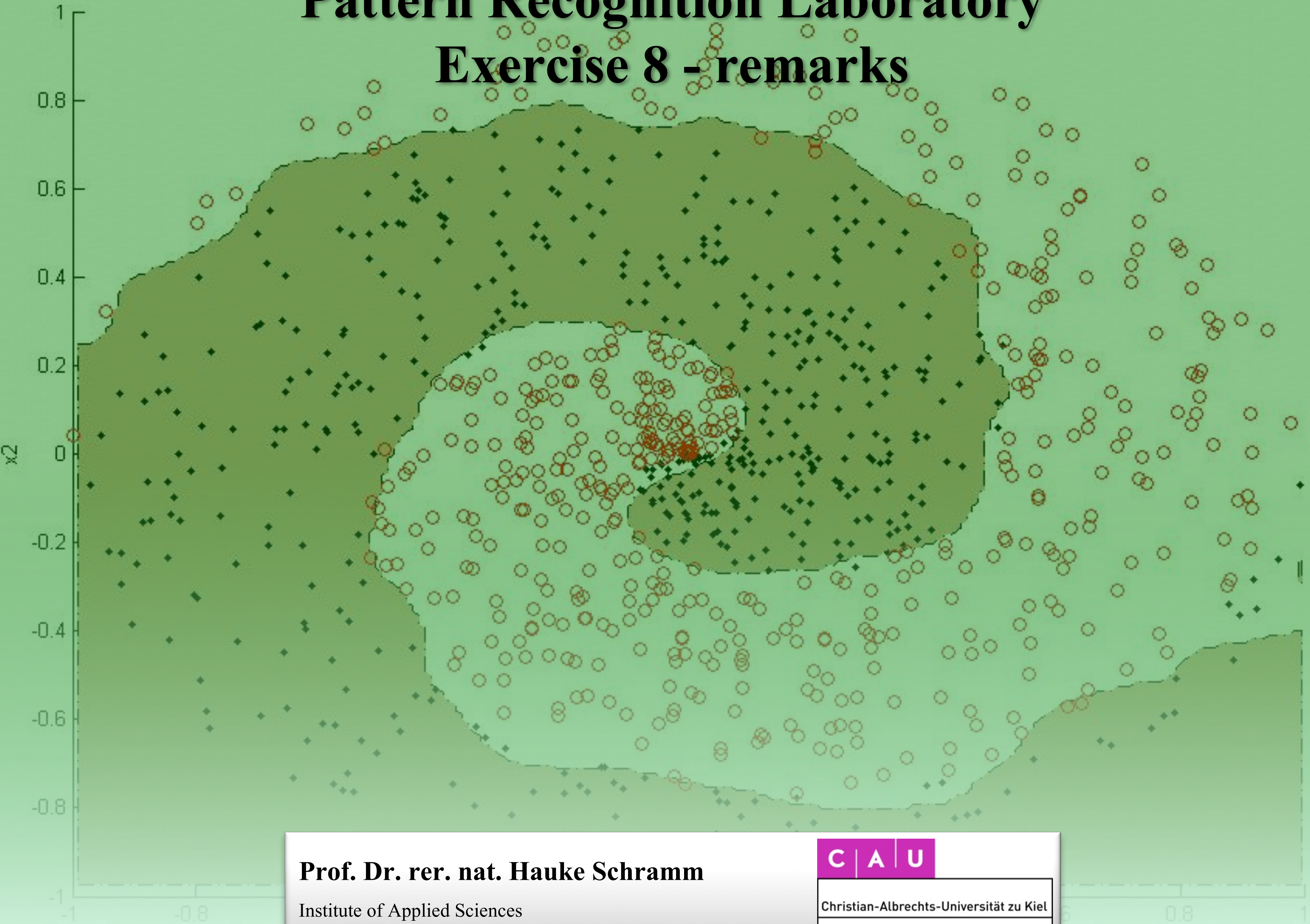
Pattern Recognition Laboratory
Exercise 8 - remarks



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Institut für Informatik

Exercise 8 - Bayes Learning

Task:

Estimation of constant c, observed under the influence of noise e_k

$$x_k = c + e_k$$

Assumptions:

- Noise is normally distributed with mean 0 and variance σ_e^2
- Constant c is a random variable with zero mean and variance σ_c^2

Given

- Prior knowledge of c (μ_c ,σ_c) and noise e (μ_e, σ_e)
- n observations of x_k → training data

Goal:

