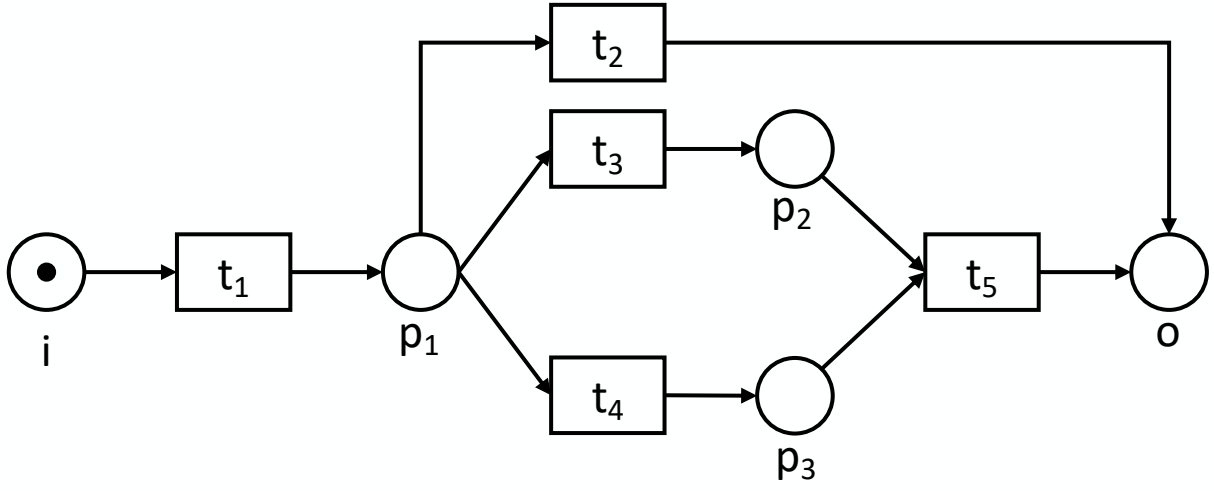
- All parts of the Petri net should potentially be reachable from its initial marking

Which one of these Petri-Nets does not contain dead transitions:



No dead transition

From the initial marking, every transition can possibly be activated.



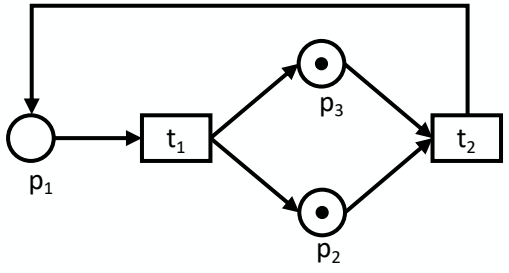
Dead transition

The transition t_5 is dead. It can never be activated.

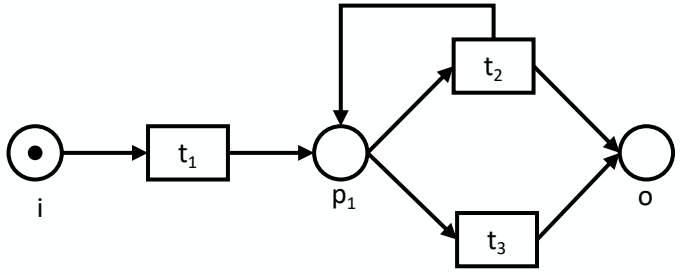
Soundness

When is a Petri net sound?
What criteria have to be met?

