Number Systems

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Positional Number Systems

Positional Number Systems

- **In positional number system, some number b is selected as base and symbols or digits are assigned to numbers between 0 and b-1**

- **The value of each digit in a number can be determined using –**
  - **The *digit***
  - **The *position of the digit* in the number**
  - **The *base* of the number system (where the base is defined as the *total number of digits* available in the number system)**

Decimal Number System

- **In decimal number system *base is 10* and the basic symbol or *digits are 0,1,2,3,4,5,6,7,8,9***
- **Each digit in the number is associated with a power of 10, according to its position in the number.**
- **Example: $3942 = 3x10^3 + 9x10^2 + 4x10^1 + 2x10^0$**

Binary Number System

- **The base is *2***
- **The digits are *0 and 1***
- **Example: $11010 = 1\times2^4 + 1\times2^3 + 0\times2^2 + 1\times2^1 + 0\times2^0$**

Hexadecimal Number System

- **The base is *16***
- **The digits are *0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F***
- **The hex letters *A through F* denote numbers *ten to fifteen,* respectively.**
- **Example: 1A = $1 \times 16^1 + 10 \times 16^0$**

Conversion between Number Systems

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Hexadecimal to Decimal

Convert $8A2D_h$ to decimal

$$
\begin{aligned}
8A2D_h &= 8\text{x}16^3 + A\text{x}16^2 + 2\text{x}16^1 + D\text{x}16^0 \\
&= 8\text{x}16^3 + 10\text{x}16^2 + 2\text{x}16^1 + 13\text{x}16^0 \\
&= 8\text{x}16^3 + 10\text{x}16^2 + 2\text{x}16^1 + 13\text{x}16^0 \\
&= 32768 + 2560 + 32 + 13 \\
&= 35373_d
\end{aligned}
$$

Binary to Decimal

Convert 1101_b to decimal

$$\begin{aligned} 1101_b &= 1\times2^3 + 1\times2^2 + 0\times2^1 + 1\times2^0 \\ &= 8 + 4 + 0 + 1 \\ &= 13_d \end{aligned}$$

Decimal to Hexadecimal

Convert 11172_d to hexadecimal

16 | 11172

16 | 698 — 4

16 | 43 — A (10)

16 | 2 — B (11)

0 — 2

$2BA4_h$

Decimal to Binary

Convert 95_d to binary

2	95	
2	47	1
2	23	1
2	11	1
2	5	1
2	2	1
2	1	0
	0	1

1011111_b

Hexadecimal to Binary

Convert $2B3C_h$ to binary

2	B	3	C
0010	1011	0011	1100

$2B3C_h = 10101100111100_b$

Binary to Hexadecimal

Convert 1110101010_b to hexadecimal

11 1010 1010

0011	1010	1010
3	A	A

$1110101010_b = 3AA_h$

Hexadecimal Addition and Subtraction

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Hexadecimal Addition

Add $5B39_h$ and $7AF4_h$.

- $9+4 = 13$. In Hex, we represent 13 as D_h.

$$\begin{array}{r} 5B39_h \\ 7AF4_h \\ \hline \boxed{D_h} \end{array}$$

- 3+F meaning $3+15=18$. In Hex, we represent 18_d as 12_h. So, we place 2 and have 1 as carry.

$$\begin{array}{r} {\scriptstyle 1} \\ 5B39_h \\ 7AF4_h \\ \hline \boxed{62D_h} \end{array}$$

- $1+B+A = 22$. In hex, we represent 22_d as 16_h. So, we place 6 and have 1 as carry.

$$\begin{array}{r} {\scriptstyle 1} \\ 5B39_h \\ 7AF4_h \\ \hline \boxed{62D_h} \end{array}$$

Hexadecimal Addition continued..

$$\begin{array}{r} 1 \\ 5B39_h \\ 7AF4_h \\ \hline \boxed{62D_h} \end{array}$$

- Now, 1+5+7=13 which is D_h is hex.

$$\begin{array}{r} 5B39_h \\ 7AF4_h \\ \hline \boxed{D62D_h} \end{array}$$

- Thus $5B39_h + 7AF4_h = D62D_h$

Hexadecimal Subtraction

Subtract $BA94_h$ from $D26F_h$.

Binary Addition and Subtraction

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Binary Addition

Add 1001_b and 1100_b .

How integers are represented in the computer?