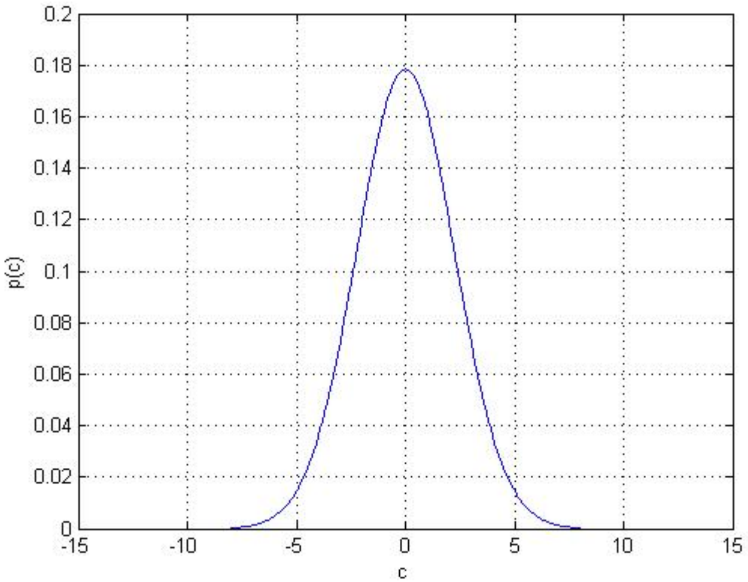
"Good" estimation of c, taking into account the available knowledge sources.

Exercise 8 - Bayes Learning

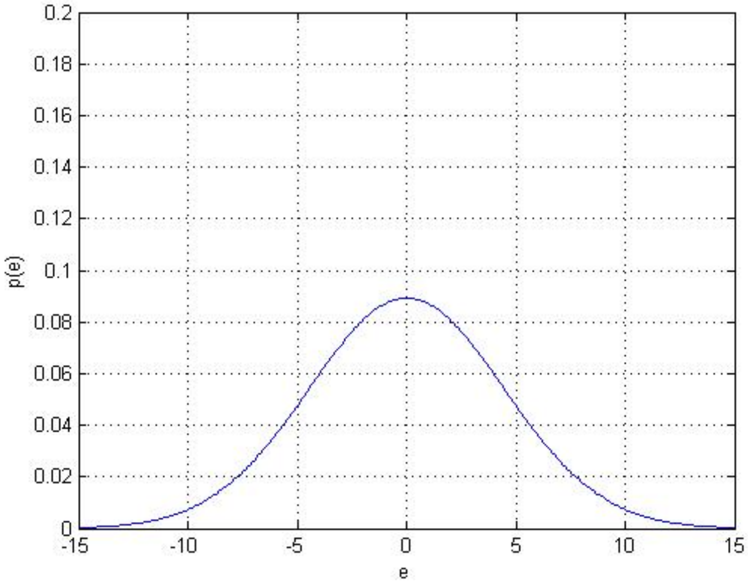
<u>Given</u>

- Prior knowledge of c → $p(c|\mu_c, \sigma_c^2)$
- Prior knowledge of e → $p(e|\mu_e, \sigma_e^2)$

$$p(c|\mu_c = 0, \sigma_c^2 = 5)$$



$$p(e|\mu_e = 0, \sigma_e^2 = 20)$$



Exercise 8 - Bayes Learning

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Exercise 8 - Bayes Learning

Generation of training data

- Constant is assumed to have a fixed value: $c = 30$

  --> prior knowledge is quite wrong!

- For different n ({1 2 4 8 16 32 64 128 256 512 1024})

  Generate n training samples using $x_k = c + e_k$

- For each training set with n samples:

  Estimate and plot

  - Log (likelihood): $log(p(D|\mu))$
  - Log (likelihood * prior): $\boxed{log(p(D|\mu) \cdot p(\mu))}$

Maximized for MAP parameter estimation

Exercise 8 - Bayes Learning

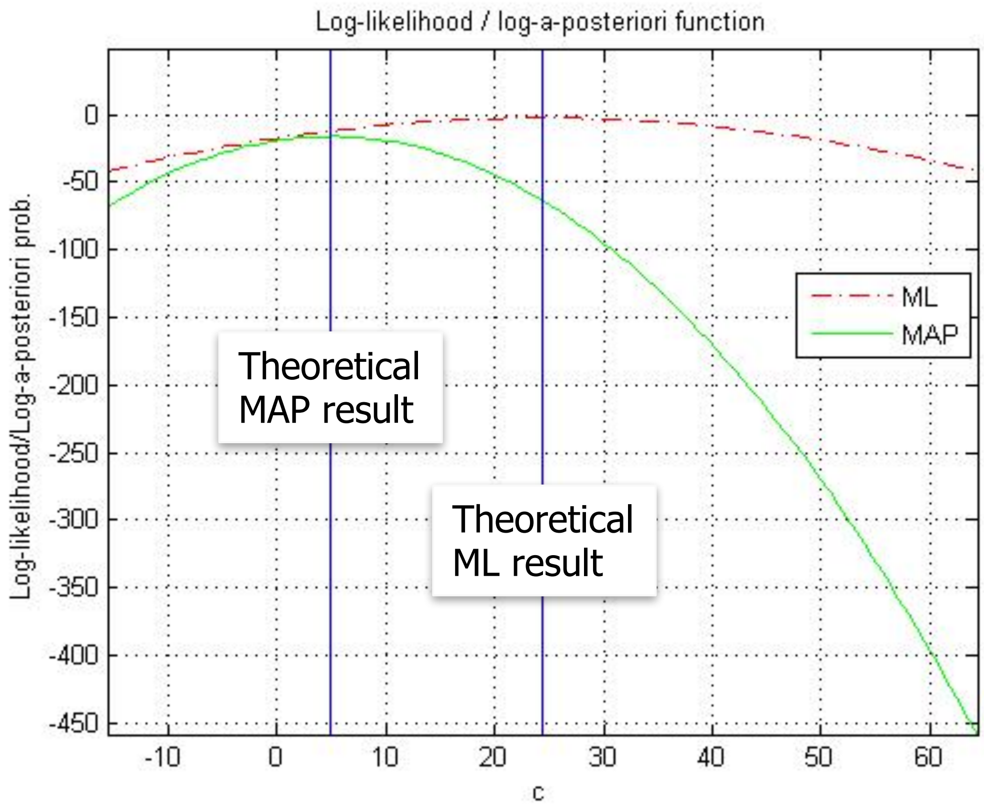
For each training set with n samples:

Estimate and plot log (likelihood), log (likelihood * prior):

1. n = 1

e =

24.4132



Exercise 8 - Bayes Learning

For each training set with n samples:

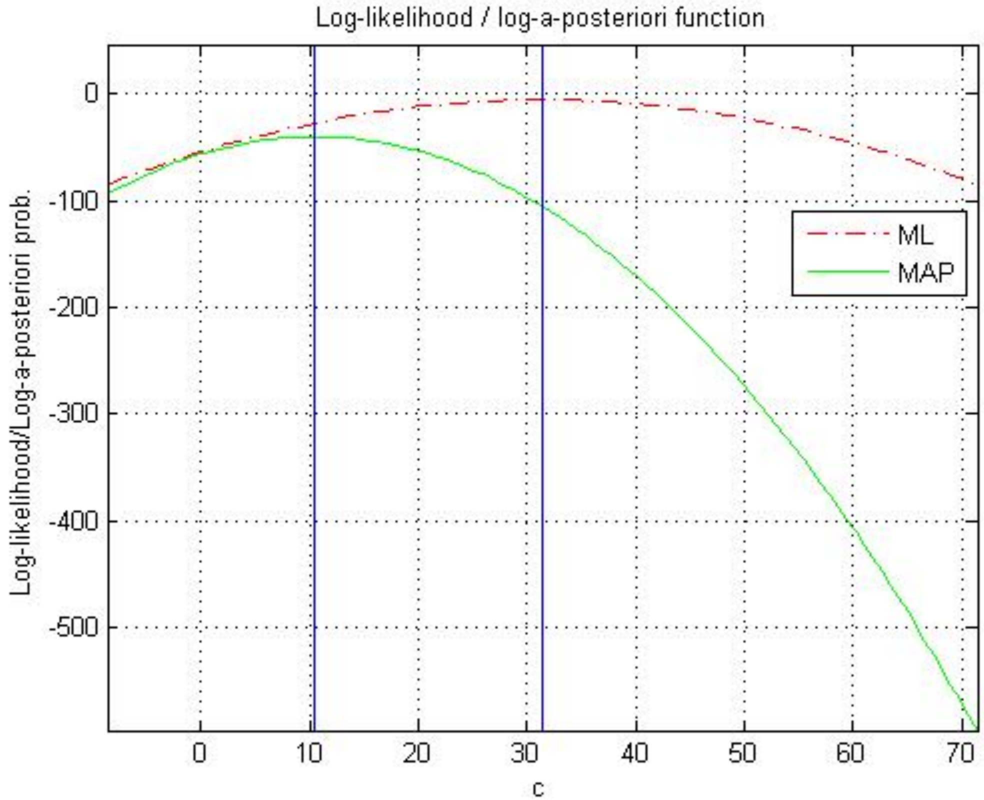
Estimate and plot log (likelihood), log (likelihood * prior):

2. n = 2

e =

34.0240

28.8097



Exercise 8 - Bayes Learning

For each training set with n samples:

Estimate and plot log (likelihood), log (likelihood * prior):