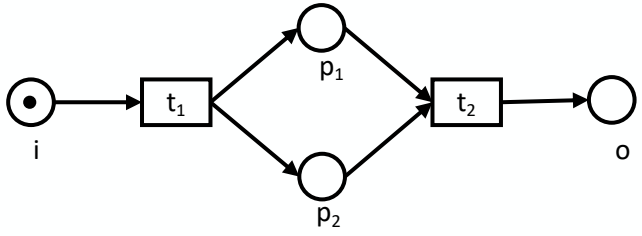
Safe



Option to complete

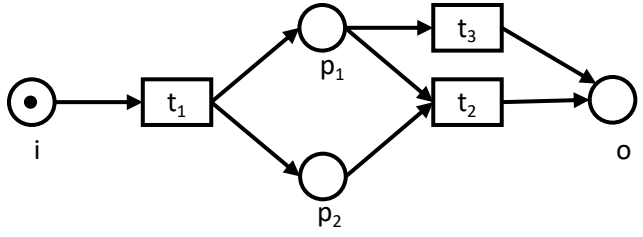


Proper completion



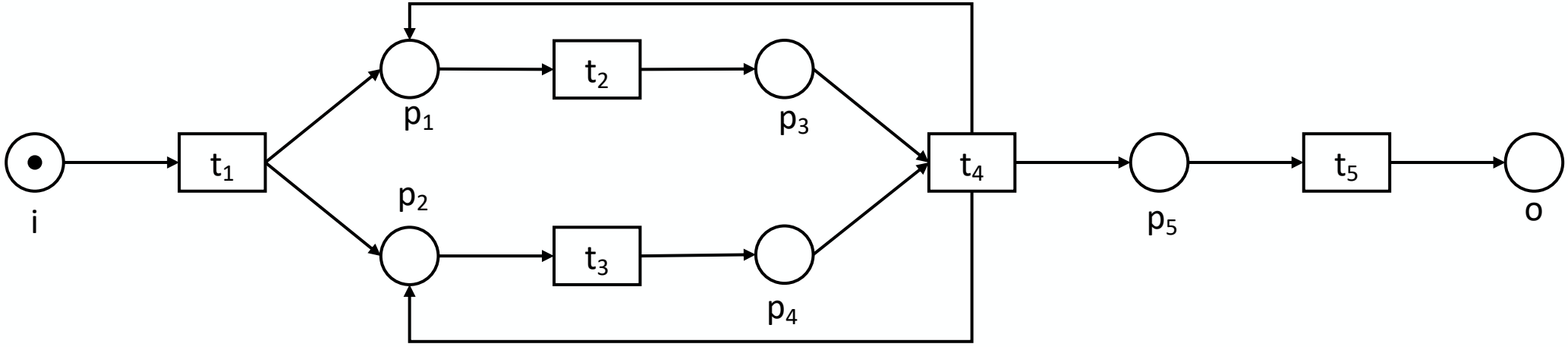
No dead transitions

(initial marking)



Soundness
Exercise 1

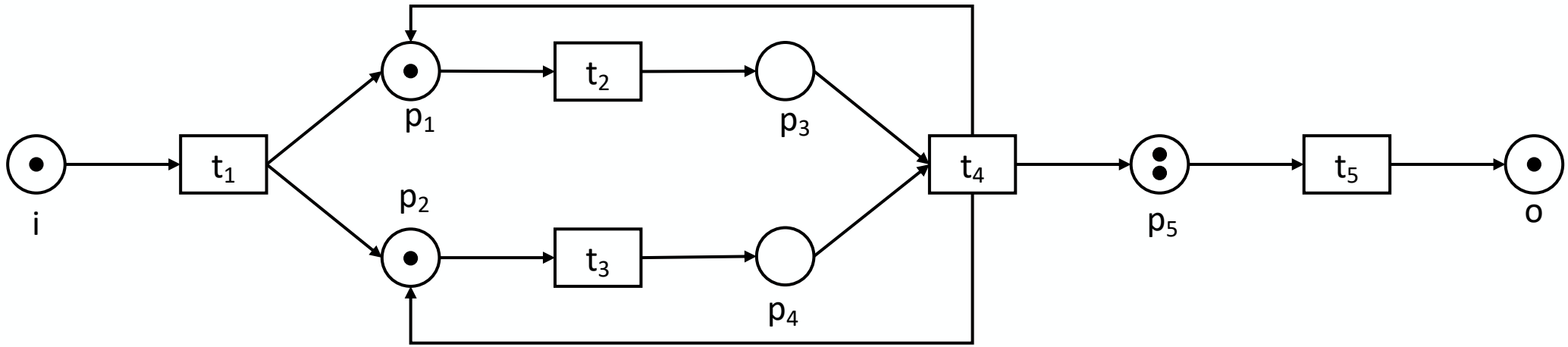
Is the following Petri Net sound?



Soundness
Exercise 1

Kiel University
Christian-Albrechts-Universität zu Kiel

Is the following Petri Net sound?



✗ Safe: The place p_5 can accumulate more than one token.

✗ Proper completion: Because of the AND-split in t_4 at least two tokens will always remain in the net when the sink place is marked.

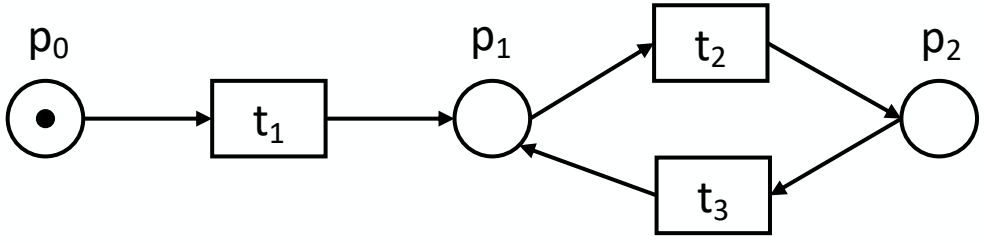
✓ Option to complete: No matter the sequence or combination transitions are enabled, it is always possible to mark the sink place.

✓ No dead transitions: Given the initial marking of the Petri net, all parts of the model are potentially reachable.

