Solutions

Theory Sheet 7

Solution T-7.1: Maximum-likelihood estimation for exponential distribution

Set up the likelihood function

$$L(\lambda) = \prod_{i=1}^{n} f_X(x_i|\lambda) = \prod_{i=1}^{n} \lambda e^{-\lambda x_i}$$

and the log-likelihood function

$$\begin{aligned}
\ln L(\lambda) &= \sum_{i=1}^{n} \ln(f_X(x_i|\lambda)) = \sum_{i=1}^{n} \ln(\lambda e^{-\lambda x_i}) \\
&= \sum_{i=1}^{n} \{\ln \lambda - \lambda x_i\} = \sum_{i=1}^{n} \ln \lambda - \sum_{i=1}^{n} \lambda x_i \\
&= \ln \lambda \sum_{i=1}^{n} 1 - \lambda \sum_{i=1}^{n} x_i = n \ln \lambda - \lambda \sum_{i=1}^{n} x_i
\end{aligned}$$

Setting the first derivative of the log-likelihood function to zero

$$\frac{d}{d\lambda} \ln L(\lambda) = \frac{n}{\lambda} - \sum_{i=1}^{n} x_i = 0$$

leads to

$$\frac{n}{\lambda} = \sum_{i=1}^{n} x_i$$

and thus

$$\lambda = \frac{n}{\sum_{i=1}^{n} x_i} = \frac{1}{1/n \sum_{i=1}^{n} x_i}$$

So the Maximum-likelihood estimate of the free parameter is given by

$$\lambda = \frac{1}{\mu_x}$$