2. n = 4

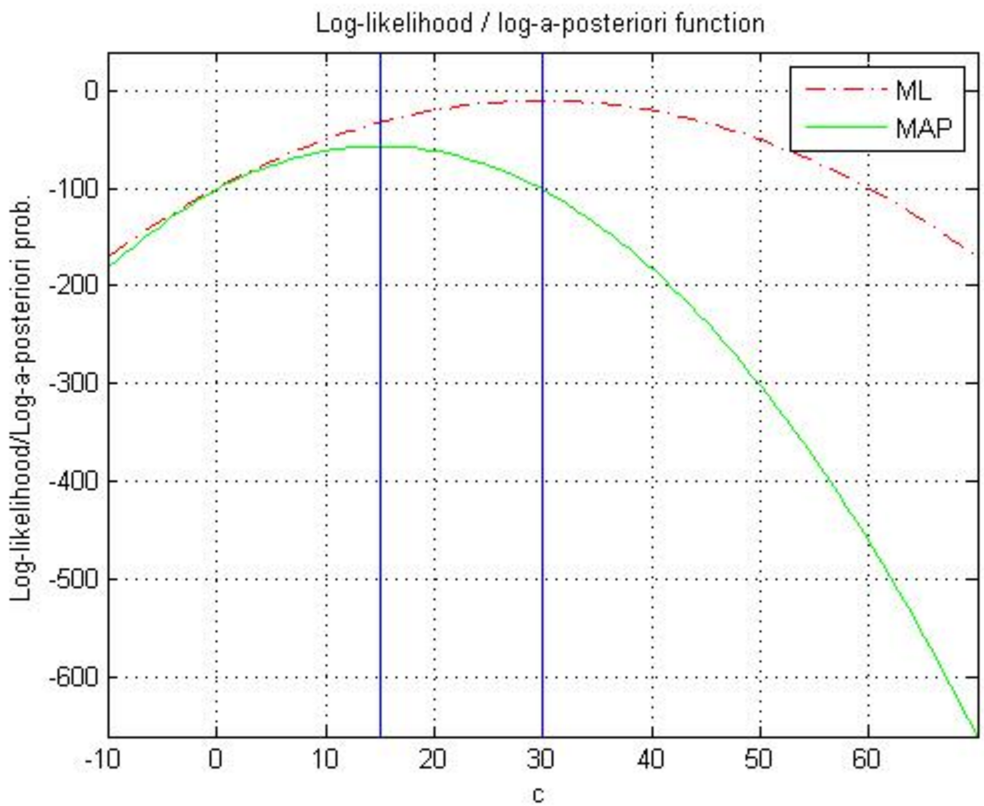
e =

34.5851

30.2318

26.8810

28.0725



Exercise 8 - Bayes Learning

For each training set with n samples:

Estimate and plot log (likelihood), log (likelihood * prior):

2. n = 8

e =

28.8391

24.3278

20.7094

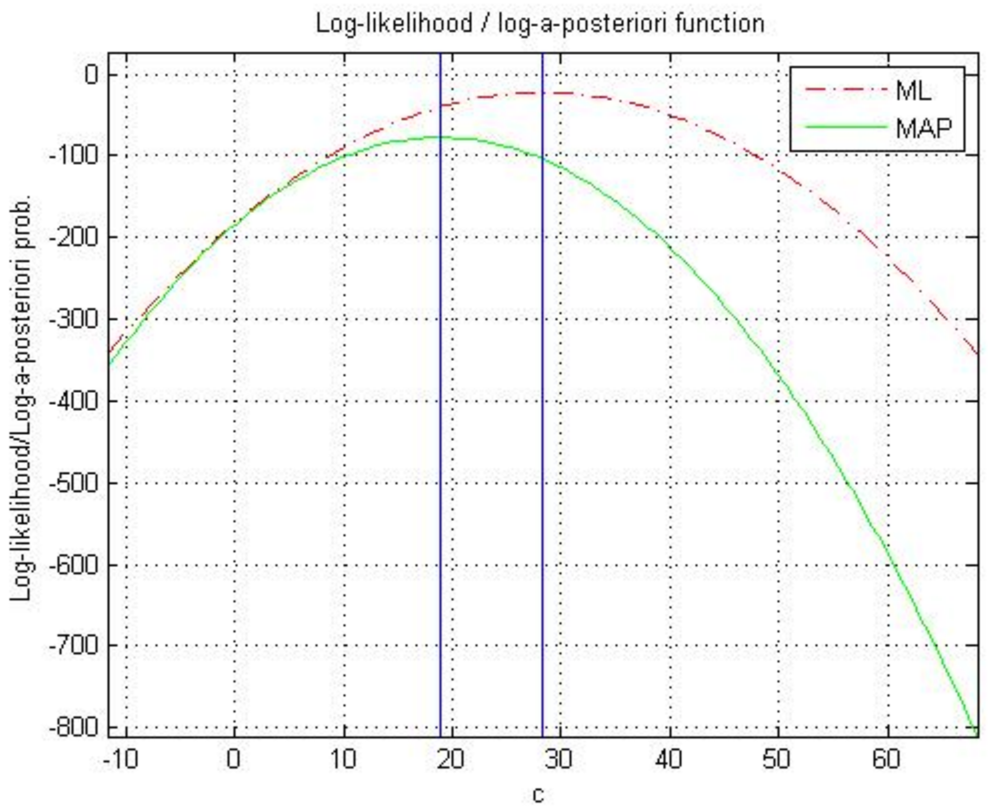
28.5715

31.0383

27.6976

29.5379

35.3295



Exercise 8 - Bayes Learning

For each training set with n samples:

Estimate and plot log (likelihood), log (likelihood * prior):