Transitions

A transitions is called **live** under a given marking m,
if it is not dead under any reachable marking m'.

A Transition is called dead, given a marking m
if it cannot be activated by any reachable marking m' or m itself.



	Live	Dead
t_1		
t_2		
t_3		

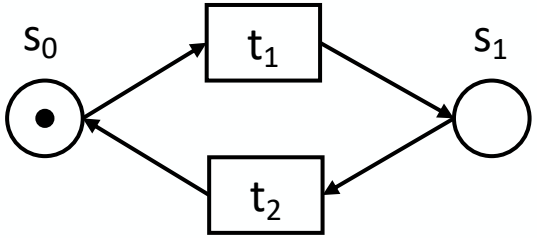
Kiel University
Christian-Albrechts-Universität zu Kiel

Markings

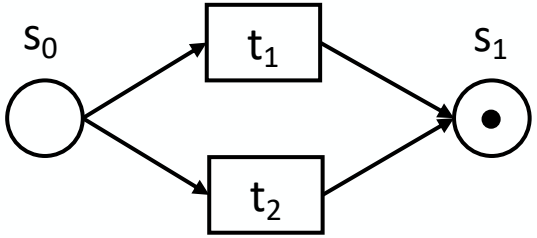
A marking is called **live**, if all Transitions are live.

A marking is called **dead**, if all transitions are dead.

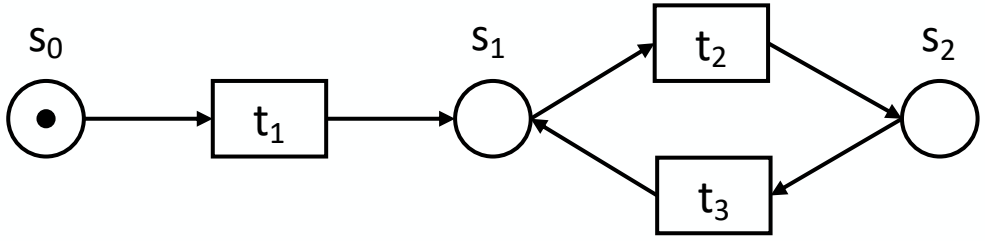
A marking is called **deadlock-free** if no dead marking can be reached.



Marking is live



Marking is dead

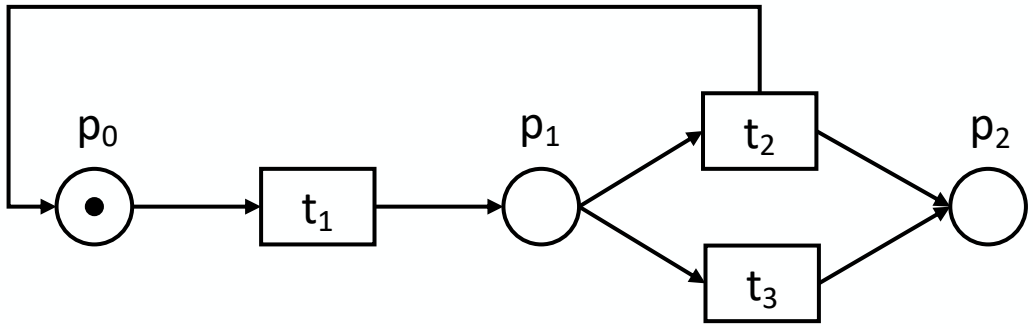


Marking is neither live nor dead

Kiel University
Christian-Albrechts-Universität zu Kiel

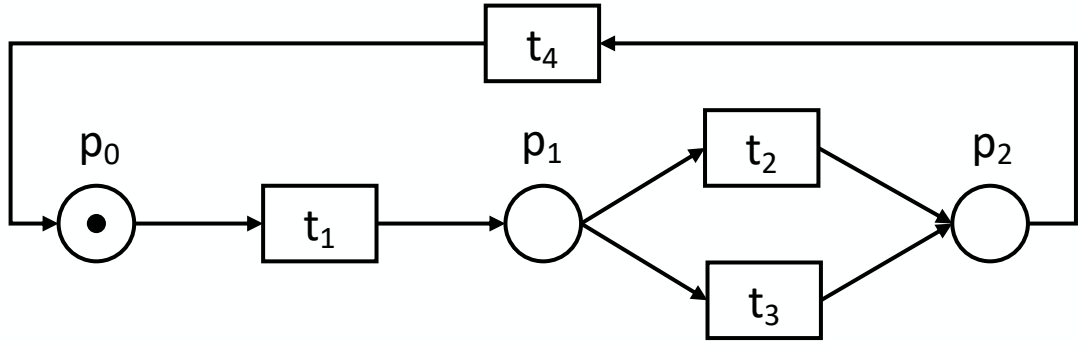
A marked Petri net with the initial marking m_0 is called **reversible,** if m_0 is reachable from every reachable marking.