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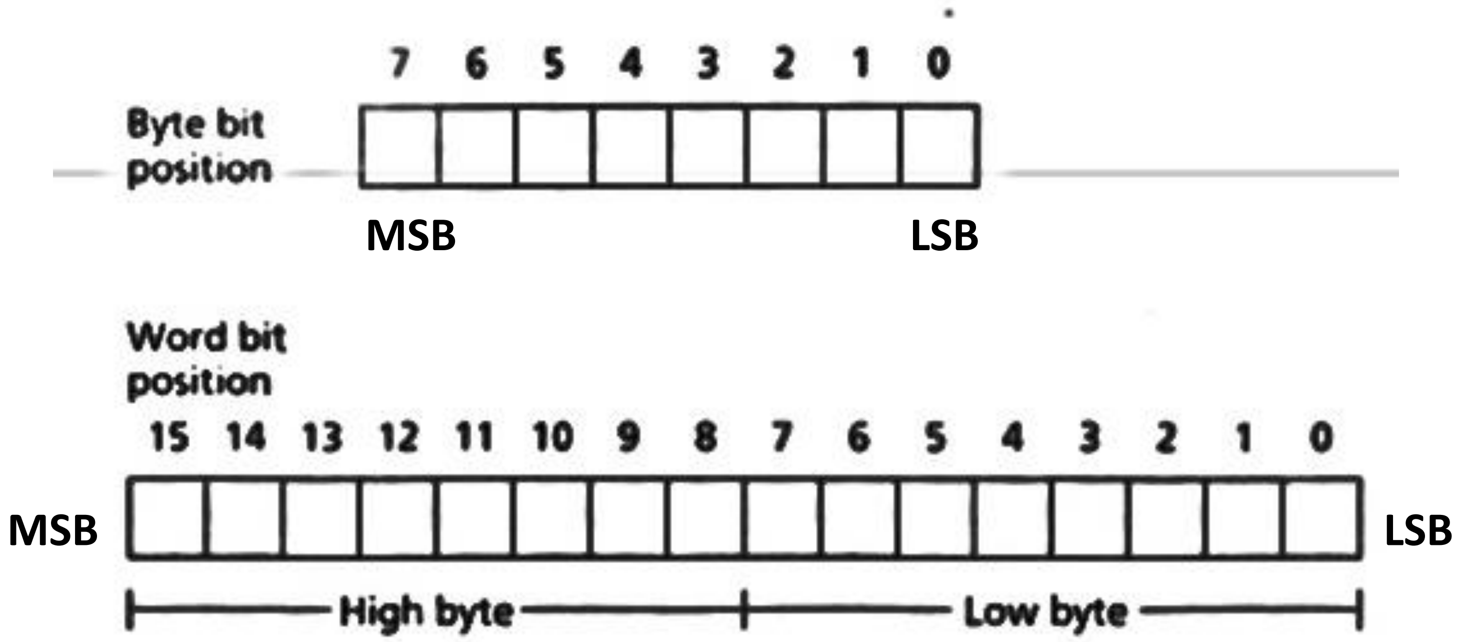
MSB and LSB

- **Most Significant Bit (*MSB*) is the *leftmost bit*.**

  In a word, the msb is bit 15; in a byte it is 7.

- **Least Significant Bit (*LSB*) is the *rightmost bit*. That is, bit 0.**



One’s Complement

The one's complement of an integer is obtained by *complementing each bit*; that is, replace each 0 by a 1 and each 1 by a 0.

Example 2.6 Find the one's complement of 5 = 0000000000000101.

Solution: **5 = 0000000000000101**

One's complement of 5 = 1111111111111010

Note that, if we add 5 and its one's complement, we get 1111111111111111.

Two’s Complement

To get the two's complement of an integer, just *add 1 to its one’s complement.*

Example 2.7 Find the two's complement of 5.

Solution:

One's complement of 5 = 1111111111111010

+ 1

Two’s complement of 5 = 1111111111111011 = $FFFB_h$

Note that, As 5 and it’s two’s complement add up to 0, the two's complement of 5 is the correct representation of -5.

Two’s complement of two’s complement of a number gives the number itself.

Unsigned Integers

- Represents a magnitude, so it is *never negative.*
- appropriate for representing quantities that can never be negative, such as *addresses of memory locations, counters, and ASCII character codes*
- none of the bits are needed to represent the sign, and so *all 8 bits in a byte, or 16 bits in a word, are available to represent the number.*
- The largest unsigned integer that can be stored in a byte is 11111111= FF_h = 255. This is not a very big number, so we usually *store integers in words.*
- The biggest unsigned integer a 16 bit word can hold is 1111111111111111 = $FFFF_h$ = 65535.
- The number is *odd if the LSB is 1.*
- The number is *even if the LSB is 0.*

Signed Integers

- **It can either be *positive or negative.***
- **Most significant bit (*MSB) is reserved for sign: 0 for positive and 1 for negative***
- **Negative integers are stored in a computer *as two's complement***

Signed Integers

Example 2.9: Show how the decimal integer -97 would be represented (a) in 8 bits, and tb) in 16 bits. Express the answers in hex.

Subtraction as Two's Complement Addition

The advantage of two's complement representation of negative integers in the computer is that subtraction can be done by bit complementation and addition, and circuits that add and complement bits are easy to design.

Example 2.10: Suppose AX contains $5ABC_h$ and BX contains $21FC_h$.

Find the difference of AX minus BX by using complementation and addition.

Decimal Interpretation

Unsigned Decimal Interpretation

Binary to decimal conversion

Signed Decimal Interpretation

- **If MSB is 0 then signed decimal is same as unsigned decimal**
- **If MSB is 1 take two's complement and convert it to decimal**