2. n = 16

e =

27.6209

32.2582

28.6064

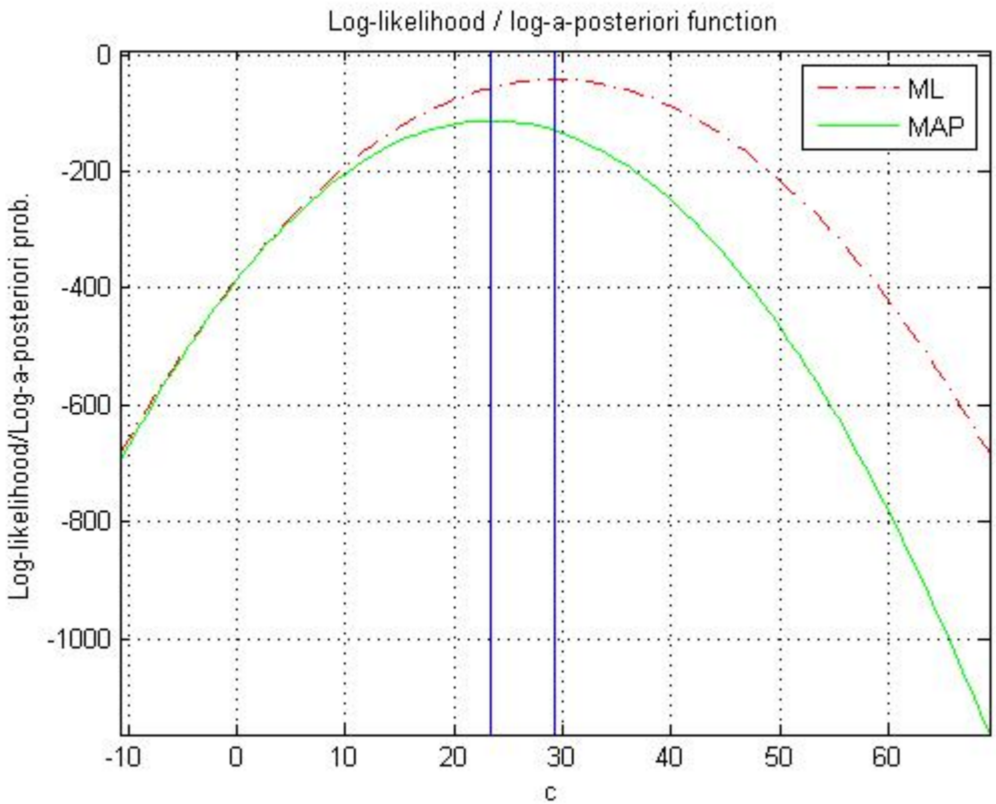
24.0197

36.4282

27.6559

26.6136

...



Exercise 8 - Bayes Learning

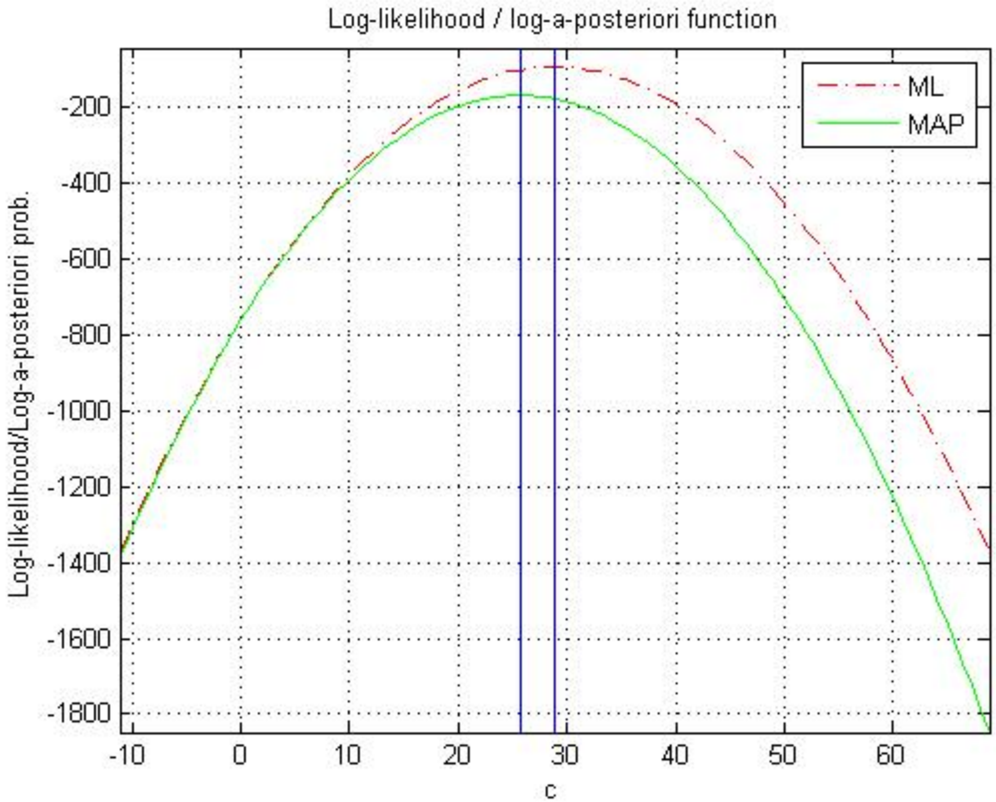
For each training set with n samples:

Estimate and plot log (likelihood), log (likelihood * prior):

2. n = 32

e =

30.9436
26.6712
33.6657
23.0147
26.6674
30.3635
24.2568
...