Are these Petri-Nets is reversible, given its initial marking:



Not reversible

The initial marking can never be restored once t_1 has been activated.



Reversible

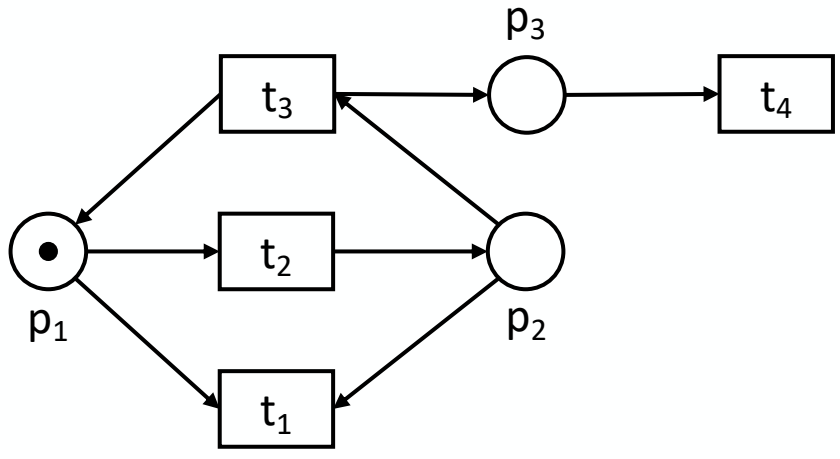
After activating t_4 the initial marking can be restored again

Liveness and Reversibility
Exercise 2

Create a Petri net with the following properties:

- The Petri net is not live
- The Petri net is reversible
- The initial marking has exactly one token
- Deadlock free
- Petri net is not bounded

One possible solution:



Workflow Nets

A workflow net is a Petri net $N = (P, T, F)$ that fulfils the following three properties:

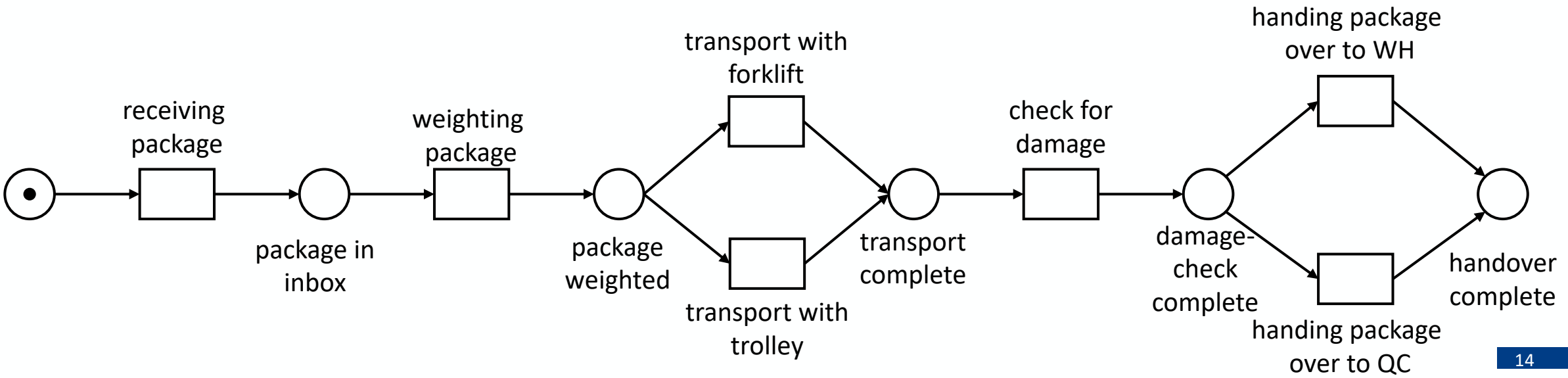
- It exists one source place such that $\cdot i = \emptyset$
- It exists one sink place such that $o \cdot = \emptyset$
- The extended Petri net with the transition t^* between i and o ($\cdot t^* = \{o\}$ und $t^* \cdot = \{i\}$) has to be strongly connected (it exists a direct path between any pair of nodes in the extended Petri net)

Workflow Nets
Exercise 3a

Kiel University
Christian-Albrechts-Universität zu Kiel

Model the following scenario as a workflow net:

When a package is received there are two different ways the package is processed depending on its weight. Heavy packages will be transported with a forklift to the warehouse. For light packages a trolley is used. If the package is damaged during transport the package has to be redirected to the quality control department. Otherwise it will be handed over to the warehouse. The process ends once the package has been handed over.



Workflow Nets
Exercise 3b

Is it possible for a workflow net to have more than one source place or sink place? Justify your answer.

Sink and source places have to be unique. A net with an additional sink or source place would no longer be *strongly connected*.