Exercise 8 - Bayes Learning

For each training set with n samples:

Estimate and plot log (likelihood), log (likelihood * prior):

2. n = 64

e =

39.0078

37.1491

26.2075

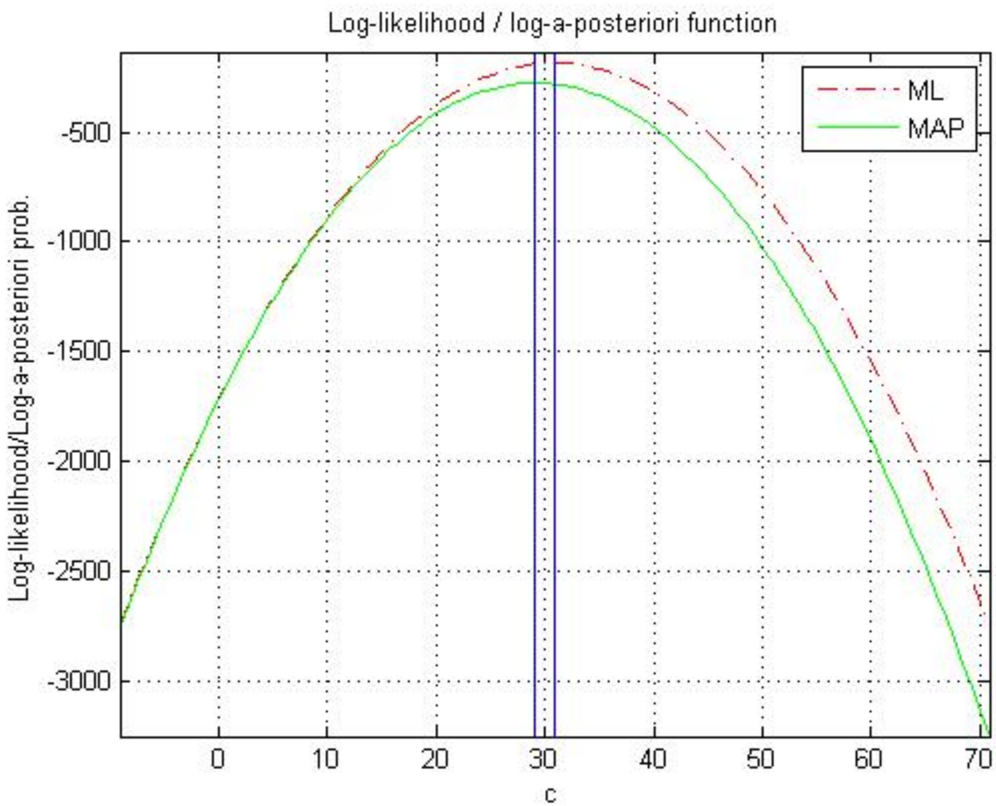
25.4153

28.8009

27.8261

35.7681

...



Exercise 8 - Bayes Learning

For each training set with n samples:

Estimate and plot log (likelihood), log (likelihood * prior):

2. n = 128

e =

32.3061

41.2414

21.6919

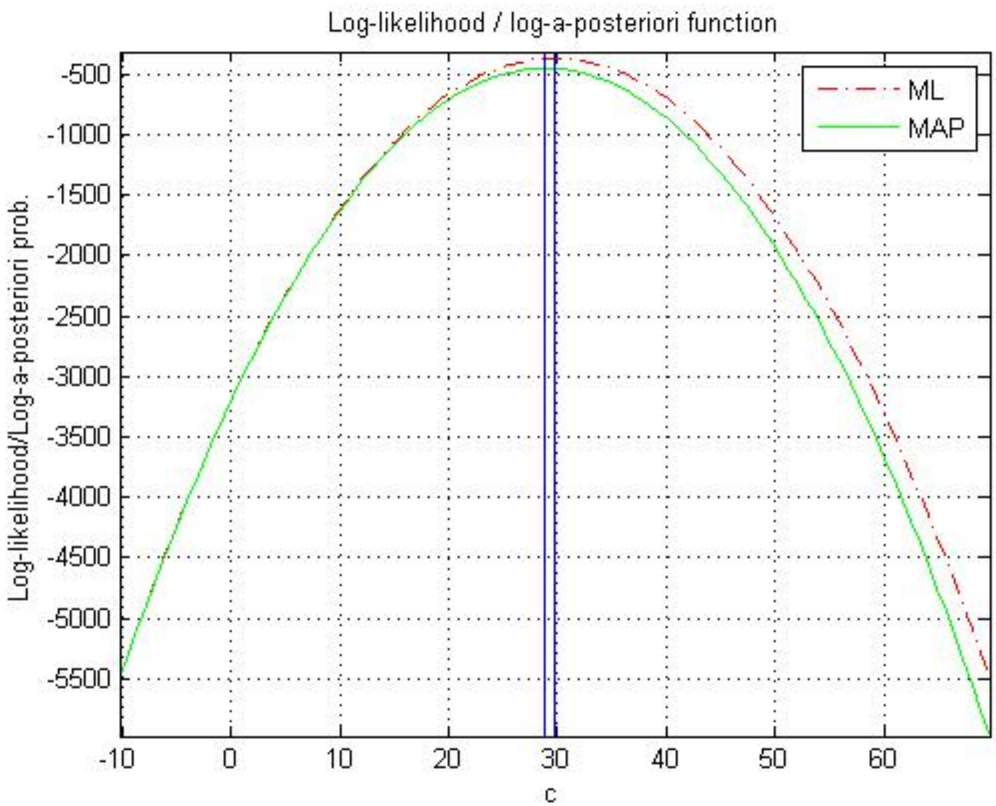
31.6026

31.1737

32.7232

27.2424

...



Exercise 8 - Bayes Learning

For each training set with n samples:

Estimate and plot log (likelihood), log (likelihood * prior):

2. n = 256

e =

35.2116

30.8420

23.6114

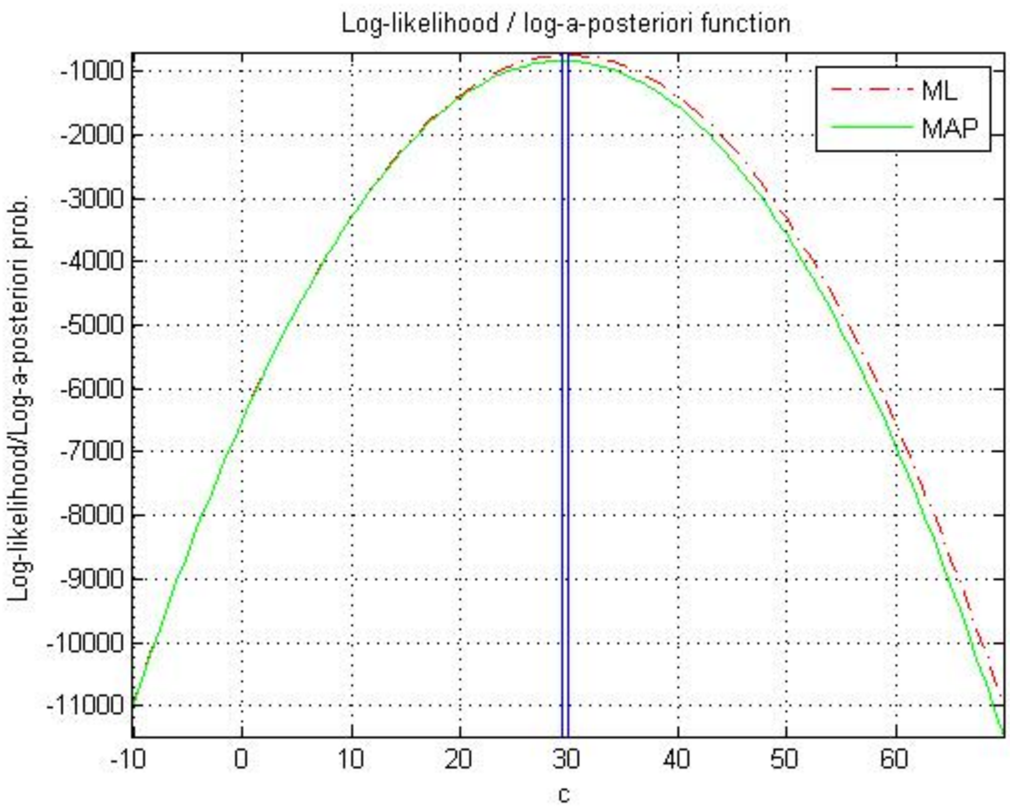
31.1572

23.1048

39.2989

33.3336

...



Exercise 8 - Bayes Learning

For each training set with n samples:

Estimate and plot log (likelihood), log (likelihood * prior):

2. n = 512

e =

27.0422

36.5392

18.6778

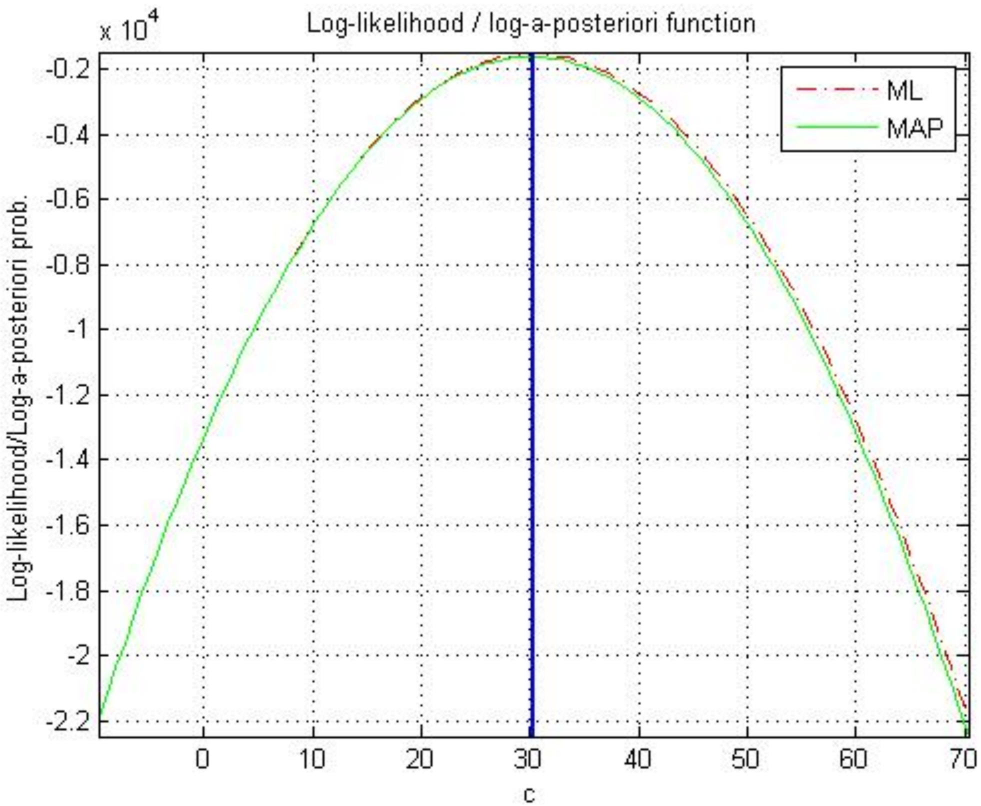
34.4218

37.4214

29.6516

29.5996

...



Exercise 8 - Bayes Learning

For each training set with n samples:

Estimate and plot log (likelihood), log (likelihood * prior):

2. n = 1024

e =

28.3212

30.5653

33.1773

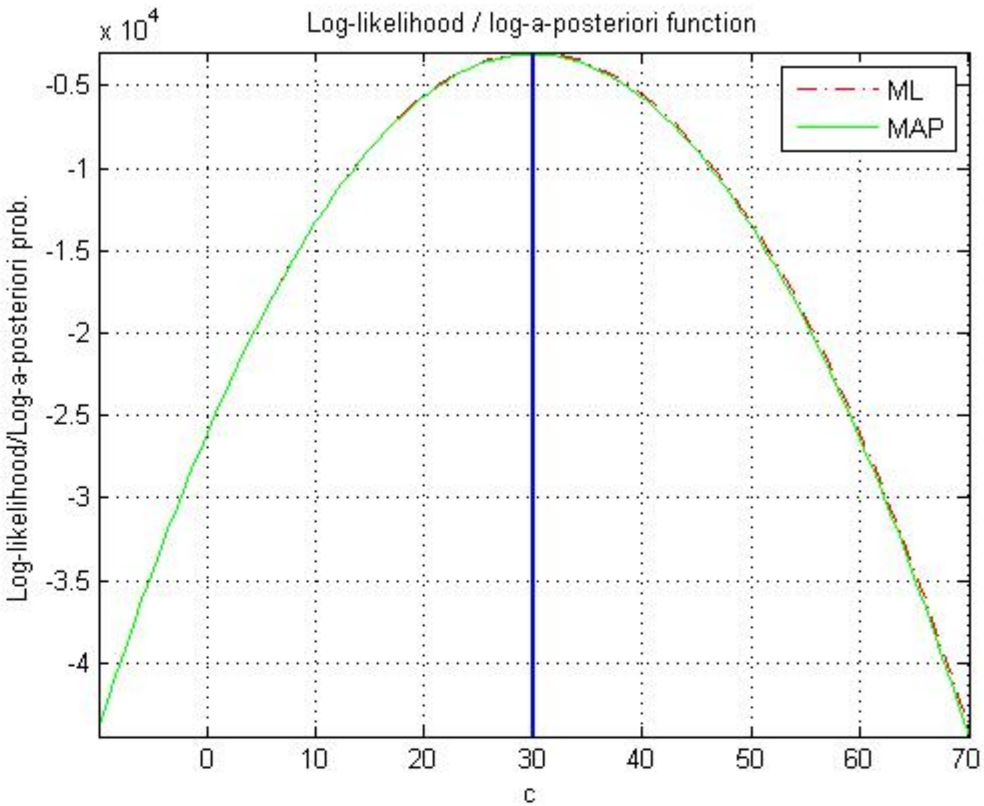
27.8407

21.0703

26.5033

26.0548

...



Exercise 8 - Bayes Learning

Result:

Both techniques have successfully estimated the "unknown" parameter c (which has been set to 30 for generation of the training data - see above).